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PRINCIPLES & PRACTICE
OF
ELECTRICAL TESTING

AS APPLIED TO APPARATUS, CIRCUITS AND MACHINES

BY

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PREFACE

THIS book is written for technical students and electrical engineers, and its aim is to present in a single volume the principles and operations of the fundamental electrical laboratory tests, the testing of circuits, and the testing of electrical machines.

The results of actual tests are given in detail as a first guide to the handling and entry of such values, and also to enable the student who has had but limited access to apparatus or machines to study from them the nature and characteristics of the latter.

As many students like to test their knowledge of a subject by solving problems relating to it, some typical examples and their answers, worked out by the slide rule, are given throughout the text.

In the construction of the present treatise the author wishes to acknowledge his indebtedness to the many excellent books, already published, on electrical engineering and testing, which have been freely consulted during the preparation of the subject-matter of the text. Thanks are also due to Mr. T. Cooney of the Royal College of Science, Dublin, for his valuable assistance in the reading and correction of the proofs.

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The remaining finger will then indicate the direction of the electromotive force induced in the conductor or coil side as it cuts the lines of magnetic force.

EXAMPLE.—Verify by this rule that the induced voltage in the effective sides of the revolving coil shown in Fig. 1 will be as indicated by the arrows pointing from *a* to *b*, and from *d* to *c* when the coil rotates clockwise. *Dots* and *crosses* are also used to indicate directions; the dot representing the point of the arrow and the cross its tail.

Lenz's rule.—The induced current acts in such a direction that its magnetic action tends to stop the motion which produces it.

EXAMPLE.—A north pole approaches in a vertical direction a horizontal coil A (Fig. 2). Show by Lenz's rule that the direction of the induced current in the coil will be as indicated. A stationary coil C, by pressing the tap key K, is equivalent to the approach of a *south* pole, and releasing K, to the removal of this pole.

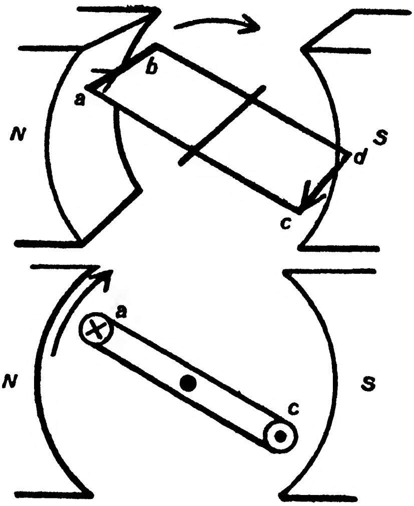

FIG. 1.—The directions of the induced voltages in a coil, rotating in a magnetic field.

The conservative principle.—The number of lines of magnetic force threading a coil or current tends to remain constant. Thus, if the lines of magnetic force in a circuit diminish, the induced current will

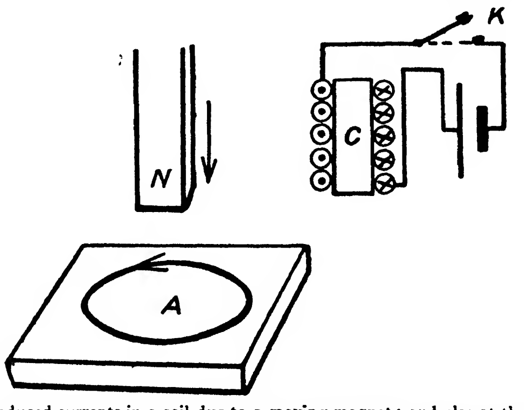

FIG. 2.—Induced currents in a coil due to a moving magnet; and also at the make and break of the circuit of a solenoid.

act so that its magnetic flux will be in the *same* direction as the decreasing flux. It will act in the opposite direction if the flux is increasing.

EXAMPLE.—Looking at one end of a coil, a current is seen to flow clockwise through it. If this current is reversed, show that the direction of the induced voltage throughout this reversal will also be clockwise, when viewed from the same end of the coil.

To find Q, the quantity of electricity produced in a coil or circuit of S turns and resistance R ohms by the complete threading of N lines of magnetic force through it.—Let N_1 be the number of lines of magnetic force threading the coil or circuit at time t_1 and N_2 at time t_2. Suppose $t_2 - t_1$ is very small. Then the quantity of electricity produced in the circuit during this small interval is, from the laws of *Faraday* and *Ohm*, expressed by

$$\frac{S}{R} \cdot \frac{N_2 - N_1}{t_2 - t_1} \cdot 10^{-8} \cdot (t_2 - t_1) \text{ coulombs};$$

that is by

$$\frac{S}{R}(N_2 - N_1)\, 10^{-8} \text{ coulombs.}$$

So for each small increment of lines to the circuit a quantity of electricity proportional to this increment is produced. Therefore, by threading the circuit with N lines of magnetic force,

$$Q = \frac{SN}{10^8 R} \text{ coulombs, } \qquad \text{(4)}$$

a relation which is much used in the theory of certain electrical experiments. As SN is the number of *magnetic linkages* produced in or removed from the circuit, the quantity is directly proportional to the number of magnetic linkages and inversely proportional to the resistance in the circuit.

This value of the quantity is independent of the time and manner of the threading, and is not affected by the lines of force due to the coil itself carrying current, as the total number of *these* lines during the operation sum to zero

EXAMPLE —A coil of 500 turns and resistance 20 ohms is joined in series with 80 ohms. The coil is then threaded by a magnetic flux of 4 kilolines. Find the quantity of electricity produced in the circuit by this threading. If the 80 ohms is shunted with 10 ohms, how much electricity will flow through the coil, and how much through the shunt, when the coil is threaded by the same flux as before? *Ans.* 200 microcoulombs; 692 and 615 microcoulombs.

Two fundamental principles of electromagnetism.—There are two important principles which are useful for solving some of the chief problems of electromagnetism. The first deals with the work done in threading a coil or circuit, carrying a current, with lines of magnetic force; and the second with the strength of the magnetic field at outside points due to a small element of a circuit carrying a current.

From elementary considerations the work done in threading a coil of S turns, carrying current I, by lines of magnetic force from a magnetic pole brought up to the coil from a distance, is proportional to $S \times I$, and also to the strength of the magnet pole.

If the coil or circuit resists the threading, mechanical work must be done to insert the lines of magnetic force, and this mechanical work will be transformed into electrical energy in the circuit. If the coil assists the threading, it can do mechanical work at the expense of some of its electrical energy. A north pole made to approach the north face of a coil is an illustration of the first case, and a south pole attracted by the north face of the coil an illustration of the second.

During this threading let n be the added lines from the external source at time t. Then e, the induced electromotive force at that instant, is given by

$$e = S\frac{dn}{dt}\,10^{-8} \text{ volts.}$$

The current in the circuit will be I′, where

$$I' = I\,.\,\frac{E+e}{E},$$

E being the steady electromotive force in the circuit.

The energy in the circuit for time dt will be

$$(E+e)\,I'dt \text{ joules,}$$

e acting in the same direction round the circuit as E, when mechanical work is done in threading against the opposition of the coil.

Now the energy contributed by the circuit is $EI'dt$ joules, so that the electrical energy contributed by the threading is $eI'dt$ joules, and this equals

$$e\,.\,I\,.\,\frac{E+e}{E}\,dt \text{ joules.}$$

By performing this threading with sufficient slowness e may be made negligible compared with E, and the electrical energy produced in the circuit by the threading during time dt is

$$eIdt\,.\,10^{8} \text{ ergs} = SIdn \text{ ergs,}$$

if I is in C.G.S. units.

For the whole threading, therefore, the electrical energy is

$$\Sigma SIdn = SIN \text{ ergs,}$$

which must also be the value of the mechanical energy from which it was transformed.

Thus, if a coil or circuit of S turns, carrying a current of I C.G.S. units, is completely threaded by N lines of magnetic force W, the work done or obtained by this threading is given by

$$W = SIN \text{ ergs.} \qquad \text{.............................(5)}$$

A conductor carrying an electric current is as a magnet, and by its field acts upon outside magnets or circuits. It would be expected that the strength of the magnetic field contributed by a small element of the conductor would be proportional to the current and the length of the element, and, for the same angle MAP (Fig. 3), be smaller the greater the length of AP.

The complete expression for the strength of the field due to a very small element AB of a circuit carrying a current i C.G.S. units is given by

$$H_{AB}=\frac{AB\,.\,i\,.\,\sin\theta}{r^2}\text{ lines per sq. cm.,}\qquad(6)$$

and its direction is at right angles to the plane PAB into the paper. Equation (6) is the second of these fundamental principles of electromagnetism.

Illustrations of the use of these principles are given in the following sections.

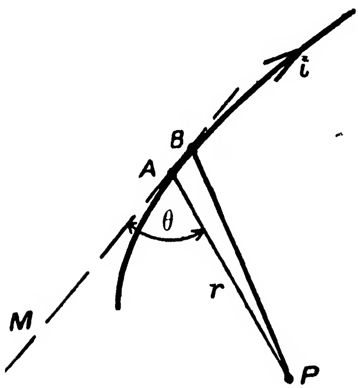

FIG. 3.—The magnetic action at P due to the elemental length AB of the circuit.

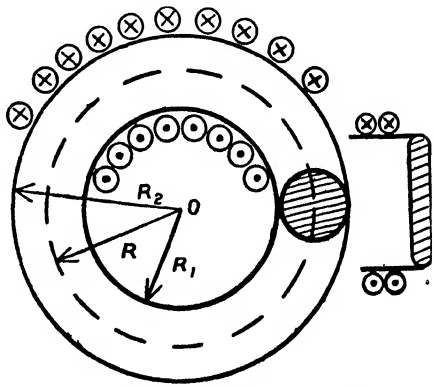

FIG. 4 —The magnetic field inside a circular solenoid.

To find H, the strength of the magnetising field inside a circular solenoid.—Let unit pole, from which 4π lines of magnetic force emanate, be carried completely round the outer path of radius R_2 (Fig 4). By this action each of the S turns of the solenoid has been threaded by 4π lines of magnetic force. Therefore, from the work principle, equation (5)

$$H_2 2\pi R_2=4\pi SI.$$

Similarly, by carrying unit pole round the inner path of radius R_1,

$$H_1 2\pi R_1=4\pi SI.$$

For the mean path or mean circumference,

$$H\,.\,2\pi R=4\pi SI,$$

or $$H=\frac{4\pi SI}{l},$$

l being the mean circumference of the solenoid.

Thus the field strength is greatest in the innermost layer, and is least in the outermost layer of the solenoid. In order to obtain a practically uniform field throughout the cross section of the spirals, the latter may be made of elongated shape as shown.

EXAMPLE.—The cross-sectional radius of the spirals of a circular solenoid is 3·5 cms., and the mean radius of the solenoid is 28 cms. It has 2000 turns or spirals, and carries 4 amperes. Obtain a graph relating H and the distance along any selected diameter of a spiral passing through O, the centre of the solenoid.

H inside a long, straight solenoid.—In the case of a solenoid of length very large compared with its cross-sectional dimensions, the

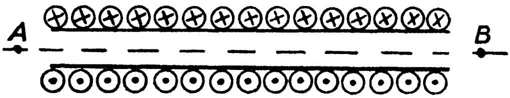

FIG. 5.—Magnetic field strength inside a long solenoid

strength of its magnetic field for outside points is practically zero. For the *same* flux of lines while concentrated within its cross section has a return cross section of infinite radius outside the solenoid. By moving unit pole from A to B (Fig. 5), the work done is Hl, and by this movement electrical energy has been produced in the circuit of the solenoid of value 4πSI. Therefore

$$H=\frac{4\pi SI}{l}. \quad \ldots\ldots\ldots\ldots(7)$$

The distance AB must be such that all the turns will have been threaded by 4π lines as the pole is moved from A to B.

For the case of the solenoid under consideration, the distances of A and B from the respective ends of it may be neglected in comparison with its length. Thus l in equation (7) may be regarded as the length of the solenoid.

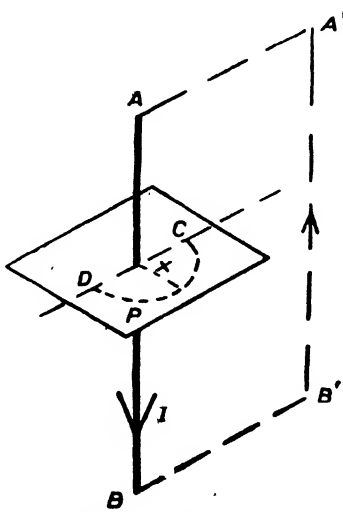

FIG. 6.—Magnetic field strength outside a long straight conductor.

The strength of the field outside a very long straight conductor carrying current.—In Fig. 6 is shown a conductor AB carrying a current of I C.G.S. units and A′B′ a very distant return conductor. Suppose unit pole is moved round the

semicircle DPC. Then the mechanical work $H \cdot \pi \cdot x$ done by this movement produces electrical energy in the circuit represented by $2\pi I$, since the rectangular coil has been threaded by half the number of lines from unit pole. Therefore

$$H = \frac{2I}{x}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The strength of the field inside a very long straight conductor carrying a current.—The cross section of the conductor is indicated in Fig. 7, and the current carried within the dotted circle of radius x is

$$I \cdot \frac{x^2}{R^2}.$$

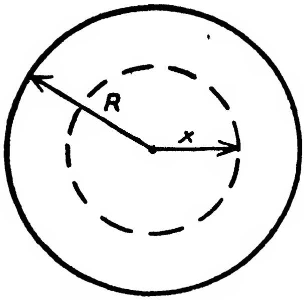

FIG. 7.—Magnetic field strength in the material of a long straight conductor.

By moving unit pole through a crevass represented by the dotted circle, mechanical work $H \cdot 2\pi x$ is done, and is transformed into electrical energy $4\pi I \frac{x^2}{R^2}$. Therefore

$$H = \frac{2I}{R^2} \cdot x. \quad \ldots\ldots\ldots\ldots (9)$$

EXAMPLE.—A solid copper conductor of great length and 1·4 cms. radius carries 1000 amperes. Obtain the graph of the strength of the magnetic field inside and outside the conductor for different distances from its axis.

The value of H inside the material of a tubular conductor of outer radius R_1 and inner R_2 is similarly found to be given by

$$H = \frac{2I}{x} \cdot \frac{x^2 - R_2^2}{R_1^2 - R_2^2}.$$

EXAMPLE.—A tubular conductor of great length has an external radius of 3 cms. and an internal one of 2 cms. It carries 1200 amperes. Obtain a graph of the strength of the magnetic field inside and outside the conductor for different distances from its axis.

The force exerted on a straight conductor carrying current in a magnetic field when placed at right angles to the direction of the latter.—AB (Fig. 8) is a conductor of length l, and A′B′ a distant *fixed* return wire. H is the strength of the magnetic field in which AB is placed. The direction of the field is from right to left and at right angles to the plane ABB′A′. Suppose AB receives a displacement x cms. parallel to itself and at right angles to the direction of the field. By this movement lxH lines of magnetic force are added to the circuit ABB′A. This produces electrical energy equal to lxHI ergs at the expense of mechanical work $F \cdot x$. Therefore

$$F = HIl \text{ dynes}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

I, the current, being in C.G.S. units.

The direction of this force acting on the conductor may be found from *Fleming's left-hand rule.* The thumb and next two fingers of the *left* hand are bent at right angles to each other. Then the thumb will indicate the direction of the force if the next finger indicates that of the field and the remaining finger that of the current.

EXAMPLE.—An armature conductor of length 20 cms., carrying 120 amperes, is situated in a magnetic field of strength 12 kilolines per sq. cm. Find in lbs. the force on the conductor, and indicate its direction for a chosen direction of current and field. *Ans.* 6·48 lbs.

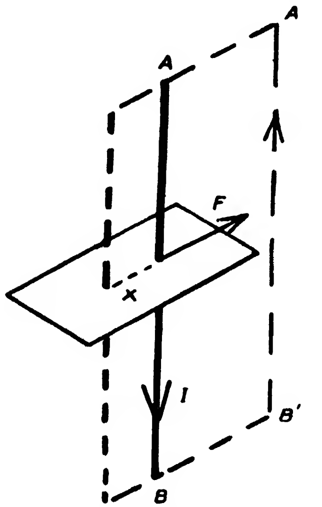

FIG. 8.—The force on a conductor, carrying current, when placed in a magnetic field.

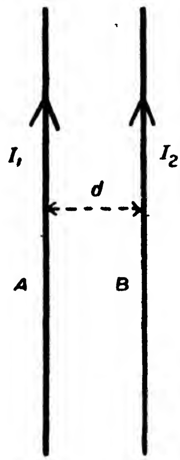

FIG. 9.—The force between two parallel conductors, each carrying a current.

The force between two long parallel conductors carrying currents, the distance between them being small in comparison with their lengths.—The strength of the magnetic field produced by conductor A (Fig. 9) at a distance d is $\frac{2I_1}{d}$. Conductor B, which carries I_2, is therefore in a field of strength $\frac{2I_1}{d}$, and from equation (10) is urged towards A with a force

$$\frac{2I_1}{d} \cdot I_2 \cdot l \text{ dynes.}$$

Thus
$$F = \frac{2I_1 I_2 l}{d}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

If the currents flow in opposite directions there will be repulsion.

When a wire is bent upon itself, that is, wound non-inductively,

the two parts will repel each other, because the directions of the current in the parts are opposite.

EXAMPLE.—Two parallel wires, 600 yards long, are placed 3 inches apart and each carries 20 amperes. What is the force of one wire upon the other? *Ans.* 0·129 lb.

The internal pressure inside a liquid conductor carrying a current.—The cross section of the liquid conductor is shown in Fig. 10, and it will be assumed that the distribution of current over it is uniform.

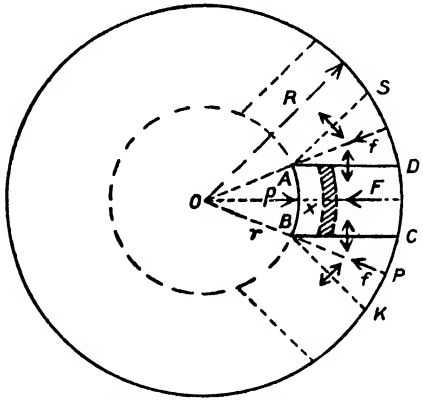

FIG. 10.—The internal pressure inside a liquid conductor carrying a current.

Consider the part ABCD, and suppose it has an axial length of one centimetre. Let p be the pressure* in dynes per sq. cm. over the surface represented by AB. Then, if AB is taken sufficiently small, the part ABCD is in equilibrium under the forces p . AB, the electromagnetic force F, and the ordinary fluid pressures along the surfaces AB, AD, DC, and CB.

Since the four latter balance,

$$p \, . \, \mathrm{AB} = \mathrm{F},$$

or,

$$p = \frac{\mathrm{F}}{\mathrm{AB}}.$$

Now the current flowing in the shaded element is equal to

$$\frac{\mathrm{AB}\, dx}{\pi \mathrm{R}^2} \, . \, \mathrm{I},$$

and the strength of the magnetic field in which the element is situated is, by equation (9), equal to

$$\frac{2\mathrm{I}x}{\mathrm{R}^2}.$$

Therefore, by equation (10), the internal force on it is

$$\frac{2\,\mathrm{I}^2\mathrm{AB}}{\pi \mathrm{R}^4} \, . \, x\, dx,$$

and the total force F on ABCD is given by

$$\mathrm{F} = \frac{2\,\mathrm{I}^2}{\pi \mathrm{R}^4} \, . \, \mathrm{AB} \int_r^{\mathrm{R}} x \, . \, dx$$

$$= \frac{\mathrm{I}^2}{\pi \mathrm{R}^4} \, . \, (\mathrm{R}^2 - r^2) \, . \, \mathrm{AB}.$$

* Due to the electric current.

The pressure p at a distance r cms. from the axis is therefore given by

$$p=\frac{I^2}{\pi R^4}\cdot(R^2-r^2) \text{ dynes per sq. cm.}$$

It will be noted that the electromagnetic force f on the wedge CBK contributes nothing to the pressure on the surface of AB, as it is balanced by the resolved parts along OP of the normal forces acting upon the faces BC and BK of the wedge. This is similarly true for the wedge ASD.

This internal pressure has no appreciable effect in solids, but in liquids the effect is quite measurable, and has been utilized in the construction of a liquid ampere meter, which may be used for either direct or alternating currents.

Example.—A liquid column has a cross section of diameter 1·0 centimetre. Find the internal pressure produced by the passage of 8 amperes through the liquid at a point 4 millimetres from the axis. What is the value of the maximum pressure? *Ans.* 0·293 and 0·815 dyne per sq. cm.

Other illustrations of the work principle of equation (5) will appear later on; the following sections illustrate the use of the *second principle* represented by equation (6).

The strength of the magnetic field along the axis of a circular coil carrying a current.—Let S be the number of turns on the coil and a

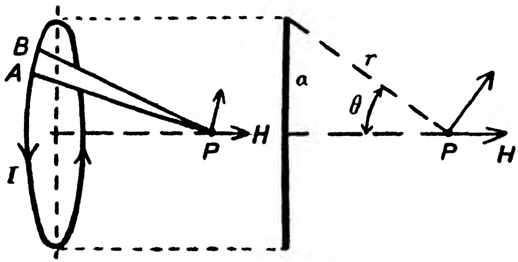

Fig. 11.—The strength of the magnetic field along the axis of a circular coil.

the mean radius of the turns. The coil is indicated in Fig. 11, and the slant distance of P from the mean circle of the turns is shown of length r.

Using equation (6) for each little element AB of the coil and summing up their magnetic strengths along the axis at P,

$$H=\Sigma\frac{AB.I.S.}{r^2}\sin\theta,$$

$$H=\frac{2\pi SIa^2}{r^3}, \qquad (12)$$

in which I is the current in C.G.S units and H is the strength of the magnetic field at P.

When $r=a$, $\qquad H=\dfrac{2\pi SI}{a}.$

If an external horizontal magnetic field of strength H_x acts at

right angles to the axis of the circular coil when its plane is vertical, a small horizontally suspended magnet on the axis will be deflected through an angle θ, such that

$$H_x \tan\theta = \frac{2\pi S I a^2}{r^3},$$

which is the law of galvanometers and ammeters of the tangent type. This equation also enables one to find the strength H_x of an unknown field.

EXAMPLE.—A small magnet is suspended as described at a point P on the axis of a circular coil in a certain horizontal magnetic field whose direction is at right angles to the axis. It is found that when the mean plane of the coil is 15 cms. from the magnet, and carries 2 amperes, the latter is deflected through an angle of 30 degrees. The coil has 50 turns and mean radius 20 cms. Calculate the strength of the magnetic field at P before and after the current was switched on. *Ans.* 2·78 and 3·21 C.G.S. units.

The strength of the magnetic field outside a straight conductor carrying current.—The strength of the field contributed by the small element dy (Fig. 12) at Q is

$$\frac{dy \,.\, I\cos\theta}{r^2},$$

and

$$y = a\tan\theta, \quad dy = \frac{a}{\cos^2\theta}\,d\theta, \quad r = \frac{a}{\cos\theta}.$$

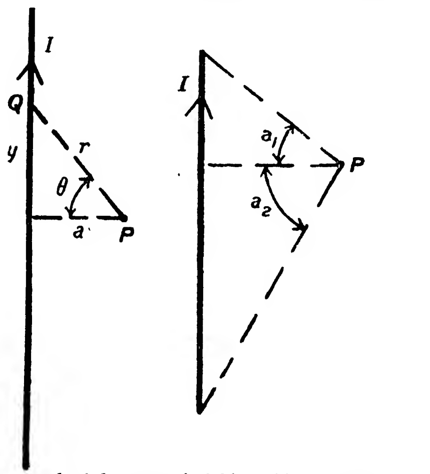

FIG. 12.—The strength of the magnetic field outside a straight conductor.

When the lengths of the conductor above and below P are very large compared with a,

$$H = 2\int_0^{\frac{\pi}{2}} \frac{I\cos\theta}{r^2}\,dy = \frac{2I}{a}\int_0^{\frac{\pi}{2}} \cos\theta \,.\, d\theta\,; \qquad \therefore\ H = \frac{2I}{a},$$

as in equation (8).

When the conductor has a limited length or one not large in comparison with a, as shown in Fig. 12, the strength of the field at P is found by the same method, and its value is given by

$$H=\frac{I}{a}\{\sin\alpha_1+\sin\alpha_2\}. \qquad (13)$$

EXAMPLE.—A square coil of 4 turns carries 12 amperes. Each side is 40 cms. in length. Obtain a graph relating strengths of the magnetic field at different points along a diagonal of the coil and their positions measured from one end of the diagonal.

EXAMPLE.—Show that the work done in carrying unit pole round any path which completely encloses a long straight conductor carrying a current I C.G.S. units is equal to $4\pi I$ ergs.

The strength of the magnetic field inside and outside a solenoid.—The solenoid may be considered as built up of a series of circular coils of which ACDB (Fig. 13) is one. The number of turns on AB is $\frac{S}{l}\cdot AB$,

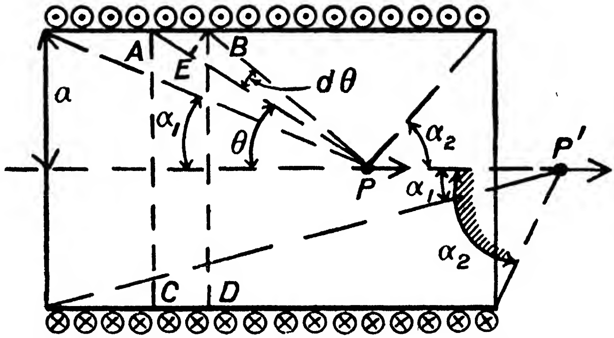

FIG. 13.—The strengths of the magnetic fields inside and outside a solenoid.

in which l is the length of the solenoid and S the total number of turns. The strength of the field at P due to this elemental coil is, by equation (12),

$$2\pi\frac{S}{l}AB\frac{a^2}{r^3}I.$$

Now $\qquad AB=\frac{BE}{\sin\theta}=\frac{r\,d\theta}{\sin\theta}$ and $r=\frac{a}{\sin\theta}$.

Therefore $\qquad H=\frac{2\pi SI}{l}\int_{\alpha_1}^{\pi-\alpha_2}\sin\theta\,d\theta,$

$$H=\frac{2\pi SI}{l}\{\cos\alpha_1+\cos\alpha_2\}, \qquad (14)$$

which is true for long and short solenoids. For long solenoids this value is very nearly constant over the cross section containing the axial point P, but this is not the case for short solenoids.

Similarly for axial points, such as P′, outside the solenoid,

$$H=\frac{2\pi SI}{l}\{\cos\alpha_1-\cos\alpha_2\}.$$

When $\frac{l}{a}$ is very large,

$$H=\frac{4\pi SI}{l}$$

for points inside the solenoid, except close to the ends, at which it becomes *half* this value.

The value of H for outside points drops off rapidly as the distance from the ends increase, and soon becomes practically zero.

EXAMPLE.—A solenoid has 2000 turns, length 40 cms., mean radius of the turns 10 cms., and current 8 amperes.

Obtain a graph relating strength of field at points along the axis of the solenoid and their distances from a suitable axial point well outside the solenoid.

EXAMPLE.—A solenoid is 400 cms. long, and the mean radius of its turns is 2 cms. Find the distance from one end of the solenoid at which the axial field inside it is only about one per cent. lower in strength than the field at the middle point of the solenoid. Also find the distance from the same end at which the field outside is only one per cent. of the strength of the field in the middle of the solenoid. *Ans.* About 10 cms. inside and 10 cms. outside the solenoid.

The work done in carrying unit pole through a solenoid in which $\frac{l}{a}$ is very great is Hl, which is, from the preceding, equal to

$$4\pi SI,$$

and is called the *magnetomotive force* of the excited solenoid. This result is in agreement with the principle of equation (5); for, by carrying unit pole through the solenoid, each turn has been threaded by 4π lines of magnetic force. Thus the two principles indicated by the equations (5) and (6) are closely related; the results from using (6) verifying those obtained from (5). Another illustration of this is as follows.

The work done in bringing unit pole from an infinite distance to the centre of a flat circular coil carrying a current.—The work done according to the method of equation (5) will be 2πIS ergs. In this case half the lines from unit pole will thread the coil in the final position. By the second principle, from which equation (12) was derived, the work done in moving unit pole along the axis of the coil from an infinite distance is

$$\int\frac{2\pi SIa^2\,dx}{r^3},$$

and $$x=a\cot\theta,\quad dx=\frac{-a}{\sin^2\theta}d\theta,\quad r=\frac{a}{\sin\theta},$$

so the work done is

$$2\pi SI\int_{\frac{\pi}{2}}^{0} -\sin\theta\, d\theta = 2\pi SI\Big[\cos\theta\Big]_{\frac{\pi}{2}}^{0}$$
$$= 2\pi SI \text{ ergs},$$

which is the same as before.

If the unit pole was moved from infinity on one side to infinity on the other side, the work done would have been $4\pi SI$, as all the lines emanating from unit pole would have threaded the coil.

The work done is independent of the path by which the unit pole is carried from an infinite distance to the centre of the coil. For it is evident that the work done on the coil, in bringing unit pole from an infinite distance along the axis to the coil, is the same as the work done by the coil in driving it back along the same path, that is, the process is reversible. It is reversible also for any other given path.

If the work done on the coil along another path is greater than that along the axial path, then, by bringing the pole up along the axis and letting it return by this other path, a certain amount of work is *gained*; that is, the efficiency of the process is greater than unity, which is impossible.

Magnetic reluctance, flux density, and permeability. A cylinder of length one cm. and cross section one sq. cm. of non-magnetic material offers *unit magnetic reluctance* to the passage of lines of magnetic force through it. If the material is magnetic, especially if soft iron, the reluctance is much below unity; it offers much less resistance to the passage of lines of magnetic force through it.

A unit cylinder, if non-magnetic, when placed in a magnetic field of strength H, has H lines passing through it when parallel to the direction of the field; while, if a like cylinder of iron is substituted for the other, a great many more lines, called *lines of magnetic induction*, in addition to the H lines will pass through the iron.

This total number B crossing one sq. cm. of the magnetic material is called the *flux density* in the material produced by the magnetising field H. The reluctance of this material is given by $\frac{H}{B}$, and $\frac{B}{H}$ is termed the *magnetic permeability* for the magnetising field H and denoted by μ. As μ depends on the value of H, it will be more representative to use suffixes, thus:

$$\mu_H H = \frac{B_H}{H},$$

and the reluctance of unit cylinder of a magnetic material is

$$\frac{1}{\mu_H}.$$

The reluctance or reluctivity R of a cylinder of magnetic material, whose length is l cms. and cross-sectional area a sq. cms., is that of a cylinders in parallel, each of cross section one sq. cm. and of reluctance

$$\frac{l}{\mu_H}.$$

So that
$$R=\frac{l}{a\mu_H}.$$

A long solenoid has a magnetising field of

$$H=\frac{4\pi SI}{l},$$

and a rod of magnetic material placed inside it adds another system of lines, those of magnetic induction, which run in the same direction as the H lines, but are much more numerous.

EXAMPLE.—From the following results of a test on a sample of iron calculate the permeability and reluctivity of the iron, and graph these values against the corresponding values of H.

H.	B.	H.	B.
2	2000	20	14000
3	4000	30	15000
4	6000	60	17000
5	8000	100	18000
7	10200	140	18700
10	11700	200	19700
14	13000		

The law of magnetic circuits.—This relates magnetic flux, reluctivity, and the magnetomotive force of the circuit. It is expressed by

$$\phi=\frac{\text{M.M.F.}}{R}. \quad \text{.............................}(15)$$

The magnetomotive force, M.M.F., is the work done in carrying unit pole through the complete circuit of the magnetising field, and is equal to

$$\frac{4\pi SI}{10}$$

when I is in amperes.

R, the reluctivity of the circuit, is the sum of all the reluctivities of the different parts which make up the circuit.

ϕ is the total magnetic flux which is driven through this circuit by the ampere turns used. Thus

$$\phi=\frac{\frac{4\pi}{10}SI}{\Sigma R}. \quad \text{..............................}(16)$$

This law is analogous to Ohm's law in current electricity.

It is easy to show that equation (16) is true for the simple case of a circular solenoid filled with an iron core, for

$$\phi=\mu Ha \quad \text{and} \quad H=\frac{4\pi SI}{10l},$$

a being the cross-sectional area of the core, and *l* its mean length. So that

$$\phi = \frac{4\pi SI}{10} \bigg/ \frac{l}{\mu a} = \frac{\frac{4\pi SI}{10}}{R}.$$

In a great many important practical magnetic circuits the reluctance of an air gap constitutes the chief reluctance, and is very large compared with the reluctance of the iron part of the circuit.

EXAMPLE.—A circular core of round iron, 7 cms. in diameter, has a mean radius of 21 cms., and a gap of width 4 cms. is made in this ring. Compare the reluctances of the iron and air parts of the magnetic circuit, taking $\mu = 1000$ for the iron. Also calculate the ampere turns necessary to drive a flux density of 15 kilolines per sq. cm. through the iron core with and without the air gap. *Ans.* The reluctivities for the iron and air gap are respectively 0·00332 and 0·104 units; 49,200 and 1580 ampere turns.

In using equation (16) for finding the ampere turns required to drive a certain flux ϕ through a given magnetic circuit, it is necessary to have the $B - \mu$ curve of each material used in the circuit.

The flux density B in each part may be obtained by dividing ϕ by the cross-sectional area of the part, and the permeability then found from the curve. The reluctance may now be calculated and the ampere turns obtained from equation (16).

A better method of finding the ampere turns is to graph curves relating B and σ, which may be derived from the $B - H$ curves of the materials used; σ being the ampere turns necessary to drive a given flux density through one linear cm. of the material. Then, if the length and flux density is known for one part of the circuit, the ampere turns required by this part may be obtained by multiplying the value of σ corresponding to the flux density by the length of the part. The sum for all the parts gives the total ampere turns required.

σ is derived from H in the following way. Consider a solenoid full of iron. Its magnetising field $H = \frac{4\pi SI}{10l}$ produces in this core a certain flux density throughout the length *l*, and the ampere turns needed to do this are

$$\frac{10Hl}{4\pi}.$$

So that the number of ampere turns necessary to drive this flux density through one centimetre of the iron is

$$\frac{10}{4\pi} H = 0{\cdot}796H \doteqdot 0{\cdot}8H.$$

Thus, from the $B - H$ curve, by putting 0·796H instead of H, the $B - \sigma$ curve may be obtained.

In the case of air or non-magnetic material a curve is not necessary, for σ is given by 0·796 times the flux density passing through it.

EXAMPLE.—Find the ampere turns required by a magnetic circuit to produce through it a magnetic flux of 2 megalines. The circuit consists of 60 cms. of cast iron of cross section 480 sq. cms., 30 cms. of mild steel of cross section 240 sq. cms., and two air gaps, each 0·4 cm. in length of mean cross section of 260 sq. cms. Also give the ratio of the ampere turns needed for the air gaps to the total number required. The following values relating B and σ may be used.

Mild steel.

B	2000	4000	6000	8000	10000
σ	1·6	2·4	3·2	4·0	5·6

Cast iron.

B	1000	2000	3000	4000	5000
σ	1·5	3·2	4·9	7·5	11·0

Ans. 5500 ampere turns ; the ratio is 0·89.

The attraction of magnetised surfaces.—When two surfaces of iron in contact are crossed by a magnetic flux, an attractive force which increases rapidly with the flux density is exerted between them. This force diminishes greatly if the surfaces are separated by even a very small air gap, provided it is the only one in the magnetic circuit through which the flux passes. A strip of paper between the armature and pole faces of a horse-shoe magnet causes the attraction to fall to a value only a small fraction of that without the paper.

An approximately uniform pull can be made for a very short distance by nearly saturating the magnetic circuit when the armature is at its maximum separation ; then for shorter distances the same or a smaller current will produce saturation. Uniformity of pull through a short distance is best obtained by using a specially shaped electromagnet.

The attractive force of such surfaces will depend upon the number of exciting ampere turns, the magnetic quality of the materials, the size and shape of the magnetic circuit used, and the shape and size of the attractive surfaces.

The attractive force between two plane magnetised surfaces may be determined as follows.

First, **to find the strength of the axial field outside the end of a very long magnet of circular cross section.**

Let m be the strength of each sq. cm. of the pole face, h the strength of the axial field at O (Fig. 14) due to elemental ring of radius y, H the total strength of field at O, and a the distance of O from the magnetised surface. Then,

$$y=a\tan\theta,\quad dy=\frac{a}{\cos^2\theta}\,d\theta,\quad r=\frac{a}{\cos\theta}.$$

$$h = \frac{2\pi y \,.\, dy \,.\, m}{r^2} \,.\, \cos\theta.$$

$$H = 2\pi m \int_0^a \sin\theta \, d\theta = 2\pi m \Big[-\cos\theta \Big]_0^a .$$

$$H = 2\pi m\,(1 - \cos a).$$

When a is very small compared with the radius of the pole face,

$$H = 2\pi m,$$

and this value is independent of the radius of the pole face, so that for points very near to the magnetised surface,. the normal field is

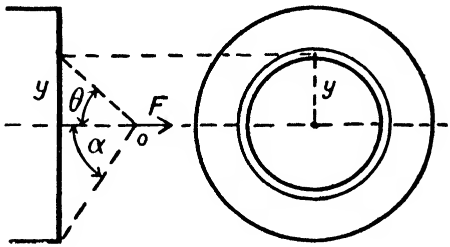

FIG. 14 —The strength of the magnetic field at a point on the axis of the magnet due to the magnetism of the pole face.

produced by the part of the surface immediately in the neighbourhood of the point.

Hence, whatever the contour of the magnetised surface, the strength of the normal field close to it is of value $2\pi m$, excepting near the contour or edges of the surface.

The force of attraction between two parallel plane surfaces close together when oppositely and uniformly charged with m units of magnetism per sq. cm. may now be found. The strength of the field between them is $4\pi m$, each surface contributing $2\pi m$ lines per sq. cm., and excepting close to the edges the field is normal to the surfaces.

FIG. 15.—Contact surfaces across which a magnetic flux is supposed to pass.

One of these surfaces will attract the magnetic charge m on *each* sq. cm. of the other surface with a force equal to

$$2\pi m \times m = 2\pi m^2 \text{ dynes.}$$

If the area of the smaller surface (Fig. 15) is A sq. cms., the total pull between the surfaces will be given by

$$P = 2\pi m^2 A \text{ dynes.}$$

When B is the flux density across these surfaces produced by a magnetising field of H, then

$$B - H = 4\pi m.$$

Practically, in every case, H is very small in comparison with B, and may be neglected, and the tractive force or pull is given by

$$P = \frac{B^2}{8\pi} A \text{ dynes},$$

or

$$P = \frac{\phi B}{8\pi} \text{ dynes},$$

where ϕ is the total flux cutting across the surfaces.

If ϕ is in megalines and B in kilolines per sq. inch,

$$P = 13{\cdot}9\phi B \text{ lbs. wt.}$$

Maximum pull for a given electromagnet.—A given electromagnet can only lift or pull up to a certain amount, because after a certain current has been reached the iron becomes saturated. It is not economical to use B greater than about 21 kilolines per sq. cm. or 135 kilolines per sq. inch, because beyond this the increment of pull gained is very small for the relatively large output of electrical energy required to produce it.

At the flux density given, good iron is fairly saturated, and the maximum pull corresponding to it is about *one ton* for every *nine square inches* of attracting surface. By inspection. therefore, the lifting power of a given magnet may be gauged, provided its exciting coils can carry the current necessary to fairly saturate the core.

EXAMPLE.—The magnet poles of electrical machines produce a considerable pull on the armature core. Calculate this pull per pole in tons for a machine in which the pole face is 45 cms. by 60 cms., and the average flux density between the pole face and the armature core is 8000 lines per sq. cm. The armature core may be regarded as smooth. *Ans.* 6·9 tons nearly.

CHAPTER II.

GENERAL PRINCIPLES—Continued.

Self and mutual inductance. The quantities representing self induction of a coil or circuit, and mutual induction of two or more coils or circuits, are functions of magnetic flux and number of turns. The product of *one line* of magnetic force and *one turn* is termed a *magnetic linkage.*

The magnetic flux produced in a coil by the passage of current I through it is proportional to I, and for the same frame is nearly proportional to S, the number of turns. In some coils, such as a circular or very long straight solenoid, the flux is proportional to S. The flux also depends upon the cross-sectional area and length of the coil. For the same frame, the number of magnetic linkages is nearly proportional to S^2I.

The total number of magnetic linkages due to a coil carrying current is generally difficult to calculate, as different turns are threaded by different values of flux.

The number of magnetic linkages threading a coil of S_1 turns, produced by a second coil of S_2 turns carrying a current I, will be for the same coil frames and positions nearly proportional to S_1S_2I.

The *coefficient of self induction*, or the self inductance of a coil or circuit, is the number of magnetic linkages produced in it by the passage of one C.G.S. unit of current or 10 amperes through the coil or circuit.

The *coefficient of mutual induction*, or the mutual inductance of two coils or circuits, is the number of magnetic linkages produced in *either* coil by the passage of one C.G.S. unit of current through the *other* coil or circuit.

A coil which has 10^9 magnetic linkages produced in it by the passage of one C.G.S. unit of current through it, or 10^8 linkages for one ampere, is said to have a self inductance of *one henry*, which is the practical unit of magnetic inductance.

Likewise two coils, one having 10^9 linkages produced in it by 10 amperes in the other, will have a mutual inductance of one henry.

EXAMPLES.—1. A coil of 2000 turns has each of these turns threaded by a flux of 8000 lines of magnetic force due to a current of 4 amperes flowing through it. Calculate its self inductance. *Ans.* 0·04 henry.

2. One of two coils has 800 turns, and the other coil when carrying 16 amperes produces through all these turns a flux of 4000 lines of magnetic force. Calculate the mutual inductance of the coils. *Ans.* 0·002 henry.

3. Two co-axial parallel plane coils A and B have a mutual inductance of 0·04 henry, and their respective self inductances are 0·08 and 0·05 henry. A steady current of 5 amperes flows through A and one of 8 amperes through B. A has 40,000 and B 20,000 turns. Calculate the magnetic flux threading A, and that threading B, first when the two currents are in like directions, and then when in opposite directions. *Ans.* 1800 and 3000; 200 and 1000.

Approximate values of the self inductances of certain coils may be calculated from their dimensions and number of turns. The simplest case is that of a very long solenoid of small cross section, or a circular solenoid whose spirals are of small cross section or elongated in shape.

Thus, let a be the mean cross-sectional area of the S turns on the solenoid and l its length; then the flux threading the turns is

$$\frac{4\pi S a}{l},$$

when one C.G.S. unit of current flows in the solenoid. Therefore

$$L=\frac{4\pi S^2 a 10^{-9}}{l} \text{ henrys.}$$

If the solenoid is full of a metal of constant magnetic permeability μ, the value of L just given must be multiplied by μ. a may be taken as the cross-sectional area of the core if μ is large.

EXAMPLE.—A ring solenoid is wound with two separate coils, one of self inductance L_1 and the other L_2. If M is the mutual inductance of the two coils, show that

$$L_1 L_2 - M^2 = 0.$$

The self inductance of a rectangular coil may be found by using equation (13), Chap. I., and integrating over the inside area, the radius of the conductor being one of the integration limits. The field inside the material of the conductors or sides of the coil should be rightly included. A simpler practical case is to find the self inductance per mile of two long parallel conductors, each of radius r inches and d inches apart from axis to axis; one conductor being a return for the other.

Using equation (8), page 8, the total number of lines of magnetic force threading the space between the conductors *per linear cm.* of the lengths of the conductors for one C.G.S. unit of current passing through them is

$$2\int_r^{d-r} \frac{2}{x}\,dx = 4\log_e \frac{d-r}{r},$$

whether d and r are in inch or cm. measure.

In addition there is the flux inside the material of the conductor, and its strength is directly as the distance from the axis. Now all this flux does not encircle the whole conductor, but in effect half of it does. The total flux threading per linear cm. of the conductor is, by equation (9), page 8, equal to

$$\int_0^r \frac{2x\,dx}{r^2} = 1.$$

If the material has a magnetic permeability μ, the flux inside the conductor per linear cm. will be μ. Taking half of this for each of the two conductors, the total number of magnetic linkages threading the *circuit* per linear cm. along both conductors is

$$4 \log_e\left(\frac{d-r}{r}\right) + \mu.$$

For the double length l cms., the self inductance is

$$4l\left\{\log_e\left(\frac{d-r}{r}\right) + \frac{\mu}{4}\right\} 10^{-9} \text{ henry}.$$

$$\text{L (per mile)} = 0\cdot644\left\{\log_e\left(\frac{d-r}{r}\right) + \frac{\mu}{4}\right\} \text{ millihenrys.}$$

Example.—An overhead outgoing copper conductor of diameter 0·5 cm. is 20 miles long, and the return conductor is parallel to the former and 50 cms. from it. Calculate the self inductance of the two conductors. *Ans.* 71·7 millihenrys.

It is sometimes of practical importance to calculate the self inductance of a solenoid from its number of turns and dimensions. This may be done by using the following formula of *Louis Cohen.**

Let r_1 be the *mean* radius of the first layer of the solenoid, r_2 that of the second, and so on. Also, let r_m be the mean radius of the turns, S the *total* number of turns, l the length, n the number of layers, and d_1 the distance between consecutive layers of the solenoid. Then, if all the lengths are in centimetres, and

$$A = n\left\{\frac{2r_m^4 + r_m^2 l^2}{\sqrt{4r_m^2 + l^2}} - \frac{8}{3\pi} r_m^3\right\},$$

$$B = [(n-1)r_1^2 + (n-2)r_2^2 + (n-3)r_3^2 + \text{etc.}],$$

$$C = \sqrt{r_1^2 + l^2} - \tfrac{7}{8} r_1,$$

$$D = [n(n-1)r_1^2 + (n-1)(n-2)r_2^2 + (n-2)(n-3)r_3^2 + \text{etc.}],$$

$$E = \frac{r_1 d_1}{\sqrt{r_1^2 + l^2}} - d_1,$$

$$F = \frac{d_1}{8}[n(n-1)r_1^2 + (n-2)(n-3)r_2^2 + \text{etc.}],$$

* *American Bureau of Standards*, 1907-1908, vol. 4, p. 389.

Cohen's formula is,

$$L = \frac{4\pi^2 S^2}{n^2 l^2} \{A + 2BC + DE - F\} 10^{-9} \text{ henrys.}$$

This value is true to one-half per cent. for values of l as low as *twice* the diameter d of the solenoid, and becomes more and more accurate as l/d increases. When this ratio becomes greater than 4, the last two terms DE and F are comparatively small, and may be neglected unless greater accuracy is desired.

A quick method for calculating an approximate value of L for solenoids in which l/d is greater than about 10 is, by *Maxwell's formula*,

$$L = \frac{4}{3} \frac{\pi^2}{n^4} \frac{S^4}{l^3} (r_2 - r_1)(r_2^3 - r_1^3) 10^{-9} \text{ henrys,}$$

S being the total number of turns, n the number of layers, l the length, r_2 the outer radius, and r_1 the inner radius of the solenoid.

EXAMPLES.—1. Calculate from Cohen's formula the self inductance of a solenoid having 400 turns, length 20 cms., outside diameter of the insulated conductor 0·2 cm., and internal diameter of the solenoid turns 20 cms. There is no insulation wrapping between the layers. *Ans.* 0·0219 henry.

2. Calculate from *Cohen's* formula, then from *Maxwell's* formula, the self inductance of a solenoid having 2 layers, length 100 cms., outside diameter of the insulated conductor 0·4 cm., and internal diameter of the solenoid turns 10 cms. *Ans* $2{\cdot}65 \times 10^{-3}$ and $2{\cdot}88 \times 10^{-3}$ henry.

The induced voltage due to self inductance. If i amperes is flowing in a circuit of self inductance L henrys at time t, the number of magnetic linkages produced in the circuit is $L10^8 i$. By Faraday's law, e, the induced voltage at time t, is given by

$$e = L \cdot \frac{di}{dt} \text{ volts.}$$

In order for I to flow, there must be an impressed or applied voltage E in the circuit, and if i increases so that $\frac{di}{dt}$ is positive, then, by the conservative principle on page 3, e must act in *opposition* to E. If i decreases so that $\frac{di}{dt}$ is negative, e must act in the *same* direction as E. In both cases the resultant voltage in the circuit is given by

$$E - L\frac{di}{dt},$$

and this, by Ohm's law, is equal to Ri; R being the resistance of the circuit. Therefore

$$L\frac{di}{dt} + Ri = E;$$

E may be of steady or variable value.

The induced voltages due to the mutual inductances of two coils.—If i_1 is the current in one coil, and i_2 that in the other, both currents in amperes,

$$e_1 = M\frac{di_2}{dt} \quad \text{and} \quad e_2 = M\frac{di_1}{dt},$$

e_1 and e_2 being the induced voltages in the respective coils.

In the case of one coil having an impressed voltage E in its circuit, and the second coil with *none*, the equations for the two circuits, according to the conservative principle and Ohm's law, will be

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} + R_1 i_1 = E,$$

$$L_2\frac{di_2}{dt} + M\frac{di_1}{dt} + R_2 i_2 = 0.$$

The growth of a current in a circuit containing self inductance and resistance.—On closing switch S (Fig. 16), the current takes time to

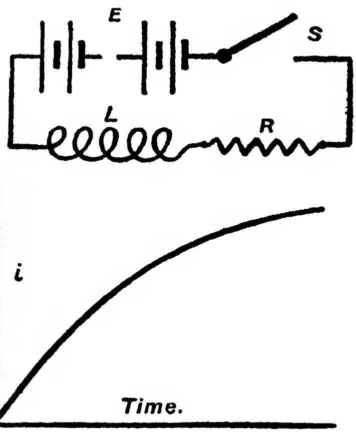

FIG. 16.—The growth of current on making a circuit.

reach its steady value of $I_0 = \frac{E}{R}$. Let t, the time, be measured from the instant of closing S; then, at the instant when the current is of value i *amperes*,

$$E - L\frac{di}{dt} = Ri,$$

or

$$L\frac{di}{dt} + Ri = E. \quad \text{.................................} (a)$$

Therefore $$\frac{di}{dt}\Big/\left(\mathrm{E}-\mathrm{R}i\right)=\frac{1}{\mathrm{L}},$$

and integrating $$-\frac{1}{\mathrm{R}}\cdot\log_e(\mathrm{E}-\mathrm{R}i)=\frac{t}{\mathrm{L}}+\mathrm{K},$$

where K is an arbitrary constant.

$$\therefore\ \mathrm{E}-\mathrm{R}i=\mathrm{A}e^{-\frac{\mathrm{R}}{\mathrm{L}}t}.$$

A is an arbitrary constant whose value depends upon, from what instant time is measured. Now $i=0$ when $t=0$, so that $\mathrm{A}=\mathrm{E}$.

Therefore $$i=\frac{\mathrm{E}}{\mathrm{R}}\left\{1-\frac{1}{e^{\frac{\mathrm{R}}{\mathrm{L}}t}}\right\},$$

and from this equation, $$t=2.3\frac{\mathrm{L}}{\mathrm{R}}\log_{10}\frac{\mathrm{I}_0}{\mathrm{I}_0-i}.$$

$\frac{\mathrm{L}}{\mathrm{R}}$ is defined as the *time constant* of the circuit, because the time taken by the current to reach any stated proportion of the final current is proportional to $\frac{\mathrm{L}}{\mathrm{R}}$.

Thus, for a circuit containing L and R in series, the time the current takes to reach 0·8 of its final value is $2{\cdot}3\log_{10}5$ times $\frac{\mathrm{L}}{\mathrm{R}}$.

Fig. 16 shows the gradual rise of the current in a circuit containing L and R in series.

The following table has been worked out from the last equation for the case of a circuit of *unity* time constant. x is the proportion of *final* current reached at time t seconds from the instant of closing the switch at make.

x	0·1	0·2	0·3	0·4	0·5	0·6	0·8	0·9
t	0·106	0·224	0·36	0·53	0·69	0·92	1·61	2·3

By means of this table or the graph relating x and t, the time taken for the current to attain a stated proportion of its final value in a circuit of known L and R may be quickly found. Thus, the time for the current to reach 0·8 of its final value in a circuit containing a self inductance of 8 henrys in series with 4 ohms is

$$\tfrac{8}{4}\times 1{\cdot}61=3{\cdot}22\text{ seconds.}$$

The energy stored in a magnetic field.—While the field is building up in the coil or circuit of self inductance L henrys, work is being done. Let i amperes be the current at time t; then

$$\frac{\mathrm{L}}{\mathrm{S}}\cdot di\cdot 10^8$$

is the number of lines of magnetic force added to the circuit during the interval dt when the current is i.

The work done during this interval is, by equation (5), page 5, equal to

$$S \cdot \frac{L}{S} \cdot di \cdot 10^8 \cdot \frac{i}{10} \text{ ergs} = Li \cdot di \text{ joules.}$$

Therefore the total work done in building up the field from the time when $i = 0$ to the time when $i = I$, the steady current, is

$$\int_0^I Li \cdot di = \frac{LI^2}{2} \text{ joules,}$$

and this energy is stored in the magnetic field.

EXAMPLE.—A circuit of self induction 2 henrys carries a steady current of 12 amperes. Find the energy stored in the magnetic field or fields of this circuit. *Ans.* 144 joules.

The effect of mutual inductance on the growth of current in an inductive circuit at make.—The effect will be to *increase* the time constant of the circuit, that is, delay the growth. Consider a simple case, such as a circular solenoid having two coils interleaved with each other. One coil is connected to a steady voltage E and switch, while the other is connected to a non-inductive resistance.

In this case the product of the self inductances of the two coils may be taken equal to the square of the mutual inductance, that is,

$$L_1 L_2 = M^2.$$

Also, if i_1 and i_2 are the respective instantaneous currents, and R_1 and R_2 the respective resistances of the two circuits, then at *make*

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + R_1 i_1 = E. \quad \ldots\ldots (a)$$

$$L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + R_2 i_2 = 0. \quad \ldots\ldots (b)$$

Multiply (a) by M and (b) by L_1, and subtract

$$MR_1 i_1 - L_1 R_2 i_2 = ME, \quad \ldots\ldots (c)$$

differentiate (c), $$\frac{di_2}{dt} = \frac{MR_1}{L_1 R_2} \cdot \frac{di_1}{dt}.$$

Substituting in equation (a),

$$\frac{di_1}{dt} \left\{ L_1 + \frac{M^2 R_1}{L_1 R_2} \right\} + R_1 i_1 = E,$$

which is of the same form as equation (a), page 25, and the *time constant* will therefore be

$$\frac{L_1 + \dfrac{M^2 R_1}{L_1 R_2}}{R_1} = \frac{L_1}{R_1} + \frac{L_2}{R_2},$$

which is the *sum* of the time constants of the two circuits.

If $L_1 = L_2$ and $R_1 = R_2$, the current at *make* would take twice as long to reach a certain proportion of its final value as when the short-circuited coil was absent or its switch open.

If $L_1 = L_2$ and $R_2 = \frac{R_1}{19}$, the current would take 20 times as long. In this case the copper wire of the short-circuited coil would have to be of cross-sectional area 19 times that used in the other coil, or a much larger external resistance may be used in the primary circuit.

By having such a short-circuited coil on the field magnets of an electrical machine, the building up of the field will be much more delayed, and, what is more important, the dying away of the field at break will be delayed, and the arcing at the field switch, if used, greatly lessened. This method is not in general a practical one, and less expensive and simpler devices are sometimes used.

Transient currents in divided circuits.—A transient current is generally represented by its quantity of electricity. When a circuit is divided, it is important to know how the quantity of the main transient current distributes itself in the different branches of the circuit. Thus, in Fig. 17, it may be necessary to know how much of the main quantity will pass through the branch ABD.

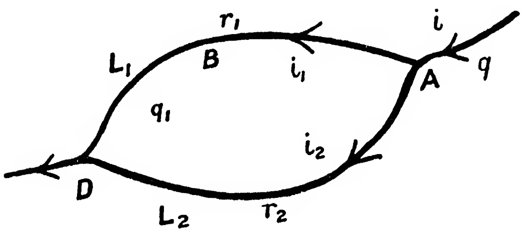

FIG. 17.—Transient currents in divided circuits.

Let i be the instantaneous value of the main current when the branch currents are i_1 and i_2. Then

$$i_1 + i_2 = i.$$

Also, the voltage across AD at the instant under consideration is expressed by

$$L_1 \frac{di_1}{dt} + R_1 i_1,$$

and also by
$$L_2 \frac{di}{dt} - L_2 \frac{di_1}{dt} + R_2 i - R_2 i_1;$$

$$\therefore \quad (L_1 + L_2)\, di_1 + (R_1 + R_2)\, i_1\, dt = L_2\, di + R_2 i\, dt.$$

Integrating for the time the transient current flows,

$$\Sigma di_1 \quad \text{and} \quad \Sigma di$$

are both zero. Therefore

$$(R_1+R_2)\Sigma i_1\,dt = R_2\Sigma i\,dt.$$

That is $$q_1=\frac{R_2}{R_1+R_2}\cdot q. \qquad\qquad (a)$$

Thus the quantity of electricity in the transient current divides in the two branches as if these possessed no self inductance.

The theorem of Gauss : electrostatics.—This is an important theorem of electrostatics, and, as it is an outcome of the law of inverse squares, certain analogous problems in electrostatics and magnetism may be solved by its use. This theorem is stated thus :

If a closed envelope contains a number of charges of statical electricity inside of it, the total normal induction over the surface of the envelope is equal to 4π times the algebraical sum of the charges.—That is

$$\text{T.N.I.} = 4\pi\Sigma q.$$

A simple illustration is that of a sphere of radius r cms. with a quantity of electricity q units at its centre. The number of lines of induction which normally cross each sq. cm. of its surface is by the law of inverse squares equal to $\frac{q}{r^2}$.

Therefore $$\text{T.N.I.} = 4\pi r^2\frac{q}{r^2} = 4\pi q.$$

Consider any small area of the envelope, such as B (Fig 18). This elemental area may be taken so small that its surface may be regarded

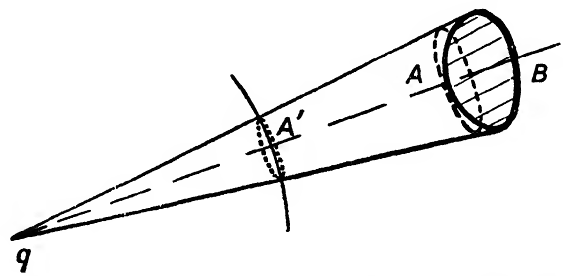

FIG. 18.—Induction across a surface due to a statical charge of electricity.

as plane. B is then the base of an oblique cone having quantity q at its apex. A is a section of this cone at right angles to the axis and touching B at one point as shown. θ is the angle between the planes A and B.

If a is the area of A, the T.N.I. over A is $\frac{aq}{d^2}$. Also, since the area of B is $\frac{a}{\cos\theta}$, and the number of lines which thread unit area of B in the *normal* direction is

$$\frac{q}{d^2}\cos\theta,$$

the T.N.I. over B is therefore

$$\frac{a}{\cos\theta}\cdot\frac{q}{d^2}\cos\theta=\frac{aq}{d^2},$$

which is the T.N.I. over A.

Now, the latter value is the same for any other cross section of the cone at right angles to the axis, such as A′.

Hence the T.N.I. for the whole surface of the envelope is the same as that for a sphere having q at its centre, that is,

$$\text{T.N.I.}=4\pi q.$$

Or, if there are more charges than one,

$$\text{T.N.I.}=4\pi\Sigma q.$$

A few illustrations of the use of this theorem will now be given.

The strength of the electrostatic field outside a long uniformly charged cylinder.—Let q be the quantity of electricity on the cylinder between A and B; the length of AB being one cm. Imagine a cylindrical envelope enclosing AB, as indicated in Fig. 19, and of radius x. Then, if F is the strength of the field at distance x from the axis of the cylinder, the T.N.I. over the whole surface of the envelope is

$$\text{F}\times 2\pi x\times 1,$$

and this, by the theorem of Gauss, is equal to $4\pi q$. Therefore

$$\text{F}=\frac{2q}{x}.$$

F is in dynes or lines of electrostatic force per sq. cm.

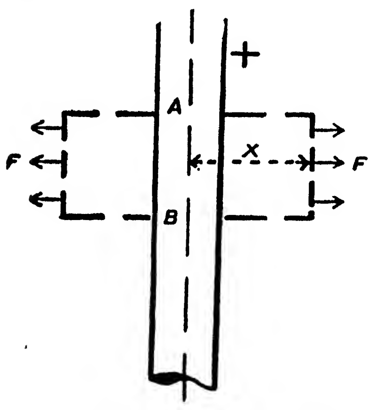

FIG. 19.—The strength of the electrostatic field outside a charged cylinder.

It should be noted that the direction of the electrostatic field is at right angles to that of the magnetic field which would be produced if a current was carried by the cylinder, and that the law of distance is the same in both cases.

The strength of the electrostatic field just outside a uniformly charged plane surface.—Consider a small cylinder of cross section one sq. cm. enveloping the charge q on one sq. cm. of the plane surface, as in Fig. 20. Then

$$\text{T.N.I.}=2\text{F}=4\pi q,$$
$$\text{F}=2\pi q.$$

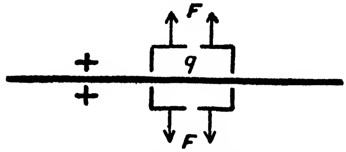

FIG. 20.—The strength of the electrostatic field just outside a charged plane surface.

The strength of the electrostatic field between two near uniformly and oppositely charged parallel plates.—Consider two cylindrical

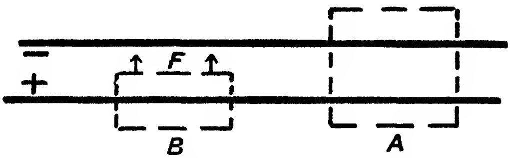

FIG. 21.—The strength of the electrostatic field between two near charged plates.

envelopes, A and B, as shown in Fig. 21. Let a be the cross-sectional area of each cylinder. Then, for A,

$$2Fa = 4\pi(q - q) = 0,$$

$$F = 0,$$

that is, there is no normal field above or below the plates.

For B,
$$Fa = 4\pi qa,$$

$$F = 4\pi q.$$

In a medium other than air, the law of inverse squares is given by

$$F = \frac{q_1 q_2}{kd^2},$$

k being a constant depending upon the nature of the medium or dielectric. So that the theorem of Gauss is more generally expressed as

$$\text{T.N.I.} = \frac{4\pi}{k}\Sigma q,$$

and the values obtained for F in the preceding cases should be divided by k, if a medium other than air is used.

Therefore, for the parallel plate condenser with a dielectric other than air,

$$F = \frac{4\pi q}{k}.$$

The capacity of a parallel plate condenser.—The difference of potential between two parallel plates, or sets of parallel plates, which form the condenser, is the work done in carrying unit charge from one plate across to the other. From the last equation, this work is

$$\frac{4\pi q}{k} \cdot d \text{ ergs},$$

q being the quantity of electricity per sq. cm. on each plate in electrostatic units, and d the distance of the plates apart in cms.

If Q is the total quantity on one set of plates, and V the potential difference between the two sets,

$$V = \frac{4\pi}{k} \cdot \frac{Q}{A} \cdot d \text{ ergs.}$$

The *capacity* of a condenser is the quantity of electricity which must be given to each set of plates to raise its potential difference one unit. Let C be this quantity. Therefore, to raise its potential difference V units, a quantity CV must be given to each plate. Hence

$$Q = VC,$$

or

$$C = \frac{Q}{V}.$$

The capacity of a parallel plate condenser is, therefore, given by

$$C = \frac{kA}{4\pi d} \text{ electrostatic units.}$$

The practical unit of electromagnetic capacity is the *Farad.* A condenser has a capacity of one farad when a potential difference of one volt across its plates gives each set of plates one coulomb of electricity. In this case,

$$Q = EC,$$

C being the capacity in farads, E the voltage, and Q the quantity in coulombs on each set of plates. The farad being a very lárge unit, capacities are usually expressed in *microfarads.*

Fig. 22.—A parallel plate condenser.

Now, one microfarad is equal to 9×10^5 electrostatic units of capacity, and therefore the capacity of a parallel plate condenser is given by

$$C = \frac{kA}{4\pi d} \times \frac{1}{9 \times 10^5} \text{ microfarads.}$$

If the condenser is of the usual form (Fig. 22), A will be the area of both sides of a plate multiplied by the number of sheets of dielectric. A is in sq. cms. and d in cms.

k is termed the *specific inductive capacity* of the dielectric, and is unity for air.

For other dielectrics, k is seen to be the ratio of the capacity of a condenser with the dielectric to its capacity when air is substituted for it.

The most satisfactory condensers are those whose dielectrics are air or mica. Cloth or paper preparations are not very satisfactory, except for certain cases. Condensers with paper or cloth dielectrics have no definite capacity, as the quantity of electricity taken by them, when connected to a given voltage, depends upon the time of

charging. Neither do they discharge fully, even when short circuited, and the residual charge takes a long time to escape.

The values of k for certain dielectrics commonly used are as follows:

Glass	6-7	India rubber	2·5
Mica	6	Ebonite	2·3
Oils	3-4	Paraffin wax	2·0
Shellac	3	Paper	1·5

These values are only approximate, and exact values, if required, should be determined experimentally and, as far as possible, under the physical conditions to which the dielectric will be exposed.

The capacity of a long concentric cable.—The cross section of a concentric cable having two tubular conductors is shown in Fig. 23. As there is no electric field inside a charged conductor, the charge on the inner conductor produces the electric field between the conductors.

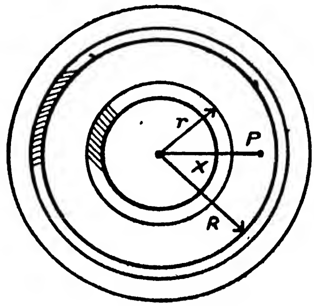

FIG. 23.—The cross-section of a concentric cable.

The strength of this field is $\dfrac{2q}{kx}$,

and $$V = F_P . dx ;$$

$$\therefore \; V = \int_r^R \frac{2q}{kx} . dx = \frac{2q}{k} \log_e \frac{R}{r},$$

q being the quantity in electrostatic units per axial *cm.* of the cable.

So that the capacity per *axial cm.* of the cable is

$$\frac{q}{V} = \frac{k}{2 \log_e \dfrac{R}{r}},$$

and one M.F. (microfarad) $= 9 \times 10^5$ electrostatic units;

$$\therefore \; C \text{ (per mile)} = 0{\cdot}039 \frac{k}{\log_{10} \dfrac{R}{r}} \text{ microfarads.}$$

EXAMPLE.—Calculate the capacity of a double concentric cable, 200 miles long, in which the outer radius of the inner conductor is 0·4 cm., and the radial thickness of the insulation between the conductors 0·8 cm. The value of k is 3. *Ans.* 49 microfarads.

EXAMPLE.—Show that the capacity per mile of two long parallel conductors, suspended in air and separated by d inches, is

$$\frac{0{\cdot}0194}{\log_{10} \dfrac{d}{r}} \text{ microfarads,}$$

r inches being the radius of each wire, and small in comparison with d. The effect of the earth is to be neglected.

A circuit with capacity and resistance; make.—A circuit has a capacity C farads in series with a resistance R ohms and an applied voltage E. When the circuit is made, the condenser gradually becomes charged to the final voltage E, and has then a quantity EC coulombs.

If i amperes is the current at time t during this charging, and q coulombs is the quantity of electricity on each set of condenser plates at the same instant,

$$E - \frac{q}{C} = Ri.$$

Now
$$\frac{dq}{dt} = i,$$

so that
$$R\frac{dq}{dt} + \frac{q}{C} = E,$$

or
$$\frac{dq}{dt} \bigg/ \left(E - \frac{q}{C}\right) = \frac{1}{R}.$$

Integrating,
$$-C \log\left(E - \frac{q}{C}\right) = \frac{t}{R} + A;$$

$$\therefore \quad E = \frac{q}{C} + Ke^{-\frac{t}{RC}}.$$

K is an arbitrary constant. If time is measured from the closing of the switch at make, then $q = 0$ when $t = 0$, so that $K = E$.

Therefore
$$q = EC\left\{1 - \frac{1}{e^{\frac{t}{RC}}}\right\}$$

or
$$q = q_0\left\{1 - \frac{1}{e^{\frac{t}{RC}}}\right\},$$

q_0 being the final charge in the condenser. Also

$$t = 2{\cdot}3RC \log_{10} \frac{q_0}{q_0 - q},$$

so that the *time constant* for such a circuit is RC.

The value of the current is $\frac{dq}{dt}$, which equals

$$EC \times \frac{1}{RC} \times \frac{1}{e^{\frac{t}{RC}}};$$

$$\therefore \quad i = \frac{E}{R} \frac{1}{e^{\frac{t}{RC}}},$$

which value, as t becomes larger and larger, gradually falls to zero.

The energy stored in a charged condenser.—Suppose a resistance R is joined across the terminals of the condenser, and the discharge takes place through it. All the energy in the condenser is transformed into heat, and its value is

$$\Sigma i^2 R\,dt \text{ joules,}$$

if i is in amperes, R in ohms, and time in seconds.

The equation of discharge is

$$\frac{q}{C} = -Ri \quad \text{and} \quad \frac{dq}{dt} = i;$$

$$\therefore \sum i^2 R\,dt = -\sum \frac{q}{C}\,dq = -\int_{q_0}^{0} \frac{q}{C}\,dq = \frac{q_0^2}{2C} \text{ joules.}$$

Then, as $q_0 = EC$,

the energy stored up in the condenser is equal to

$$\frac{CE^2}{2} \text{ joules.}$$

Example.—Calculate the energy taken from 500 volt mains by a condenser of capacity 0·8 microfarad when charged by them. *Ans.* 0·1 joule or 10^6 ergs.

Discharge of a condenser through a self inductance of negligible resistance.—In this case

$$L\frac{di}{dt} + \frac{q}{C} = 0,$$

q being the quantity in the condenser of capacity C at time t. Also

$$\frac{dq}{dt} = i;$$

$$\therefore L\frac{d^2q}{dt^2} + \frac{q}{C} = 0,$$

$$\frac{d^2q}{dt} + \frac{q}{LC} = 0.$$

Let $a^2 = \frac{1}{LC}$.

Then $$\frac{d^2q}{dt^2} + a^2 q = 0. \qquad \ldots\ldots(a)$$

The solving equation for (a) is

$$x^2 + a^2 = 0,$$

whose roots are ja and $-ja$, in which $j = \sqrt{-1}$.

The solution of (a) is

$$q = A_1 e^{jat} + A_2 e^{-jat},$$

that is,

$$q = A \sin at + B \cos at.$$

Now, when $t = 0$, $q = q_0$, the initial charge. Therefore $B = q_0$.

Also

$$i = \frac{dq}{dt} = aA \cos at - aB \sin at,$$

and $i = 0$ when $t = 0$; $\therefore$ $A = 0$.

Hence

$$q = q_0 \cos at = q_0 \cos \frac{t}{\sqrt{LC}},$$

that is,

$$q = CE \cos \frac{t}{\sqrt{LC}}.$$

and

$$i = -E\sqrt{\frac{C}{L}} \sin \frac{t}{\sqrt{LC}}.$$

The graphs of q and i with respect to time are shown in Fig. 24.

A current obeying the sine law is therefore produced in the circuit, and is subject to no damping. The original quantity in the condenser discharges through the self inductance and builds up in the

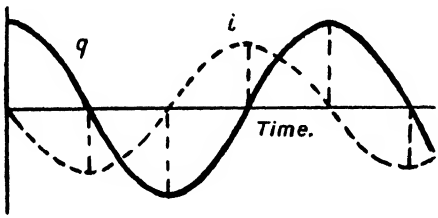

FIG. 24.—The undamped, oscillatory discharge curves for a condenser.

latter a magnetic field. This field then collapses and produces a reverse current, which charges up the condenser again to its original value, and so on.

At one instant the energy in the system is all *electrostatic* and of amount $\frac{E^2C}{2}$ joules. A quarter of a period later it is all *electromagnetic* and of amount

$$\frac{LI_m^2}{2} = \frac{L}{2} \times \frac{E^2C}{L} = \frac{E^2C}{2} \text{ joules.}$$

Thus, there is an oscillation of a given amount of energy from the electrostatic to the electromagnetic form.

The period or time to form a complete wave of current or quantity is

$$2\pi\sqrt{CL} \text{ seconds,}$$

and the frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

EXAMPLE.—A condenser of capacity 2 microfarads is charged by a voltage of 200 and then discharged through a self inductance of 5 millihenrys. Find the maximum current in the circuit and the frequency of its alternations. Also calculate the amount of energy in the circuit and the interval between the instant when all this energy is magnetic and that when it is all electrostatic. *Ans.* 4 amperes; frequency about 1600; 0·04 joule; 0·000157 sec.

A circuit without resistance is an ideal one. The presence of a very small amount of resistance will soon damp down the current by transforming the original energy of the condenser into heat. That is, the I^2R loss soon consumes the original energy of the condenser.

This damping, if R is small enough, will take a little time, and for a short time after discharge begins, the alternating current may be sufficiently undamped for certain practical purposes, namely for producing high frequency electric waves in wireless telegraphy and telephony.

Discharge of a condenser in series with self inductance and resistance.—The equation for this discharge is

$$L\frac{di}{dt} + Ri + \frac{q}{C} = 0,$$

and
$$\frac{dq}{dt} = i;$$

$$\therefore\ L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0.$$

Let
$$a = \frac{R}{2L} \quad \text{and} \quad b = \frac{1}{CL}.$$

Then
$$\frac{d^2q}{dt^2} + 2a\frac{dq}{dt} + bq = 0. \qquad \text{.........................}(a)$$

The solving equation for (a) is

$$x^2 + 2ax + b = 0,$$

whose roots λ_1 and λ_2 are respectively

$$-a + \sqrt{a^2 - b} \quad \text{and} \quad -a - \sqrt{a^2 - b}.$$

The solution for (a) is

$$q = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t},$$

A_1 and A_2 being arbitrary constants.

There are two cases to be considered, one when the roots are ***real***, that is,

when a^2 is greater than b,

and the other when the roots are imaginary, that is,

when a^2 is less than b.

First case. $$\frac{R^2}{4L^2} > \frac{1}{LC} \quad \text{or} \quad \frac{R}{2} > \sqrt{\frac{L}{C}}.$$

Let $$\sqrt{a^2 - b} = n.$$

Then $$q = A_1 e^{(-a+n)t} + A_2 e^{(-a-n)t}.$$

Now $$q = q_0 \quad \text{when} \quad t = 0;$$

$$\therefore \quad A_1 + A_2 = q_0. \qquad \text{(b)}$$

Also $$i = \frac{dq}{dt},$$

so that $$i = A_1(-a+n)e^{(-a+n)t} - A_2(a+n)e^{(-a-n)t},$$

and $$i = 0 \quad \text{when} \quad t = 0.$$

Therefore $$A_2 = -\left(\frac{a-n}{a+n}\right)A_1. \qquad \text{(c)}$$

From (*b*) and (*c*), $$A_1 = q_0 \cdot \frac{a+n}{2n},$$

$$A_2 = -q_0 \cdot \frac{a-n}{2n};$$

$$\therefore \quad q = \frac{q_0}{2n}\left\{\frac{a+n}{e^{(a-n)t}} - \frac{a-n}{e^{(a+n)t}}\right\}.$$

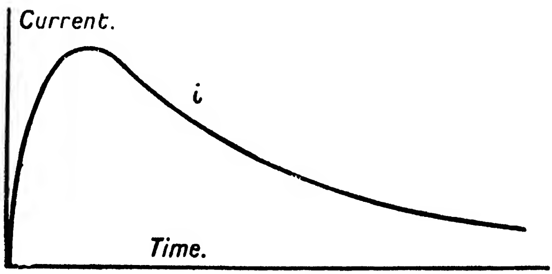

FIG. 25.—A damped, unoscillatory discharge curve of a condenser.

Since a is greater than n, the value of q starts from q_0, and as time goes on, gradually drops to zero.

The current is found by differentiating q with respect to time, and is given by

$$i = -q_0 \frac{b}{2n}\left\{\frac{1}{e^{(a-n)t}} - \frac{1}{e^{(a+n)t}}\right\}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(d)$$

As shown in Fig. 25, this current starts from zero, reaches a maximum, and then gradually dies away.

The maximum value of the current is reached when $\frac{di}{dt} = 0$, that is, when

$$(a-n)e^{nt} = (a+n)e^{-nt}$$

or

$$e^{2nt} = \frac{a+n}{a-n},$$

that is,

$$t = \frac{1}{2n} \cdot \log_e \frac{a+n}{a-n}.$$

By substituting this value of t in (d), the maximum discharge current may be calculated.

Example.—A condenser of capacity 5 microfarads is charged with 40 volts, and is then discharged through a self inductance of 0·005 henry in series with a resistance of 200 ohms. Calculate the time taken for the discharge current to reach a maximum value; also calculate the value of this maximum current. *Ans.* 0·0000963 sec.; 0·188 ampere.

Second case. $$\frac{R}{2} < \sqrt{\frac{L}{C}}.$$

Let $\sqrt{b-a^2} = m$, so that $\sqrt{a^2-b} = jm$,

$$q = A_1 e^{-(a+jm)t} + A_2 e^{(-a-jm)t},$$

$$q = e^{-at}\{A_1 e^{jmt} + A_2 e^{-jmt}\};$$

$$\therefore \; q = e^{-at}\{A \sin mt + B \cos mt\}.$$

When $t=0$, $q=q_0$. Therefore $B=q_0$. Also $i = \frac{dq}{dt}$, so that

$$i = e^{-at}\{mA \cos mt - mB \sin mt - aA \sin mt - aB \cos mt\}.$$

When $t=0$, $i=0$. Therefore $mA = aB$, so that

$$A = \frac{a}{m} q_0.$$

Hence,

$$q = q_0 \frac{a}{m} \cdot \frac{1}{e^{at}}\left\{\sin mt + \frac{m}{a}\cos mt\right\},$$

and if

$$\tan\theta = \frac{m}{a},$$

$$q = q_0 \frac{\sqrt{a^2+m^2}}{m} \cdot \frac{1}{e^{at}} \cdot \sin(mt+\theta).$$

Then i, which is equal to $\frac{dq}{dt}$, is given by

$$i = -q_0 \frac{b}{m} \cdot \frac{1}{e^{at}} \cdot \sin mt.$$

The graph of current i with respect to time is roughly indicated in Fig. 26, and shows the effect of resistance in damping out oscillations which, without this damping, would be of sinusoidal form.

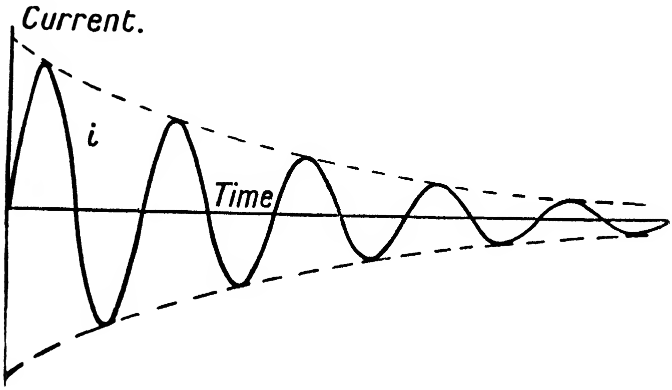

FIG. 26.—A damped, oscillatory discharge curve of a condenser.

EXAMPLE.—A condenser of capacity 5 microfarads is charged by 40 volt mains, and is then discharged through a self inductance of 0·005 henry in series with a non-inductive resistance of 20 ohms. Obtain the equation for the oscillatory discharge current, and draw its graph for a few oscillations.

CHAPTER III.

ALTERNATING-CURRENT PRINCIPLES.

ALTERNATING current can generally be produced more economically and efficiently than direct current; the alternating-current generator being of simpler construction than the direct-current machine. It is easy to build the former type for high voltages, but extremely difficult to build the latter to give even a few thousand volts. Commutation in the direct-current generator limits the production of high voltage, and in general such machines are built for voltages not exceeding about 500 volts; in exceptional cases they may be built to give 1000 volts.[1]

In long distance transmission, and even for distances of a few miles, high voltages are needed for efficient working; the higher the voltage at the generating end the greater the efficiency of the transmitting plant. For such cases, alternating currents are used, and the voltages at the generating end of the system may be as high as a hundred thousand volts, and even higher.

Alternating voltages may readily be transformed into higher or lower voltages by static transformers; they may also be transformed into direct voltages by using a machine known as a rotary converter, or the alternating voltage may be used to drive an alternating current motor coupled to a direct-current generator.

A number of advantages are gained by constructing the alternating-current generator or alternator to give as nearly as possible a simple sine curve of electromotive force. It does not, however, follow that the alternating current will be sinusoidal if the voltage obeys the sine law, for harmonics may be caused by some of the consuming devices in the circuit. Thus, at *light* loads the effect of hysteresis in the iron core of a transformer may appear in the current wave as a pronounced third harmonic and a weak fifth harmonic.

In general, the voltage wave form produced by an alternator is exactly the same shape above and below the time axis; thus, when

[1] Small direct-current generators capable of producing voltages as high as 10,000 have been constructed for the purpose of testing insulating materials with unidirectional voltage.

the lower half of the wave in Fig. 27 is displaced along the time axis to the dotted position, the dotted curve is more clearly seen to be symmetrical with the upper half. For this to be so, there can be no

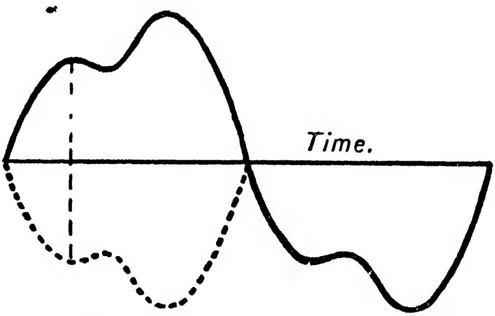

FIG. 27.—A wave having symmetrical upper and lower halves.

even harmonics in the wave, for such always produce dissymmetry between the upper and lower parts of the wave.

This may readily be shown by the synthesis of even harmonics and the first harmonic or fundamental. A simple case of a current or voltage wave containing a fundamental and a second harmonic is shown in Fig. 28. In the first diagram the harmonic starts from the

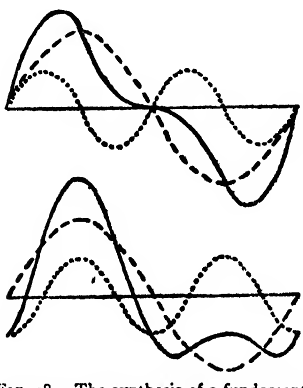

FIG. 28.—The synthesis of a fundamental wave and its second harmonic.

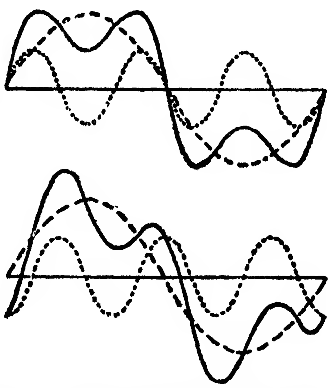

FIG. 29.—The synthesis of a fundamental wave and its third harmonic.

origin at the same time and in the same direction as the fundamental. In the lower diagram it starts one quarter of the period of the harmonic behind it. There is no symmetry at all in this latter case.

By taking other even harmonics or a number of even harmonics and building them up with the fundamental, it may be shown that the presence of even harmonics causes dissymmetry between the upper and lower parts of the wave.

This is not, however, the case with *odd* harmoncis and the fundamental. These combine to form a wave whose upper and lower parts are symmetrical. Fig. 29 shows the synthesis of the fundamental and a third harmonic, Fig. 30 that of the fundamental and the fifth harmonic.

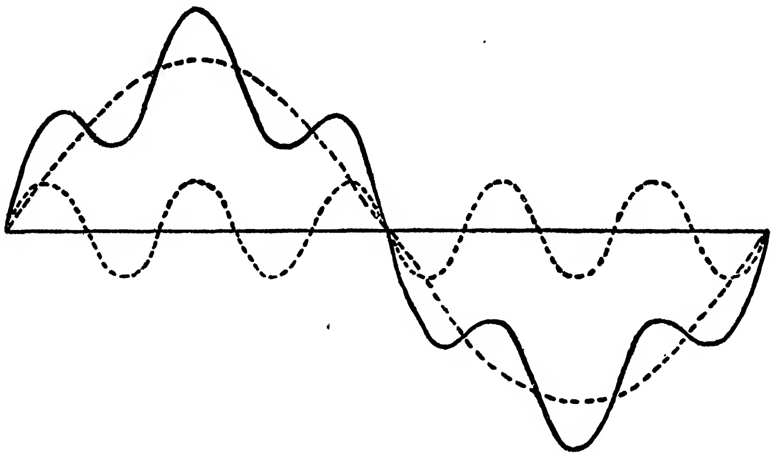

FIG. 30.—The synthesis of a fundamental wave and its fifth harmonic.

By such synthesis of wave forms it will be found that the *highest* harmonic present is represented by the number of hollows and crests in one part of the wave form. If the complex wave contains the seventh harmonic as its highest, the total number counting crests and hollows in the upper half of the wave will be seven. The highest harmonic present in the complex wave form of Fig. 31 is the eleventh.

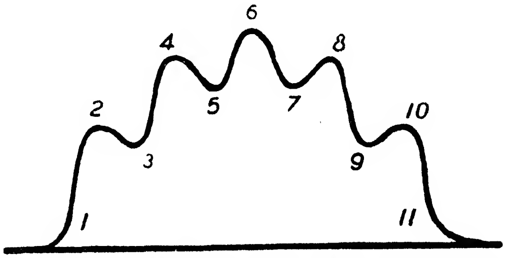

FIG. 31.—A complete wave whose highest harmonic is the eleventh.

A method of analysis is needed for the determination of the type and magnitude of each harmonic present in a complex current or voltage wave.

Measurement of alternating current and voltage.—An alternating current has a heating effect represented by

$$\sum i^2 R \, dt \times 10^7 \text{ ergs},$$

in which i amperes is the instantaneous current at time t, flowing through resistance R ohms. This value is the electrical energy transformed into heat.

In Fig. 32, MN $=dt$, M being considered so near to N that the

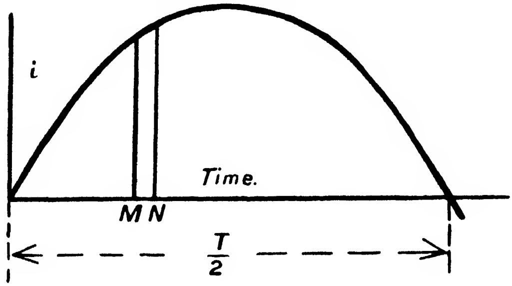

FIG. 32.—A current half wave.

current i may be regarded as of constant value throughout the time MN.

The heat energy produced for one half wave will be

$$\sum_0^{\frac{T}{2}} i^2 R\, dt \text{ joules,}$$

T seconds being the period of the alternation of the current.

Now the direct current I amperes which would produce the same heat energy in the same time is

$$I^2 R \cdot \frac{T}{2} \text{ joules;}$$

$$\therefore \quad I = \sqrt{\frac{2}{T} \sum_0^{\frac{T}{2}} i^2 dt}.$$

This square root term is expressed as the *root mean square* value of the alternating current, and its symbol is R.M.S. Thus the heating effect of the alternating current is the same as a direct current of value equal to the R.M.S. value of the alternating current.

This is true whatever the wave form or frequency of the alternations. A hot-wire ammeter calibrated for direct currents will read the R.M.S. value of an alternating current. For the same reason a hot-wire voltmeter reads either direct or alternating voltages.

An alternating current may be measured by passing it through an electro-dynamometer consisting of a moving coil in series with a fixed coil; the two coils being kept in their zero position by applying torsion to the movable coil. This torsion measures the force between the two coils, and is proportional to i^2.

The average force between the coils for one half wave will therefore be

$$\frac{2k}{T} \sum_0^{\frac{T}{2}} i^2\, dt,$$

k being a constructional constant of the instrument. This value is also the continuous average force.

The force between the coils for the direct current I is kI^2, and for these two forces to be equal, I has to be equal to the R.M.S. value of the alternating current.

Such a dynamometer calibrated for direct current may be used for measuring the R.M.S. value of the alternating current. In the latter case metal work on the instrument or in the vicinity will produce errors in the reading due to the action of the eddy currents produced in the metal, on the coils. The latter should preferably be built up in parallel sections of fine wire rather than in single sections of thick wire, for the same reason. The same remarks apply to the electro-dynamometer type of voltmeter.

A similar case to the preceding is that of the electrostatic voltmeter, the attractive force between its moving plate or plates and the fixed ones being proportional to e^2, the square of the voltage applied to the terminals of the instrument. The average force, as before, is proportional to the torsion and represented by

$$\frac{2k}{T}\sum_{0}^{\frac{T}{2}} e^2 dt,$$

and the force for a direct-current voltage E is kE^2. These are equal when E is the R.M.S. value of the alternating voltage. This type of voltmeter will therefore read either direct or alternating voltages.

Other instruments, such as Kelvin's standard balances, may be used for either direct or alternating currents if constructed for the latter case. Instruments with soft iron plungers working in solenoids are generally calibrated for either direct current or else for alternating current. If calibrated for the latter and used with the former, or *vice versa*, a discrepancy of a few per cent. may be noted between the readings for the same value of alternating and direct current.

Relations between R.M.S., average, and maximum values of sine curves.—A simple sine curve is shown in Fig. 33. Its instantaneous,

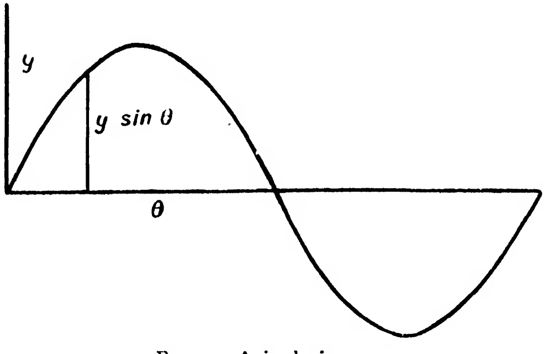

FIG. 33.—A simple sine wave.

average, maximum, and R.M.S. values are represented respectively by y, Y_{av}, Y_m, and Y.

Then, $$Y_{av}=\frac{1}{\pi}\int_0^\pi Y_m \sin\theta\, d\theta=\frac{Y_m}{\pi}\Big[-\cos\theta\Big]_0^\pi=\frac{2}{\pi}Y_m.$$

$$Y^2_{\text{R.M.S.}}=\frac{1}{\pi}\int_0^\pi Y_m^2 \sin^2\theta\, d\theta=\frac{Y_m^2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^\pi=\frac{Y_m^2}{2}.$$

$$\therefore\ Y=\frac{1}{\sqrt{2}}\cdot Y_m.$$

Y_m is termed the *amplitude* of the wave.

Thus the *average* value $=\frac{2}{\pi}$ times the *maximum* value.

" R.M.S. " $=\frac{1}{\sqrt{2}}$ " " "

" R.M.S. " $=\frac{\pi}{2\sqrt{2}}=1{\cdot}111$ times the *average* value.

$$\textit{Amplitude factor}=\frac{\text{Maximum value}}{\text{R.M.S. value}}=1{\cdot}414 \text{ for a sine curve.}$$

$$\textit{Form factor}=\frac{\text{R.M.S. value}}{\text{Average value}}=1{\cdot}111 \text{ for a sine curve.}$$

A *flat* and a *peaked* half wave are shown in Fig. 34.

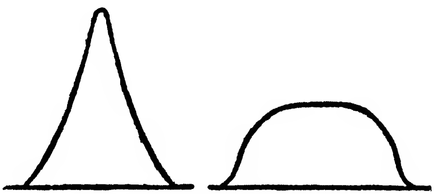

FIG. 34.—Peaked and flat half waves.

The flat curve has a small amplitude factor and a small form factor, while the peaked curve has both these factors large.

The reading on a hot-wire ammeter or dynamometer of an alternating current which contains a fundamental of amplitude 50 amperes, a third harmonic of 10 amperes, and a fifth harmonic of 12 amperes, is

$$\frac{1}{\sqrt{2}}\cdot\sqrt{50^2+10^2+12^2},$$

because each component produces its own heating or magnetic effect in the instrument.

Instantaneous values of current and voltage will be represented by i and e. Maximum values by I_m and E_m. Average values by

I_{av} and E_{av}. R.M.S. values by I and E. Unless otherwise stated these values will be considered as expressed in amperes and volts.

The theory of the simple alternator.—The simplest illustration of the generation of alternating current is shown represented in Fig. 35. N, the pole of a magnet, is moved vertically up and down between two points *a* and *b* above a flat coil placed on a horizontal board.

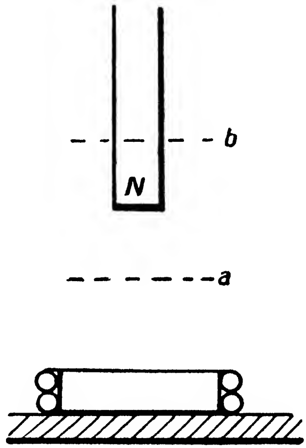

Fig. 35.—An illustration of a simple alternator.

This motion requires mechanical energy, for the coil repels the magnet as it approaches and attracts it when it recedes. The electrical energy induced in the coil is the transformation of this mechanical energy.

By applying Lenz's Rule to this case, it is evident that an alternating current is produced in the coil of frequency equal to the number of double strokes of the magnet. The greater the speed the greater this induced current, and more mechanical power is required to drive N.

In practice, instead of a single pole moving up and down above the coil a set of poles are rotated across the face of a set of armature coils connected in series or otherwise arranged, and induced currents are produced in the armature winding.

The simplest illustration for quantitative deductions is the rectangular or circular coil rotating uniformly between two magnet poles as indicated in Fig. 36. In this case, if time is measured from the

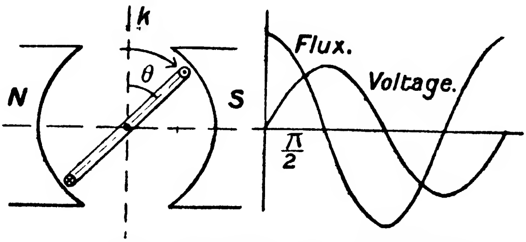

Fig. 36.—The flux and voltage graphs of a simple alternator.

vertical position of the coil,

$$\theta = \omega t \text{ radians,}$$

t being the time the coil moves through an angle θ from the dotted line.

By applying *Fleming's* right-hand rule to this case, it is readily shown that the induced voltage is in one direction for the first half revolution reckoned from *k*, and in the opposite direction for the

second half. A complete voltage wave is therefore produced per revolution of the coil.

The value of this voltage may be found by *Faraday's* Law. Thus, if ϕ_m is the maximum flux threading the S turns of the coil, when its plane is vertical,

$$\phi = \phi_m \cos \omega t,$$

and
$$e = -S \cdot \frac{d\phi}{dt} \cdot 10^{-8} = S\phi_m \omega 10^{-8} \sin \omega t.$$

If a = mean area of the turns of the coil, and b the strength of the magnetic field,

$$\phi_m = ab.$$

Also
$$\omega = 2\pi f,$$

f being the frequency, that is, the number of complete waves of induced voltage produced per second.

$$\therefore \quad e = Sab \cdot 2\pi f 10^{-8} \sin 2\pi ft.$$

In this case f = the number of revolutions per second.

When t is zero the flux is a maximum, and the induced voltage is zero. When ωt is $\frac{\pi}{2}$ the flux is zero, and the voltage is a maximum. The physical reason for this is, that when the coil is threaded with ϕ_m, the effective sides of it are merely sliding along the lines of magnetic force and not cutting them, but when the flux is zero these sides are cutting the lines at right angles, and therefore most effectively.

It should be noted as a matter of great importance that *the induced voltage lags* 90 *degrees behind the flux which produces it*; and this statement will be found to be of general application. This follows from the fact that the flux obeys the cosine law and the voltage the sine law, or the matter may be considered as follows:

In Fig. 36 are shown the flux and voltage curves; the former having a maximum value when $t = 0$. At this instant the induced voltage e is zero, and maximum when $t = \frac{\pi}{2\omega}$ seconds, at which time the flux is zero. It is required to find whether the e curve ascends or descends, that is, whether it starts above or below the time axis. This e curve must ascend as shown, because the flux is beginning to fall in value, and by the conservative principle on page 3, the direction of e must be such that it tends to keep the flux threading the coil constant in value. It can only do this by acting in the direction shown. Thus the *lag* of e behind ϕ, which produces it, is $\frac{\pi}{2}$ radians or 90 degrees.

Example.—A flat coil of 500 turns of fine wire, and of mean area 600 sq. cms., revolves uniformly between two opposite poles at 1600 R.P.M. The strength of the magnetic field is 800 C.G.S. units. Calculate the maximum, average, and R.M.S. values of the voltage induced in the coil. *Ans.* 400; 256; 284 volts.

The voltage induced in a coil threaded by an alternating sinusoidal magnetic flux.—Let ϕ_m be the maximum value of the flux threading the coil, and f the frequency of its alternations. At a certain instant the coil of S turns is threaded by ϕ_m lines, and $\frac{1}{2f}$ second later by $-\phi_m$ lines.

Therefore, by *Faraday's* law,

$$E_{av} = 2\phi_m S 10^{-8} \Big/ \frac{1}{2f} = 4\phi_m S f 10^{-8} \text{ volts,}$$

and the R.M.S. value

$$E = 4{\cdot}44\phi_m S f 10^{-8} \text{ volts.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

In the case of a transformer (Fig. 37), P is the primary coil of S_1

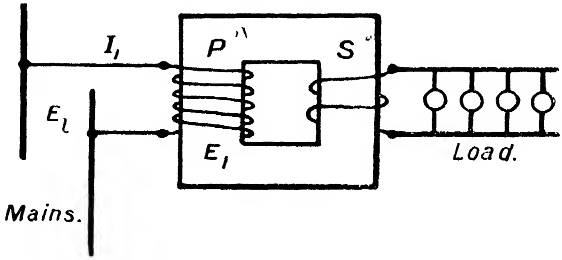

FIG. 37.—A single-phase transformer on load.

turns and S the secondary coil of S_2 turns. The latter is loaded. The induced voltage in each of the coils is given by the preceding formula.

Consider coil P. Within it are the three voltages—the applied voltage E_l, the induced voltage E_1, and the voltage drop $r_1 I_1$. Even at full load $r_1 I_1$ is small compared with E_l, so that E_1 is nearly equal and opposite to E_l, because their resultant is the voltage drop. Therefore, very nearly for all loads,

$$E_l = E_1 = 4{\cdot}44\phi_m S_1 f 10^{-8} \text{ volts,}$$

and since E_l is constant, ϕ_m, the flux in the core of a transformer, is nearly constant for all loads.

EXAMPLE.—A transformer has a primary coil of 400 turns and a secondary of 100 turns. The cross-sectional area of the iron of its core is 80 sq. cms. Its primary coil is connected to mains of 500 volts and frequency 50. Calculate the maximum value of the flux density in the core and the voltage across the open terminals of the secondary coil. *Ans.* About 7000 lines per sq. cm.; 125 volts.

The current produced in a circuit fed by a sine voltage.—Consider a circuit having only non-inductive resistance R ohms. This is fed from mains across which a voltage of $E_m \sin \omega t = e$ operates. By Ohm's law the current i is given by

$$i = \frac{e}{R} = \frac{E_m}{R} \sin \omega t$$

The i and e curves (Fig. 38) are therefore in phase with each other.

The *power*, P watts, supplied to the circuit is the sum of all the small amounts of energy represented by

$$ei\,dt = i^2 R\,dt$$

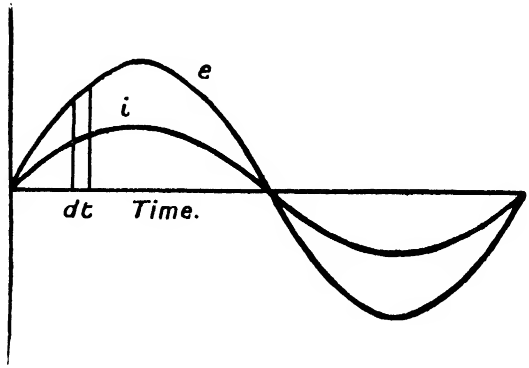

FIG. 38.—Sine voltage and current curves.

throughout a complete wave multiplied by f, the number of waves produced in a second. Therefore

$$P = f\frac{E_m^2}{R}\int_0^{\frac{2\pi}{\omega}} \sin^2 . \omega t . dt,$$

$$= f\frac{E_m^2}{R}\left[\frac{t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\frac{2\pi}{\omega}}$$

$$= f\frac{E_m^2}{R} . \frac{\pi}{\omega} = \frac{E_m}{\sqrt{2}R} \times \frac{E_m}{\sqrt{2}},$$

$$P = EI \text{ watts.}$$

That is, the power consumed by the circuit is the product of the R.M.S. values of the current and voltage.

Consider a circuit with only self inductance L henrys.—In this case a back voltage e_b is produced in the circuit due to L, and by Ohm's law the resultant voltage is equal to Ri; but R is zero, so that e_b must exactly balance e, the applied voltage at every instant, as in Fig. 39.

Also e_b is produced by the magnetic flux which is in phase with, and due to the current in the circuit. Now e_b lags 90 degrees behind this flux, and therefore the same amount behind the current i, or, what is the same thing, i *leads* e_b by 90 degrees. So that the current wave shown dotted in the figure *lags* 90 degrees behind the applied voltage e wave when there is only self inductance in the circuit.

The maximum value ϕ_m of the flux produced in the circuit is given by

$$\phi_m = \frac{LI_m}{S} \cdot 10^8,$$

S being the number of turns. Now, such an alternating flux produces an induced voltage

$$E_b = \frac{4\pi}{2\sqrt{2}} LI_m f,$$

according to equation (*a*) on page 49. Therefore

$$E = E_b = 2\pi f LI = L\omega I.$$

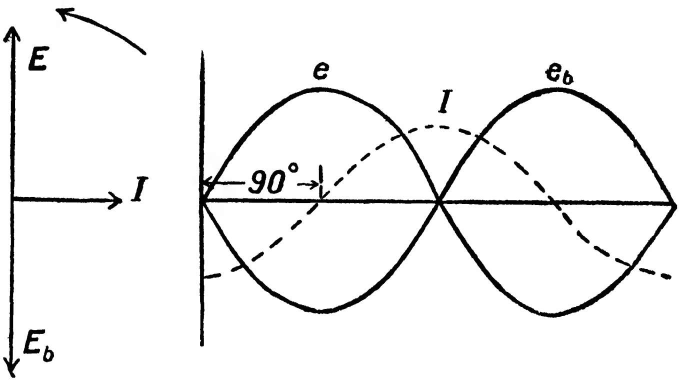

FIG. 39.—The voltage and current graphs of a circuit containing self inductance, and also its vector diagram for a sine voltage.

This is an important relation, and it should be noted that E is the value of the applied voltage across the terminals of the inductance necessary to drive the current through it. $L\omega$ is a term expressed in ohms, and is called the impedance of the circuit. It obeys Ohm's law, as shown by the equation,

$$I = \frac{E}{L\omega}.$$

EXAMPLE.—An alternating current of 2·1 amperes and frequency 50 is flowing through a circuit containing only two self inductances in series, one of value 0·02 henry and the other 0·08 henry. Calculate the voltage across the terminals of each inductance. *Ans.* 13·2 and 52·8 volts.

The vector diagram for R.M.S. values is also shown in Fig. 39. The rotation is anti-clockwise, and I is seen to lead E_b by 90 degrees, and lag behind E by the same amount. ϕ, the magnetic flux in the circuit, is in phase with the current.

There is *no* power consumed in this circuit, for summing up the small amounts of energy represented by $ei\,dt$, for a half or complete

wave gives zero. This is also evident from the fact that, R being zero, there is no heating in the circuit, and therefore no power consumed.

Though no power is consumed in the circuit, yet power is *supplied* to it, but the latter returns as much to the mains as it receives from them. Thus, during time $\frac{1}{4f}$ second the circuit is receiving energy from the mains to build up its magnetic field, while in the next $\frac{1}{4f}$ second, it is restoring an equal amount to the mains by the falling of the flux from ϕ_m to zero.

In practice it is impossible to get a pure self inductance, especially when it has to be of large value; the resistance of the wire, and especially the hysteresis and eddy actions in the iron core, if used, causing the transformation of a certain amount of electrical energy into heat.

For decreasing current in alternating circuits, *non*-inductive resistances are very wasteful of electrical energy, and in general are never used where economy is essential. Choking coils having a minimum resistance and suitable self inductance are used instead, with but very little loss of power.

EXAMPLE.—The primary of a transformer, having 400 turns, is connected to 346 volt mains of frequency 50, and the no-load current, that is the current in the primary coil, when the secondary is open, is 6 amperes. Neglecting its small resistance, calculate the self inductance of the primary coil and the maximum flux density in the core; the cross-sectional area of the latter being 60 sq. cms. *Ans.* 0·184 henry and 6,500 lines per sq. cm.

A circuit containing L and R.—This circuit is illustrated in Fig. 40. The applied voltage divides itself into two parts, one to balance e_b in the self-inductance part of the circuit, and the other to drive the current through R. Therefore, vectorially,

$$e = e_b + \mathrm{R} \cdot i,$$

and e_b is produced by the flux which is in phase with the current i, and is 90 degrees in phase behind it.

The vector diagram for this circuit is also shown in the same figure. E_b is 90 degrees behind I, and its value is $\mathrm{L}\omega\mathrm{I}$. RI is one of the components of the applied voltage and the other is OA, which balances E_b.

Therefore E, the applied voltage, is the resultant of OA and RI, and of value

$$\mathrm{E} = \mathrm{I}\sqrt{\mathrm{R}^2 + (\mathrm{L}\omega)^2},$$

that is, the product of the current and the *impedance* of the circuit.

It will be noted that RI is also the resultant of the applied and back voltages.

The diagram also shows that the current lags an angle θ behind E of value given by

$$\tan\theta = \frac{L\omega}{R}.$$

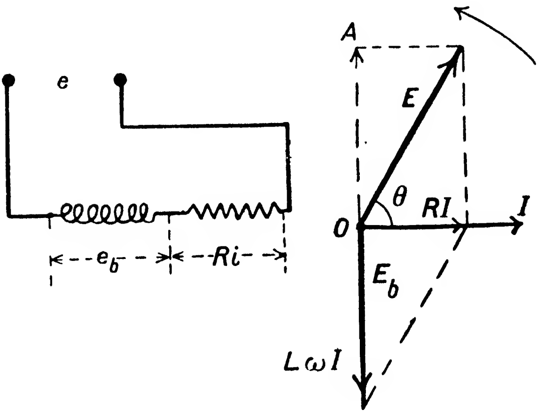

FIG. 40.—A circuit containing resistance and self inductance; and its vector diagram for a sine voltage.

Expressed trigonometrically, when

$$e = E_m \sin \omega t,$$

$$i = \frac{E_m}{\sqrt{R^2 + (L\omega)^2}} \cdot \sin(\omega t - \theta), \quad \text{.....................} (a)$$

in which $\omega = 2\pi f$ and $\tan\theta = \frac{L\omega}{R}$.

Again, since $$I = \frac{E}{\sqrt{R^2 + (L\omega)^2}}$$

and $$\cos\theta = \frac{R}{\sqrt{R^2 + L\omega)^2}},$$

$$I = \frac{E}{R}\cos\theta,$$

or $$\cos\theta = \frac{RI}{E}.$$

The power supplied to the circuit is used up entirely in the resistance part of the circuit and none in the self-inductance part.

$$\therefore \; P = I^2R = (IR)I = EI\cos\theta;$$

cos θ is called the *power factor* of the circuit.

The current flowing in the circuit has two components, one in phase with e, the other 90 degrees behind it, as shown in Fig. 41.

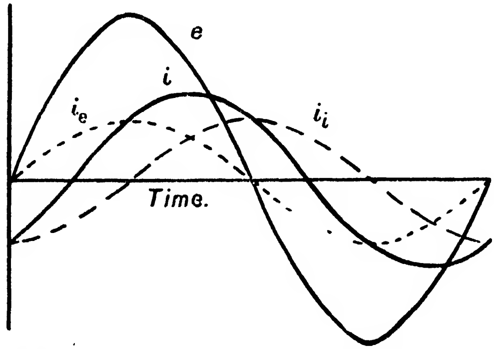

FIG. 41.—Idle and effective components of a current lagging behind a sine voltage.

The former i_e is the effective component, and the latter i_i the idle or wattless component. These two components appear by expanding the value of i in equation (a),

$$i = I_m\{\sin \omega t \cos \theta - \sin \theta \cos \omega t\},$$

$$i = I_m \cos \theta \,.\, \sin \omega t + I_m \sin \theta \,.\, \sin\left(\omega t - \frac{\pi}{2}\right).$$

The ratio of the *idle* to *effective* component is $\tan \theta$, for the maximum value of the first is $I_m \sin \theta$ and of the second $I_m \cos \theta$.

EXAMPLE.—An alternating voltage of 200 at frequency 50, supplies current to a circuit of resistance 4 ohms, and a self inductance of 0·02 henry. Find the input current, the lag, the power factor of the circuit, the wattless current, the effective current, and the power supplied to the circuit. *Ans.* 26·9 amperes; 57·5 degrees; 0·537; 22·7 amperes; 14·44 amperes; 2·9 K.W.

Circuit containing only capacity, C farads.—The circuit is shown in Fig. 42. As e, which is equal to $E_m \sin \omega t$, rises in value from zero above the time axis, it acts in the direction indicated by the arrow, and the current i flows into the plates of the condenser along the same direction until the maximum value of e, namely E_m, is reached.

The current at this time is then zero, and the condenser of capacity C begins to discharge in the opposite direction, that is, i is reversed while e retains its first direction. Also, since Ri is zero, e_b, the back voltage of the condenser, exactly balances e, the applied voltage, at each instant.

The curves of e, e_b, and i have therefore the relative positions shown in Fig. 42, and it will be noted that i *leads* e by 90 degrees.

If q is the quantity of electricity in the condenser at time t, q is equal to Ce, that is,

$$q = CE_m \sin \omega t$$

and $$i = \frac{dq}{dt} = CE_m\omega \cos \omega t;$$

$$\therefore \quad I_m = CE_m\omega,$$

$$I = CE\omega.$$

Also, $$E_b = E = \frac{I}{C\omega}.$$

It will be noted that E is the applied voltage necessary to drive the current I through the impedance

$$\frac{I}{C\omega}.$$

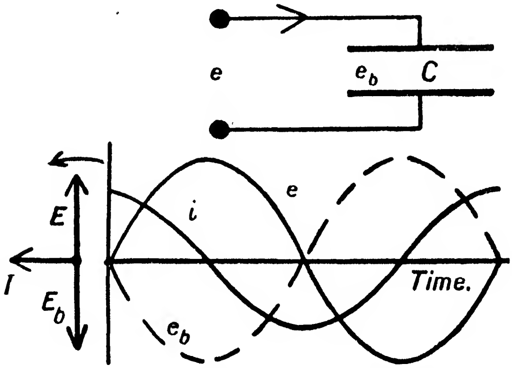

FIG. 42.—A circuit containing capacity; and its vector diagram and graphs for a sine voltage.

In this circuit no energy is consumed; that supplied being stored in the condenser during one quarter of the wave, that is, in time $\frac{I}{4f}$ second, and in the next $\frac{I}{4f}$ second returned to the mains. This is also evident from the fact that there is no I^2R loss.

In practice the condensers used as capacities have a dielectric hysteresis loss which appears in the form of heat. Some condensers, especially if the frequency and voltage are high, become seriously heated when subjected to alternating voltage for some time. It is difficult to build large condensers, which can work with even moderate currents, without considerably heating after a time.

Circuit containing R, L, and C.—This circuit and its vector diagram is shown in Fig. 43. The applied voltage divides itself throughout the circuit, one component balancing the e_b of the self inductance, a second balancing the e_b of the capacity, while the third drives the current i through R.

Now the current is 90 degrees *behind* the component which balances the e_b of self inductance, and the current is 90 degrees *ahead* of the

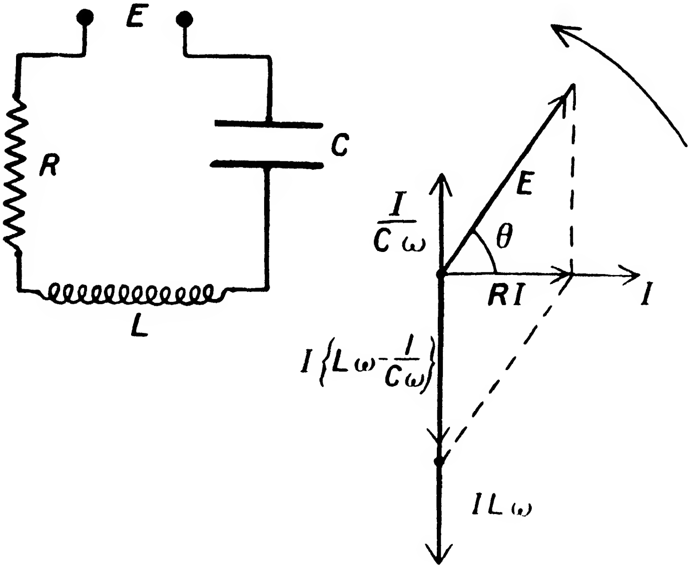

FIG. 43.—A current containing resistance, capacity, and self inductance; and its vector diagram for a sine voltage.

component which balances the e_b of capacity. The vector diagram is therefore as shown in the figure.

From this diagram

$$E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2},$$

$$\tan\theta = \frac{L\omega - \dfrac{1}{C\omega}}{R}.$$

It follows as before that

$$I = \frac{E}{R}\cos\theta.$$

Also, there is no loss of power either in the self inductance or the capacity part of the circuit, so that P, the power supplied to the circuit, is given by

$$P = I^2R = (IR)I = EI\cos\theta.$$

The instantaneous values of voltage and current are

$$e = E_m \sin\omega t,$$

$$i=\frac{E_m}{R}\cos\theta\,.\sin(\omega t-\theta),$$

or

$$i=\frac{E_m}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}\sin(\omega t-\theta).$$

If $L\omega$ is greater than $\frac{1}{C\omega}$, the current i *lags* behind the voltage e, and *leads* the current if less than $\frac{1}{C\omega}$. If $L\omega=\frac{1}{C\omega}$, the current i is in phase with e.

In this last case *resonance* is said to occur in the circuit. This sometimes becomes a source of danger, as the introduction of a capacity or self induction in a circuit may cause an excess current to flow through it.

If the applied voltage contains harmonics, each of these may be regarded as acting independently of the others through the given circuit, and the actual current flowing will be the vector sum of the individual currents.

Resonance rarely occurs with the fundamental current, but more often occurs with the harmonics. The condition for resonance to occur with the nth harmonic is that

$$nL\omega=\frac{1}{nC\omega}.$$

It is, of course, not essential that these terms should be exactly equal to produce excess current in some section of the net work of a supply system. They may be far from equality, and yet produce such current, especially if R is small.

A given network may work well for a certain type of machine, but if a new machine which has certain harmonics in its voltage curve is substituted, resonance may occur in one or more sections of the network.

EXAMPLES.—1. A circuit contains a resistance of 40 ohms, a self inductance of 0·7 henry, a capacity of 20 *microfarads*, and it is fed by an alternating voltage of 200 at frequency 50. Calculate the current, the phase difference, the power factor, and the power supplied to the circuit. Also obtain the voltages a voltmeter would indicate if placed across each of these three parts of the circuit. *Ans.* 2·77 amperes ; 56·3 degrees ; 0·555 ; 308 watts ; 111, 610, and 444 volts.

2. It was found that when a new alternator was put on to a supply system of frequency 50, that the fuses of a certain section were at once blown. This section had a capacity effect of about 0·083 micro-farad, and a self inductance of about 0·72 henry. If the capacity or self induction were removed from the section the trouble ceased. The cause of the breakdown was therefore attributed to the presence of harmonics in the voltage curve of the new machine. Determine which harmonic was the cause of the trouble. *Ans.* The thirteenth harmonic.

Impedances in series.—Let $L_1C_1R_1$ be the self inductance, capacity, and resistance of the first impedance; $L_2C_2R_2$ for the second, and so on. Also let E_1 be the applied voltage across the first impedance, E_2 across the second, and so on.

The phase difference θ_1 for the first impedance may be calculated from

$$\tan\theta_1 = \frac{L_1\omega - \dfrac{1}{C_1\omega}}{R_1},$$

and θ_1 and $\cos\theta_1$ read from tables. Then E_1 may be calculated from

$$E_1 = I\sqrt{R_1^2 + \left(L_1\omega - \frac{1}{C_1\omega}\right)^2} = \frac{R_1I_1}{\cos\theta_1}.$$

This is repeated for the other impedances, and the vector diagram drawn as shown in Fig. 44. The applied voltage E across the whole set of impedances is the resultant of all the components, and θ is the phase difference between current I and applied voltage E.

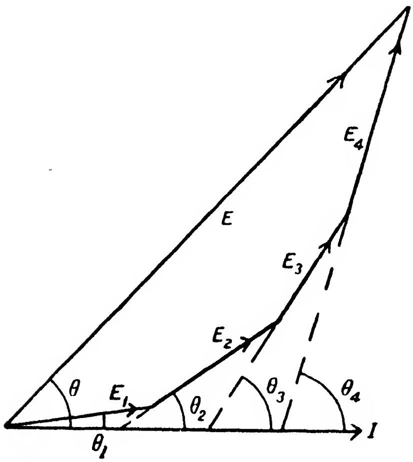

FIG. 44.—Graphical construction for impedances in series.

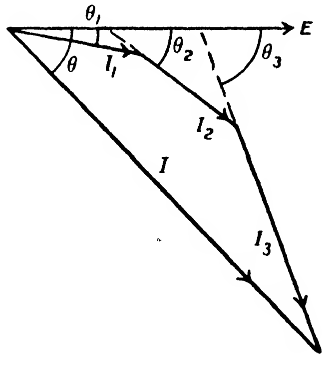

FIG. 45.—Graphical construction for impedances in parallel.

Impedances in parallel.—Let L_1, C_1, and R_1 be the self inductance, capacity, and resistance of the first impedance; L_2, C_2, and R_2 for the next, and so on. The phase difference for each branch or impedance may be calculated from

$$\tan\theta_1 = \frac{L_1\omega - \dfrac{1}{C_1\omega}}{R_1},$$

and θ_1 and $\cos\theta_1$ read from tables.

Then the current is given by

$$I_1 = \frac{E}{R_1} \cos \theta_1,$$

E being the applied voltage across the impedances. This is repeated for the others.

The vector diagram may then be drawn as in Fig. 45, each current being set off at its phase difference with respect to E. The resultant of all these currents is the main current I, and its phase difference θ is obtained from its position with respect to that of E.

I may be more exactly determined by calculation thus,

$$I^2 = \{I_1 \sin \theta_1 + I_2 \sin \theta_2 + \text{etc.}\}^2 + \{I_1 \cos \theta_1 + I_2 \cos \theta_2 + \text{etc.}\}^2,$$

and

$$\tan \theta = \frac{I_1 \sin \theta_1 + I_2 \sin \theta_2 + \text{etc.}}{I_1 \cos \theta_1 + I_2 \cos \theta_2 + \text{etc.}}.$$

If this system is in *series* with a second similar one across the mains of voltage E, voltage E_1 will be supposed to act across the first and E_2 across the second. I, the main current through them, will be the same. Determine as before the value of I, and $\tan \alpha_1$ for the first system. I will be in terms of E_1. Thus

$$E_1 = k_1 I.$$

α_1 is the phase difference between E_1 and I. Repeat for the second system, obtaining α_2, and

$$E_2 = k_2 I.$$

From Fig. 46, it follows that

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos(\alpha_2 - \alpha_1),$$

$$E^2 = I^2 \{k_1^2 + k_2^2 + 2k_1k_2 \cos(\alpha_2 - \alpha_1)\}.$$

I may therefore be calculated, and also E_1 and E_2.

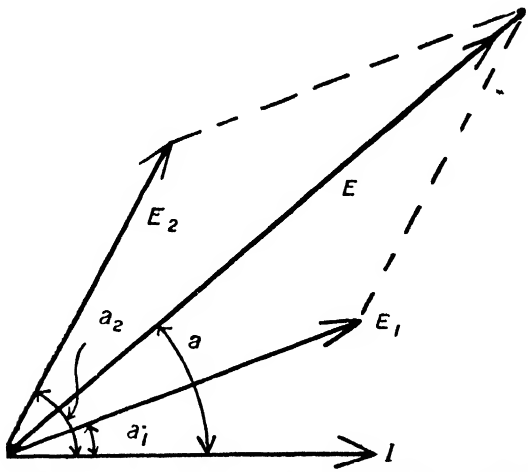

FIG. 46 —The vector diagram for two complex impedances in series.

The current in any branch may now be found; for instance, that for the third branch of the first system is given by

$$I_3 = \frac{E_1}{R_3} \cos \theta_3,$$

and θ_3 has already been calculated.

In a similar manner three such systems may be dealt with, and so on.

It is evident that resonance will occur in any branch of these systems or network if

$$L\omega = \frac{1}{C\omega},$$

and excess current may pass through it, especially if R is small.

EXAMPLES.—1. A circuit containing a self inductance of 0·2 henry, and a capacity of 15 microfarads, is shunted by a non-inductive resistance of 40 ohms in series with a self inductance of 0·12 henry. The voltage across these two parallel circuits is 20 at frequency 50. Calculate the current from the mains and each branch current; also the phase difference between the applied voltage and the main current. *Ans.* 0·289 ampere, 0·134 for the first branch, 0·364 for the second branch, and 23·6 degrees lag.

2. A condenser of capacity 2 microfarads is shunted with a non-inductive resistance of R ohms, and is supplied with alternating voltage at frequency 50. Calculate the angle of lead of the main current with respect to the applied voltage for values of R equal to 500, 1000, 2000, and 5000 ohms. *Ans.* 17·4, 32·2, 51·5, and 72·3 degrees.

3. Two networks are in series across mains of voltage 500 and frequency 50. The first has three branches numbered 1, 2, and 3. The second has two branches 4 and 5. These branches are as follows:

Branch.	R ohms.	L henry.	C microfarads.
1	20	0·4	20
2	30	—	40
3	12	0·2	—
4	8	0·3	50
5	6	0·05	—

Calculate the current in each branch, the voltage across each network, and the main current and its phase difference with respect to the applied voltage. *N.B.*—The final work may be tested for accuracy by comparing I . 500 . cos θ with

$$I_1^2\, 20 + I_2^2\, 30 + I_3^2\, 12 + I_4^2\, 8 + I_5^2\, 6\,;$$

I being the main current, θ the phase difference of the latter with respect to the applied voltage across the networks, and I_1, I_2, etc.,

the respective branch currents. If the work is correct this comparison should show equality.

Ans. 13·87, 6·396, 8·455, 5·15, and 9·58 amperes are the respective branch currents commencing with branch number one. The voltage across the first network is 542 and across the second 161. The main current is 14·66 amperes, and leads the applied voltage of 500 by 24 degrees.

An alternating current in a conductor : skin effect.—A high frequency alternating current passes mainly through the outermost layers of the conductor and very little through the more central part. At the very high frequencies used in wireless telegraphy this effect is much more pronounced, the mere skin of the conductor carrying the bulk of the current. It follows that the resistance of a conductor or coil is greater for alternating currents of sufficiently high frequency than for direct currents.

There are two magnetic fields produced by the passage of a current through a conductor; an internal and an external field. Such a conductor, if straight, may be regarded as a large number of filaments, each parallel to the axis of the conductor, connected in parallel, and fed across their ends with a certain voltage.

If this voltage is direct, then at *make* the current flows along these filaments, but having only to build up the external field around the skin filaments its value will be greater in them than in the axial filaments around which it has to build the internal as well as the external field. In other words, the equivalent self inductance of the inner filaments is greater than the more outer ones, and the growth of the current is more delayed in the former than in the latter.

Thus, the current at a certain instant just after make, is a maximum at the skin, of smaller value at the centre, and of intermediate values between skin and centre.

If the voltage is alternating, and of sufficient frequency, the currents in the central filaments have not time to rise to any appreciable value before the voltage is reversed, and the bulk of the current will pass through the outer layers.

In a conductor of high magnetic permeability such as iron, the *internal* field will be much stronger and the skin effect consequently much increased.

Also, the greater the diameter of the conductor the greater will be the internal field in comparison with the external field, for the latter will be smaller while the former will be unchanged for a given conductor. This is proved as follows:

Consider a long straight conductor of length l cms., carrying a current I C.G.S. units.

If a is the radius of the conductor, the value of the *internal* field at point x cms. from the axis is, by equation (9), page 8,

$$\frac{2\mathrm{I}x}{a^2}.$$

The total internal field is therefore

$$2Il\int_0^a \frac{x}{a^2}dx = Il,$$

neglecting the end values of the conductor. That is, the number of lines of magnetic force encircling the axis inside the material of the conductor is I per axial cm. of its length. This number is therefore independent of its radius.

A conductor of finite length has an external field of finite value; this is evident from equation (13), page 13, α_1 and α_2 becoming practically zero when the point is taken sufficiently far enough from the conductor. That is, the magnetic field outside a certain distance becomes practically zero.

As this external field is summed up or integrated between limits, one of which is the radius of the conductor, it follows that the larger the radius for the same length and current the smaller will be the external field. Also, though this radius limit is small it is important, because the external field is the strongest just outside the conductor. Thus the greater the diameter the greater the skin effect.

If the wire is of small radius the external field is very large in comparison with the internal field and the skin effect is much smaller.

Specific resistance plays a part also in skin action. As the time constant of a filament is its self inductance, divided by its resistance, the current would penetrate more quickly to the interior of the conductor the greater its specific resistance.

Thus the skin effect will increase with the frequency, diameter, and magnetic permeability, but will decrease with increase of the specific resistance of the material.

For low frequencies not exceeding about 50, and with conductors of diameter not exceeding about one centimetre, this effect is very small. It will be appreciable if harmonics are present with the fundamental.

At high frequencies the effect is large; the resistance of copper wire 2 millimetres in diameter to an alternating current of frequency a million has been found to be about 8 times that for a direct current, and for the same frequency a copper wire of 0·04 millimetre in diameter has a similar ratio of 1·5.

Eddy current action.—The relative motion of lines of magnetic force and conducting materials causes, in general, the production of eddy currents in the latter. If the material is magnetic as well as conductive, magnetic hysteresis will also occur.

A material threaded by an alternating magnetic flux has eddy currents produced in it. A conductor moving in a *uniform* magnetic field has no eddy currents produced in it, but their production takes place when the field varies in strength.

These eddy currents produce heating in the material and cause a loss of electrical energy. An expression for this loss may be determined as follows. Consider a plate of thickness t cms. subject to an

alternating flux density of maximum value B_m and frequency f, acting at right angles to its cross section.

This plate is shown in Fig. 47, and it will be assumed that t is very small compared with its length l. Consider an elemental rectangular shell *abcd* of the plate. This shell may be regarded as a closed coil of one turn having a cross-sectional area of $a\,dx$ and length $2l$; *ab* and *dc* being negligible compared with *bc*.

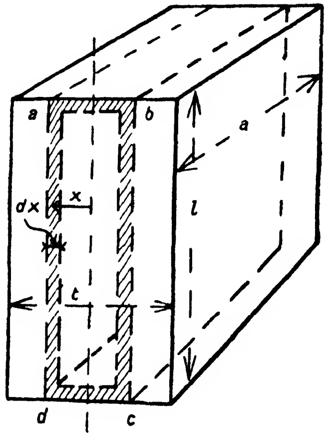

FIG. 47.—Eddy currents in a metal block.

The resistance of this coil is therefore equal to R, where

$$R = \frac{2\sigma l}{a\,dx},$$

and E the voltage induced in the coil by the alternating flux is, by equation (*a*), page 49, given by

$$E = 4{\cdot}44 \times 2xlB_m f 10^{-8}.$$

The current flowing in the coil is $\frac{E}{R}$ and the electrical power representing the eddy loss in the coil is

$$\left(\frac{E}{R}\right)^2 R = \frac{E^2}{R} \text{ watts};$$

this is equal to

$$\frac{8{\cdot}88^2 l^2 B_m^2 f^2 10^{-16} \times x^2}{\dfrac{2\sigma l}{a\,dx}},$$

$$39{\cdot}5 \times 10^{-16} \frac{la}{\sigma} B_m^2 f^2 \,.\, x^2\,dx = K \,.\, x^2\,dx.$$

Summing up for the total loss P_e, gives

$$P_e = K \int_0^{\frac{t}{2}} x^2\,dx = \frac{Kt^3}{24};$$

$$\therefore\ P_e = \frac{1{\cdot}65 \times 10^{-16}}{\sigma} \times B_m^2 f^2 t^2 \times \text{volume},$$

volume being equal to *lat*.

The eddy-current loss per *cubic centimetre* is therefore

$$\frac{1{\cdot}65 \times 10^{-16}}{\sigma} (B_m f t)^2 \text{ watts}.$$

This loss is reduced by laminating the sheets of metal or con-

ductors. For consider a *solid* cube of one cm. edge and an equal cube of the same material built up of n laminations. In the first case the eddy loss is proportional to t^2, that is, may be represented by unity. For the same B_m and f in the second case, the loss in *each* lamination of the cube is proportional to $\left(\frac{t}{n}\right)^2$, and the total loss is represented by

$$n \cdot \frac{t^2}{n^2}, \text{ that is, by } \frac{1}{n}.$$

Thus neglecting the spacing between the laminations, the ratio of the eddy loss in the laminated material, to that in the solid is $\frac{1}{n}$; therefore the thinner the lamination the smaller the loss.

In practice these laminations would be lightly insulated from each other by varnish, enamel, or thin paper. The length of metal of the core plates for electrical machines along the direction at right angles to the plane of the insulation, is about 0·9 times the gross length.

Eddy action in conductors.—If a changing magnetic flux cuts across conductors, eddy currents are produced in them. In Fig. 48 is shown part of a conductor, of which the portion AB is supposed to be threaded with a varying flux whose direction is at right angles to the plane of the paper. Eddy currents are produced and flow as indicated by the dotted rectangle.

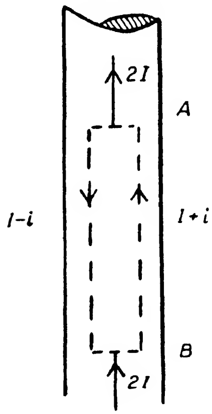

Fig. 48.—Eddy currents in a conductor.

If the conductor carries a current 2I and the eddy currents are represented by a current i, then on one side of the conductor between A and B is a current $I - i$ and a current $I + i$ on the other side.

The copper loss or heating loss at a given instant for the part AB is

$$(I+i)^2 \cdot 2r + (I-i)^2 \cdot 2r = (I^2+i^2)\,4r,$$

in which r is the resistance of the conductor between A and B; it is assumed that one half of the cross section carries $I + i$ and the other $I - i$.

Now this loss would be $(2I)^2 r = 4I^2 r$, if the eddy action was absent. Therefore, the latter increases the copper loss, and in effect is equivalent to an increase of resistance of the conductor.

Conductors in electrical machines are often cut by such fluxes, and the eddy current action may increase the copper loss in the conductors by 20 to 40 per cent. in some cases, unless the conductors are suitably laminated.

Fig. 49 shows a transformer in which the low tension coil is nearest the core, and the high tension coil on the outside, which is generally the case. When current flows in the coils the main flux passes right

round the laminated iron core, but the leakage fluxes pass out of the core as shown, and return across the turns of the coils. Eddy currents

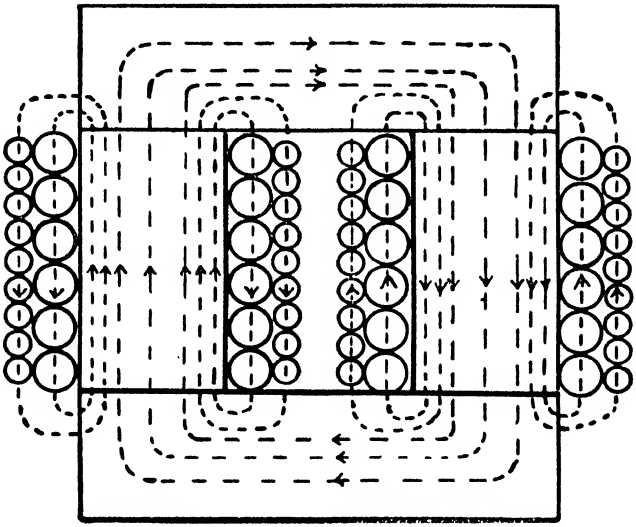

FIG. 49.—The main and leakage fluxes of an *unloaded* transformer.

are therefore produced in the turns, and the copper loss of the transformer is increased.

To lessen this loss the coils may be made up of fine wires or copper strips in parallel; the strips or wires being lightly insulated from each other.

This eddy action will increase with the flux strength, the frequency, and the diameter, but decrease with increase of specific resistance.

CHAPTER IV.

ALTERNATING-CURRENT PRINCIPLES—Continued.

In this chapter methods of measurement, the arrangement of three-phase circuits, and the analysis of wave forms will be considered.

Determination of self inductance and power factor.—The self inductance L of part or the whole of an inductive circuit may be determined from measurements of R its resistance, I the current, E the voltage across its ends, and f the frequency of the current.

R is best obtained by sending a *direct* current through the circuit, of about the same value as the alternating current to be used in the test, and measuring the current and voltage of the circuit. The latter divided by the former gives the resistance.

From readings taken when alternating current is used in the circuit, and

$$I = \frac{E}{\sqrt{R^2 + (L \cdot 2\pi f)^2}},$$

or

$$L = \frac{I}{2\pi f I}\sqrt{E^2 - (IR)^2},$$

L may be calculated.

The power supplied to the circuit is I^2R watts, and is also expressed by $EI\cos\theta$. Therefore the power factor of the circuit is given by

$$\cos\theta = \frac{RI}{E}.$$

This method of obtaining the power factor is satisfactory, if R can be measured accurately. In cases where eddy action and skin effect are present in the conductors of the circuit, R as found by the direct-current method would be too small, and the power factor would work out less than its true value.

Also, if the inductive part of the circuit had an iron core, the power supplied to the circuit would be

$$I^2R + P_0,$$

P_0 being the iron losses in the core. It follows that, on account of these losses, the power factor will again work out too small by the method under consideration.

The three-voltmeter method of measuring power and power factor in a given inductive circuit.—This method is an application of the vector law of voltages for a circuit containing impedances in series. It is an advantage to include an ammeter in the circuit. The inductive circuit AB (Fig. 50) under test is shown as part of another circuit, and it is required to find the power used by it, when supplied with a given current, and also its power factor.

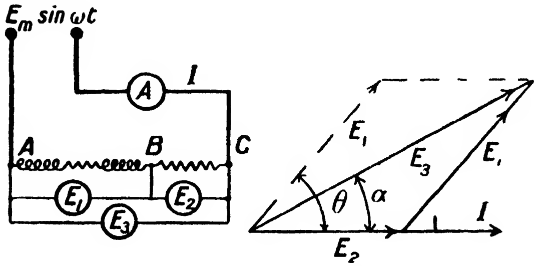

FIG. 50.—Three-voltmeter arrangement for measuring power and power factor on a single-phase circuit.

To do this AB is placed in series with a non-inductive resistance BC, and the voltmeters are arranged as shown. Instead of using three, a single one, fitted with leads having metal contact pieces, may be applied to the three parts of the circuit and their voltages quickly obtained.

The vector diagram is also shown, and the power factor *cos θ* may readily be found from it. The power consumed by AB is

$$IE_1 \cos\theta.$$

The power factor may be more accurately calculated from the relation

$$\cos\theta = \frac{E_3^2 - E_2^2 - E_1^2}{2E_1E_2}.$$

In this test accurate instruments, especially the voltmeters, should be used and carefully read, as an error in one or more of the voltages may considerably affect the value of *cos θ*. For example, the *true* readings for such a test were

$$E_3\ 110, \quad E_2\ 40, \quad \text{and} \quad E_1\ 80.$$

In taking the readings, however, an error was made, and the values recorded were

$$E_3\ 108, \quad E_2\ 40, \quad \text{and} \quad E_1\ 80.$$

The true power factor of AB from the first set of readings is

$$\cos\theta = 0{\cdot}64,$$

and from the second set $\cos\theta = 0{\cdot}57.$

Thus an error of about 2 per cent. in the reading of one of the voltmeters causes an error of about 11 per cent. in the value of the power factor. This cause may largely account for the discrepancy which sometimes occurs between the wattmeter and three-voltmeter methods of determining power factor.

The power factor $\cos\alpha$ of the circuit AC may also be calculated from

$$\cos\alpha = \frac{E_2^2 + E_3^2 - E_1^2}{2E_2E_3},$$

and the power supplied to AC is

$$E_3 I \cos\alpha.$$

EXAMPLE.—The true readings for a three-voltmeter test were

$$E_1\ 220, \quad E_2\ 150, \quad \text{and} \quad E_3\ 320;$$

but the readings recorded were

$$E_1\ 220, \quad E_2\ 155, \quad \text{and} \quad E_3\ 310.$$

Calculate the true power factor of AB and that from the recorded readings. *Ans.* 0·477 and 0·347.

The dynamometer type of wattmeter.—This very general type of wattmeter is much used in the measurement of power and power factor. It consists of one or more fixed coils for carrying the current of the supply power, and a movable coil, in series with a high non-inductive resistance, to be placed across the feeding voltage terminals.

Both the current and volt coils may be connected to the secondary side of step-down current transformers, for measuring large currents and high voltages.

The volt coil is brought to the zero position by means of a spring and torsion head, and the reading on the torsion scale is proportional to the magnetic force between the two coils.

In other cases the volt coil moves against torsion, and the needle fixed to this coil moves over a calibrated scale.

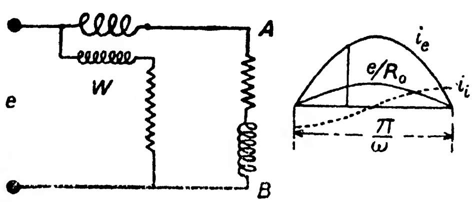

FIG. 51.—Wattmeter arrangement for measuring power on a single-phase circuit.

The method of connecting the wattmeter to read P, the power supplied to an inductive circuit AB, is shown in Fig. 51. If an ammeter is placed in the circuit and a voltmeter across its terminals,

the power factor of AB is given by $\frac{P}{IE}$; IE being the product of the ammeter and voltmeter readings.

Consider the case of a sinusoidal current and voltage. The force exerted between the movable and fixed coils of the wattmeter W at a given instant is proportional to the product of the current in the former and the effective component of the current in the fixed coil.

If $$e = E_m \sin \omega t,$$

then $$i = I_m \sin (\omega t - \theta),$$

and the effective component of i, which is in phase with e, is

$$i_e = I_m \cos \theta \sin \omega t.$$

The idle, wattless, or quadrature component i_i in the current coil of W, and the current in the volt coil, have no average mutual force between them, because during one quarter of a period the turning effort is in one direction and opposite in the next quarter.

The instantaneous value of the current in the volt coil of W is $\frac{e}{R_0}$, the resistance of the volt coil circuit being R_0. This current, on account of R_0 being very large in comparison with the self inductance of the volt coil multiplied by the frequency, is practically in phase with e.

The average force throughout a half wave is therefore proportional to

$$\frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} \frac{E_m I_m}{R_0} \sin^2 \omega t \,.\, \cos \theta \, dt = \frac{\omega}{\pi}\frac{E_m I_m}{R_0} \cos \theta \left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right]_0^{\frac{\pi}{\omega}}$$

$$= \frac{1}{R_0}\frac{E_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta = \frac{EI\cos\theta}{R_0},$$

and EI $\cos \theta$ is the power supplied to the circuit.

Now, the reading of the instrument is proportional to the average force between its coils, so that the reading multiplied by a constant gives the power supplied to the circuit.

Such a wattmeter is, however, not limited to the case of sinusoidal values of current and voltage. Its readings are independent of the wave form and also the frequency of the supply mains. The treatment of the general case is as follows:

Let i and e be the instantaneous values of the current and voltage of the inductive circuit whose supply power is to be measured by the wattmeter. The instantaneous power is ie and the total power supplied to the circuit is

$$f \,.\, \int_0^T ei \, dt,$$

T being the period of the wave of current or voltage and f the

frequency, and $ei\,dt$ the electrical energy supplied to the circuit during time dt.

Now i passes through the current coil of the wattmeter, and $\frac{e}{R_0}$ through the volt coil, and the instantaneous mutual force between the coils is $k.\frac{ei}{R_0}$, 'in which k is a constant dependent on the constructional values of the coils.

The average force between the coils throughout a period, is the average force between the coils, and is equal to

$$\frac{1}{T}\int_0^T \frac{kei\,dt}{R_0}.$$

Also, as f is the number of complete waves per second,

$$\frac{1}{f}=T,$$

so that the force between the coils is

$$\frac{k}{R_0}f\int_0^T ei\,dt,$$

that is, $\frac{k}{R_0}$ times the power supplied to the circuit.

Therefore, whatever the wave form, provided it is periodic, and whatever the frequency, the wattmeter reading multiplied by a contant gives the power supplied to the circuit.

A limitation must in practice be placed upon the frequency, because at high frequencies, skin and eddy action become important; also the self induction of the volt coil multiplied by a high frequency will produce a lag of the current in it, behind the voltage operating across the ends of the volt-coil circuit.

It has been assumed that the volt-coil circuit has a negligible self inductance, that is, the current in this circuit is in phase with the voltage across its ends. This assumption is practically justifiable, provided the resistance of the non-inductive part of the volt-coil circuit is sufficiently high.

Consider the case when the current in the volt coil lags θ_0 behind the voltage, and let L_0 be its self inductance and R_0 the resistance of the volt-coil circuit. Thus, in both coils of W the current lags behind the voltage of the circuit.

Let $A=\sqrt{R_0^2+(L_0\omega)^2}$, so that the current i_0 in the volt coil is given by

$$i_0=\frac{E_m}{A}\sin(\omega t-\theta_0),$$

and in the current coil by

$$i=I_m\sin(\omega t-\theta).$$

Now, if time is reckoned $\frac{\theta_0}{\omega}$ seconds later, that is, from C (Fig. 52) instead of from O, OC in time measure being $\frac{\theta_0}{\omega}$ second, these equations become

$$i_0 = \frac{E_m}{A} \sin \omega t,$$

$$i = I_m \sin \{\omega t - (\theta - \theta_0)\}.$$

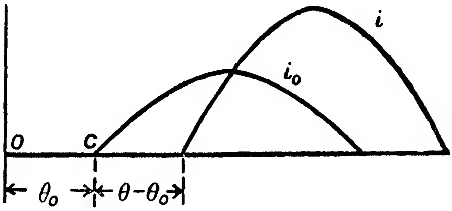

FIG. 52.—The current in the volt coil of a wattmeter lagging behind the voltage across it.

The current i has two components, one of which only is effective for producing turning effort in conjunction with i_0, namely the component in phase with i_0 and of value

$$I_m \cos (\theta - \theta_0) \sin \omega t.$$

The average mutual force on the two coils is therefore proportional to

$$EI \frac{\cos (\theta - \theta_0)}{A},$$

or, since $$\cos \theta_0 = \frac{R_0}{\sqrt{R_0^2 + (L_0\omega)^2}} = \frac{R_0}{A},$$

to $$\frac{EI}{R_0} \cos \theta_0 \cos (\theta - \theta_0),$$

which is proportional to the torsion scale-reading.

The *actual power*, which is $EI \cos \theta$, is therefore represented by the *reading* of W multiplied by a number which is the product of two coefficients, one of which is constant and the other of value

$$\frac{\cos \theta}{\cos \theta_0 \cos (\theta - \theta_0)} = \frac{1}{\cos \theta_0 + \sin \theta_0 \tan \theta} \times \frac{1}{\cos \theta_0}.$$

In general $\cos \theta_0$ may be taken as unity, as the cosine of even a few degrees is very nearly equal to one.

This variable coefficient K may therefore be taken as of value

$$\frac{1}{1 + \sin \theta_0 \tan \theta},$$

and the reading of W is therefore dependent upon the power factor *cos* θ of the circuit under test.

Values of K for different phase differences θ, or different power factors, are worked out for the case of θ_0 being 2 *degrees*, in the following table:

θ	20	30	40	60	70	80
$\cos\theta$	0·94	0·865	0·765	0·5	0·342	0·174
K	0·987	0·98	0·97	0·943	0·91	0·824

and the graph of $\cos\theta$ and K is shown in Fig. 53.

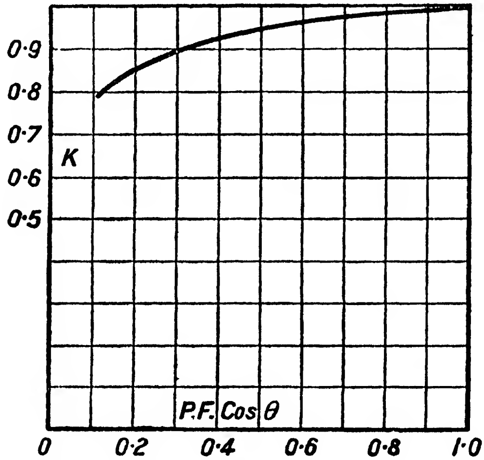

FIG. 53.—The correction coefficient for a wattmeter when using it on loads of different power factors.

Thus, if the wattmeter has been calibrated for unity-power factor, it will read too high on circuits of lower power factors. For instance, a reading on the wattmeter of 500 watts would be indicated for a circuit using only $500 \times 0{\cdot}824 = 412$ watts at a power factor of 0·174, if θ_0 of the instrument was *two* degrees.

Again, since $$\tan\theta_0 = \frac{L_0\omega}{R_0} = \frac{L_0 \cdot 2\pi f}{R_0},$$

the value of θ_0 will depend upon the frequency of the alternating current, and the higher the frequency the smaller will K become. Thus, if the wattmeter is calibrated for unity-power factor and a frequency of 50, it will have a double error, each causing the reading to be too high if the instrument is used for higher frequencies and lower power factors.

It follows that, in constructing this type of wattmeter, the value of $\frac{L_0}{R_0}$ should be made as small as possible, consistent with the necessary

sensitiveness of the instrument, and also, that when calibrated, the frequency and power factor of the load should be recorded.

The power factor of certain alternating-current machines may range from 0·2 at no load to 0·9 or higher at full load. That of an electrical circuit from unity down to a very small value. The input power of a transformer or induction motor at low power factors may be appreciably over-rated by a badly designed wattmeter of the type under consideration, if calibrated by direct current, or alternating non-inductive loads, or at frequencies lower than that used in the transformer.

A capacity effect is also produced in the volt-coil circuit due to the non-inductive resistance, the wire of which doubled back on itself, acting as a condenser shunted with its own resistance. This effect may be reduced to an inappreciable extent by subdividing the coil into a sufficient number of sections.

A small error is involved by the current coil being included in the load circuit, but the small copper loss due to it, if appreciable, may be subtracted from the wattmeter reading.

When used, the instrument should be kept well away from metal work.

Convenient wattmeter sets are constructed containing an ammeter, voltmeter, and wattmeter, so that the power may be read by the latter, and the power factor calculated by dividing the wattmeter reading by the product of the readings of the ammeter and voltmeter.

The production of single-phase currents.—The simplest type of alternator has already been considered. A more representative type is shown in Fig. 54. It is a four-pole single-phase alternator, and the number of coils in the winding is taken as one half the number of poles.

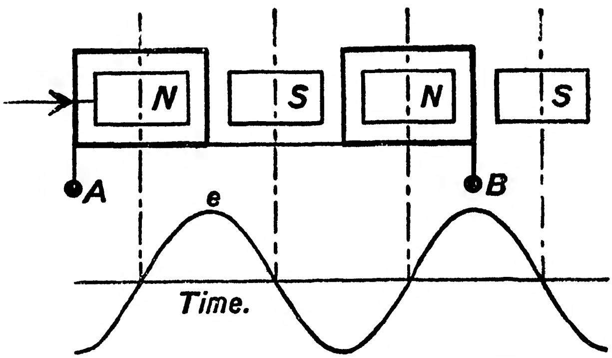

Fig. 54.—A four-pole single-phase alternator and its voltage curve.

As is usually the case the poles revolve and the winding is fixed. Suppose the poles move in the direction of the arrow; then, by applying Lenz's rule for induced currents or voltages, an alternating voltage is found to act across AB.

The voltage curve is shown, and it will be noticed that there is an alternation of voltage every time the poles move through a distance equal to twice the pole pitch. That is, if f is the frequency of alternation, or the number of waves produced per second, p the number of poles, and n the revolutions per second of the poles,

$$f=\frac{np}{2}.$$

The value of the voltage induced in the winding is proportional to the strength of the poles, the number of turns of the winding, and the speed of the poles. By using as many coils as poles, the voltage will be double of that produced when only half of them are used, but f will be the same in both cases.

The production of three-phase currents.—Three sets of coils are used in this case, as shown in Fig. 55, and the voltage curves are

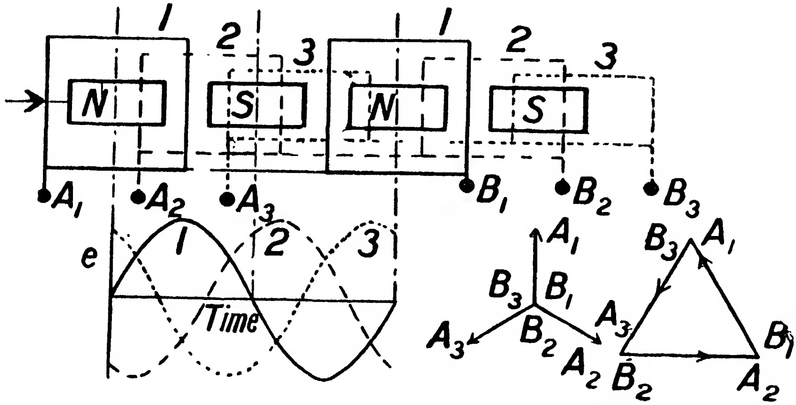

FIG. 55.—A four-pole three-phase alternator and its voltage curves.

displaced 120 electrical degrees from each other. Thus, if the equation of the first phase winding A_1B_1 is

$$e_1=E_m \sin \omega t,$$

that of the second phase A_2B_2 is

$$e_2=E_m \sin (\omega t-120),$$

and that of the third phase A_3B_3 is

$$e_3=E_m \sin (\omega t-240).$$

If at a given instant e_1 is acting from B_1 to A_1, then at that instant e_2 is acting from B_2 to A_2 and e_3 from B_3 to A_3. The frequency f is also given by

$$f=\frac{np}{2}.$$

The terminals A_1B_1 and A_2B_2 and A_3B_3 may be kept independent, and each pair may be used to supply single-phase currents; this would require *six* cables.

It is usual to connect the phases as *star* or Y; or *mesh* or delta. These two methods of connecting are shown in the figure, and the arrows indicate the directions of the induced voltages in the phases at a given instant. These arrows must always be associated with the trigonometrical values of the voltages e_1, e_2, and e_3 already stated.

The voltage across A_1 and A_2 in the case of the star, is $e_1 - e_2$ in the direction from A_2 to A_1, that is, equal to

$$E_m\{\sin \omega t - \sin(\omega t - 120)\} = \sqrt{3}E_m \cos(\omega t - 60).$$

Across A_2 and A_3 it is $\sqrt{3}E_m \cos(\omega t - 180)$ in the direction from A_3 to A_2, and across A_3 and A_1 of value $-\sqrt{3}E_m \cos(\omega t - 120)$ from A_1 to A_3.

These voltages taken in order may also be expressed by their equivalent forms

$$\sqrt{3}E_m \sin(\omega t + 30),$$

$$\sqrt{3}E_m \sin(\omega t - 90),$$

$$\sqrt{3}E_m \sin(\omega t - 210) \quad \text{or} \quad -\sqrt{3}E_m \sin(\omega t - 30),$$

and the phase differences are seen to be 120 degrees. Thus the star arrangement is equivalent to the mesh and *vice versa*. This is illustrated in Fig. 56, in which each diagram shows the direction and

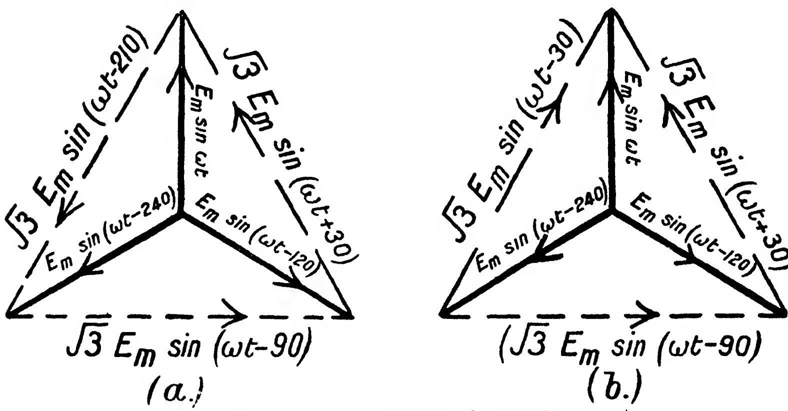

FIG. 56.—Star and equivalent mesh connections.

values of the voltages of the star and equivalent mesh connections. Diagram (*b*) is equivalent to that of (*a*).

In the case of the mesh (Fig. 55) the voltage across A_1A_2 is that of the phase. It should be noted that the voltage across B_1A_1 in the direction shown is $E_m \sin \omega t$, due to its own phase, and it is also

$E_m \sin \omega t$, due to the voltages in the other two phases and in the same direction. For

$$E_m \sin(\omega t - 240) + E_m \sin(\omega t - 120) = -E_m \sin \omega t,$$

and the direction is from B_3 to A_2 through B_2, but since the sign is minus, $E_m \sin \omega t$ acts from A_2 to B_3 through B_2. Similarly for the other phases.

The total voltages acting through the mesh is also zero, for

$$E_m \{\sin \omega t + \sin(\omega t - 120) + \sin(\omega t - 240)\} = 0.$$

Power formula for a balanced three-phase load.—A balanced three-phase load is one which has the three phases equally loaded and of the same power factor. Consider the case of a *star*-connected generator and a star-connected load. This is shown in Fig. 57.

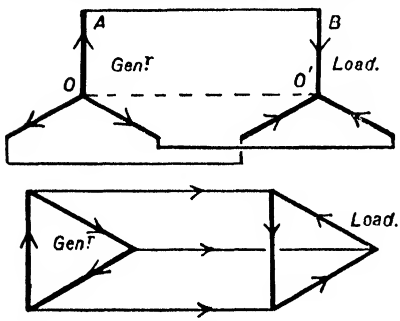

Fig. 57.—A three-phase generator on balanced loads, star and mesh connected.

Suppose OO′ are joined by a wire. From symmetry, each phase of the generator will send current along this connection proportional to its voltage, and the direction of each of the three currents will be from O′ to O. The resultant current along O′O will therefore be

$$kE_m \{\sin \omega t + \sin(\omega t - 120) + \sin(\omega t - 240)\} = 0.$$

Thus the potential at O is the same as at O′, and joining O to O′ with a cable of large or negligible resistance makes no difference in the distribution of the currents through the system. In other words, each of the three circuits such as OABO′ is independent of each other, and may be regarded as a single-phase circuit having a load BO′.

Let I_l be the line current, E_1 the terminal voltage across OA, that is BO′, and $\cos\theta$ the power factor of the load BO′. Then the power supplied to this load is

$$I_l E_1 \cos \theta.$$

Therefore the total power supplied to the whole load is

$$3 I_l E_1 \cos \theta = \sqrt{3} E_l I_l \cos \theta,$$

the line voltage being $\sqrt{3}$ times the phase-terminal voltage.

Consider now the case of the mesh load, fed by a mesh-connected generator (Fig. 57). In this case there are also three circuits, which may be regarded as independent of each other. Fig. 58 shows these circuits separated from each other.

If a_3 is joined to b_2 and A_3 to B_2, the current distribution in the

rectangle $A_1a_1b_1B_1$ will be the same whether the side a_1A_1 is in contact with b_3B_3, and b_1B_1 with a_2A_2, or separated as shown. This follows from the fact that the potential difference across b_3a_2 at any instant is equal to that across a_1b_1, and in the same direction.

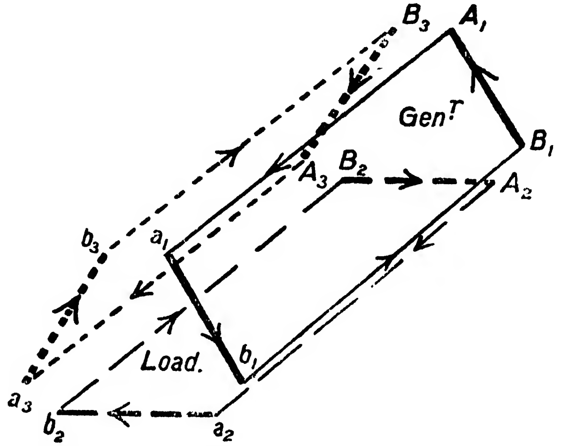

FIG. 58.—A mesh-connected system, equivalent to three independent circuits.

The current along A_1a_1 and b_1B_1 is represented by $kE_m \sin \omega t$. That along A_2a_2 and b_2B_2 by $kE_m \sin(\omega t - 120)$, and that along A_3a_3 and b_3B_3 by $kE_m \sin(\omega t - 240)$. Therefore, if a_1A_1 and b_3B_3 are replaced by one cable the current in it will be

$$kE_m\{\sin \omega t - \sin(\omega t - 240)\} = \sqrt{3}kE_m \cos(\omega t - 120).$$

By using a single cable instead of two, the cross-sectional area of the copper required is only $\frac{\sqrt{3}}{2} = 0{\cdot}866$ times that needed for two independent cables.

The *line* current I_l is therefore $\sqrt{3}$ times the *phase* current for a *mesh* load. If *cos θ* is the power factor of one-phase load, the total power supplied to the mesh load will be

$$3I_1E_l \cos\theta = \sqrt{3}I_lE_l \cos\theta.$$

Since a star load is equivalent to a mesh load, and *vice versa*, the same expression for the power P supplied to the three-phase balanced load is also true whether the mesh load is fed by a star-connected generator, as in Fig. 59, or *vice versa*. The general expression for a balanced three-phase load is, therefore,

$$P = \sqrt{3}E_lI_l \cos\theta.$$

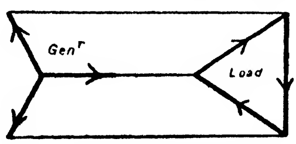

FIG. 59.—A three-phase generator on a mesh-connected load.

EXAMPLE.—The full-load input of a three-phase induction motor is 40 kilowatts, the full-load current 74 amperes, and the line voltage is 346. Calculate the full-load power factor. *Ans.* 0·9.

Methods of measuring power on a three-phase system.—The simplest case is that of a three-phase star load, the latter being non-inductive, and the star or neutral point accessible. By using an ammeter in each line, and a voltmeter across each line and the star point, or a single voltmeter which by switches may be placed across each limb of the load in turn, the power taken by each phase may be measured. The total power will be given by

$$P = E_1I_1 + E_2I_2 + E_3I_3.$$

The load may be either balanced or unbalanced.

If the load is balanced and inductive the method is shown in Fig. 60. Three times the reading of the wattmeter will give P the total supply power. If an ammeter is placed in the line and a voltmeter across the line and star point, the power factor may be obtained from

$$P = 3EI_l \cos\theta,$$

E being the phase voltage read on the voltmeter and I_l the line current read on the ammeter.

If the load is unbalanced, three wattmeters may be used. In this case the current coil of the first is placed in one supply cable, and its volt coil across this cable and the neutral point. The second and third wattmeters are similarly arranged for the second and third cables. The supply power is the sum of the readings of the three wattmeters.

In the case of the star point not being accessible or absent, as in the case of a mesh load, the input power may be measured by one wattmeter if the load is balanced, but two wattmeters are required if the load is unbalanced.

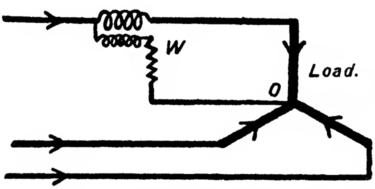

FIG. 60.—An arrangement for power measurement on a star-connected load.

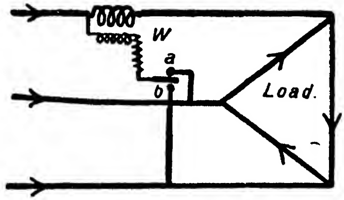

FIG. 61.—The one-wattmeter method of measuring power on a mesh-connected load.

One-wattmeter method of measuring power supplied to a balanced load ; neutral inaccessible.—The method is illustrated in Fig. 61, and applies either to a balanced star or mesh load. The current coil is placed in one line and the volt coil connected in turn to each of the other lines. Then the sum of the two readings of the wattmeter will give the power supplied to the load.

Consider the case of star mains feeding a star-connected balanced load, as shown in Fig. 62, and suppose the current coil of the watt-

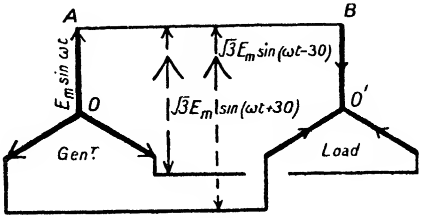

FIG. 62.—A star load with inaccessible neutral.

meter is in line AB. This coil will therefore carry a current

$$I_m \sin(\omega t - \theta),$$

produced entirely by $E_m \sin \omega t$, as circuit OABO′ is quite independent of the rest of the system.

The line voltages reproduced from Fig. 56, *diagram* (*b*), are shown, and these operate in turn across the volt coil of the wattmeter on adjusting the switch of Fig. 61.

In the first case, the voltage across the ends of the volt circuit of the instrument is $\sqrt{3}E_m \sin(\omega t + 30)$, and the current coil carries $I_m \sin(\omega t - \theta)$. Therefore P_1, the reading of the wattmeter, is given by

$$P_1 = \sqrt{3}EI_l \cos(\theta + 30),$$

E and I_l being the R.M.S. values of the phase voltage and line current. Thus

$$P_1 = E_l I_l \cos(\theta + 30).$$

Similarly, for the second case,

$$P_2 = E_l I_l \cos(\theta - 30).$$

The sum of the two readings will therefore be

$$I_l E_l' \{\cos(\theta + 30) + \cos(\theta - 30)\} = 2 \cos\theta \cos 30 I_l E_l$$
$$= \sqrt{3} I_l E_l \cos\theta,$$

and this has been shown to be the value of the power supplied to a three-phase balanced load.

The two readings of the wattmeter are proportional to

$$\frac{\cos(\theta + 30)}{\cos(\theta - 30)}.$$

When θ is 30 degrees, the first reading P_1 is *one half* that of the second, P_2, and when θ is zero, that is, the load is non-inductive, the values P_1 and P_2 are equal.

By expanding this ratio,

$$\frac{P_1}{P_2}=\frac{\sqrt{3}-\tan\theta}{\sqrt{3}+\tan\theta},$$

from which $$\tan\theta=\sqrt{3}\times\frac{P_2-P_1}{P_2+P_1}. \qquad (a)$$

The power factor of the load may either be calculated from

$$P_1+P_2=\sqrt{3}\,I_l E_l \cos\theta,$$

or by means of equation (a).

The two-wattmeter method of measuring power supplied to a three-phase load.—This method has the great advantage of being independent of the wave form, frequency, and the nature of the load, whether balanced or unbalanced. It is of general application to all three-phase loads supplied with periodic currents.

In Fig. 63 is shown the connections for a three-phase load. The

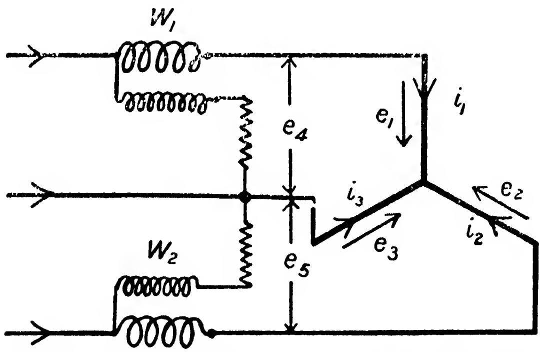

FIG. 63.—The two-wattmeter method of measuring power supplied to a Star-connected load.

directions of the instantaneous currents and phase voltages are indicated by the arrows.

The total instantaneous power used by the load is given by

$$p=e_1i_1+e_2i_2+e_3i_3,$$

and the sum of the instantaneous currents is zero,

$$i_1+i_2+i_3=0.$$

Eliminating i_3, as it does not pass through the coils of the wattmeters,

$$p=i_1(e_1-e_3)+i_2(e_2-e_3)$$
$$=i_1e_4+i_2e_5,$$

$$\therefore\ \frac{1}{T}\int_0^1 p\,dt=\frac{1}{T}\int_0^T i_1e_4\,dt+\frac{1}{T}\int_0^T i_2e_5\,dt,$$

$$P=P_1+P_2,$$

that is, the total power is equal to the sum of the readings of the two wattmeters.

If one of the wattmeters indicates a negative reading, this must be subtracted from the other, to give P. In this case, if the wattmeter's scale is unidirectional, the connections of the volt-coil circuit should be reversed.

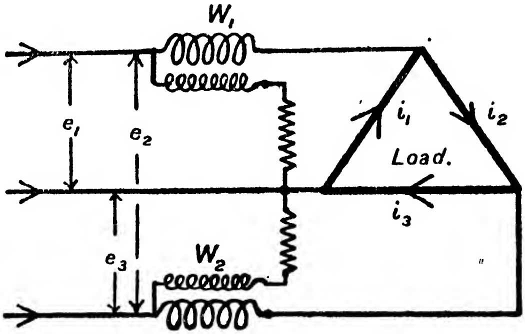

FIG. 64.—The two-wattmeter method of measuring power supplied to a mesh-connected load.

Similarly for the case of a *mesh* load (Fig. 64),

$$p = e_1 i_1 + e_2 i_2 + e_3 i_3,$$

and

$$e_1 + e_2 + e_3 = 0\ ;$$

$$\therefore\ p = e_1(i_1 - i_2) + e_3(i_3 - i_2),$$

$$P = P_1 + P_2,$$

the total power supplied being equal to the algebraical sum of the readings of the two wattmeters.

Determination of frequency.—If the alternator supplying the system is accessible, the frequency may be determined from the formula

$$f = \frac{np}{2},$$

in which p is the number of poles of the machine and n the revolutions per second.

Another method is to use a length of steel wire stretched as on a monochord, and a horse-shoe magnet M placed as shown in Fig. 65.

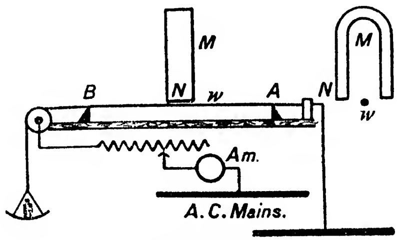

FIG. 65.—The monochord arrangement for the determination of frequency.

When an alternating current passes through the wire, it will be subjected to an up-and-down force. Thus it will be pushed down when the current goes from A to B, and pulled up for the reverse

direction. There will be a swing to and fro of the wire for every wave of the current.

The length and tension of the wire are adjusted until the natural frequency of the wire is the same as that of the alternating current. When this happens the wire oscillates up and down with considerable amplitude, and a paper rider placed on the middle part of it will be thrown off. This vibrating length l is measured and the magnet removed.

Keeping the wire under the same tension and the current passing through it, a second adjustment of its length is made until the wire can be made to emit a note in unison with a tuning fork of known frequency N. A paper rider placed on the wire will be thrown off, if the shank of the fork, when sounding, is held in contact with the board along which the wire is stretched. This will happen for the case of unison, and may be used when the ear is not trained to detect notes in unison.

If the second length is L, $f=\frac{NL}{l}$.

The frequency may be directly determined by a *frequency meter*, which in general consists of a set of prongs of graded lengths each fixed at one end, and having a natural period corresponding to a certain frequency.

These prongs are operated by means of electromagnets carrying the alternating currents whose frequencies are required. Only the prong which has the same frequency as that of the alternating current vibrates, and its end is seen to move up and down at a certain place on the frequency scale.

If the frequency of the alternating current lies between the values of two adjacent prongs, both of these will vibrate, one perhaps more than the other.

The frequency may also be determined by feeding a small two- or three-phase induction motor of suitable voltage from the mains, whose frequency is required, and finding its speed N R.P.M. The frequency is then given by

$$f=\frac{Np}{120}.$$

p is the number of poles of the induction motor, and is equal to twice the number of stator coils per phase.

This method is, however, limited to a small range, as the motor is designed for a certain frequency and should not be supplied with currents of much lower frequency, as they may be excessive. Currents of higher frequency have not this disadvantage. For instance, if the motor were designed for a frequency of 50, it could probably be used for measuring frequencies ranging from 40 to 80.

A stroboscopic disc may be used at one end of the motor shaft for more accurate determinations. This disc is divided into twice as many sectors as there are poles in the machine, and are painted alternately black and white.

If the motor is three phase and has a total number of stator coils equal to six, the motor will have

$$2 \times \tfrac{6}{3} = 4$$

poles. The disc will then have 4 black and 4 white sectors.

A metallic filament lamp is fixed near the disc and supplied with current from the mains which feed the motor.

If the disc shows a slow apparent rotation of the sectors in the opposite direction to that of the rotor, while the speed N_1 is being registered, then N_0, this slow rotation in revolutions per minute, should be determined. The frequency required is then given by

$$f = \frac{N_0 + N_1}{120} \cdot p.$$

Analysis of wave forms.—An electric machine may have a complex voltage wave form, or a circuit, a complex current wave. In certain cases it is important to analyse these waves into their components. The wave form may be obtained by an oscillograph.

Methods of analysis are derived from Fourier's theorem, which states that the equation of any complex periodic wave may be represented by taking a sufficient number of the following terms :

$$y = A_1 \sin \theta + B_1 \cos \theta + A_2 \sin 2\theta + B_2 \cos 2\theta + A_3 \sin 3\theta + B_3 \cos 3\theta + \text{etc.}$$

As in general only odd harmonics are present in alternating currents and voltages,

$$y = A_1 \sin \theta + B_1 \cos \theta + A_3 \sin 3\theta + B_3 \cos 3\theta + A_5 \sin 5\theta + B_5 \cos 5\theta + \text{etc.},$$

in which y is the instantaneous voltage or current, and θ is equal to ωt or $2\pi ft$.

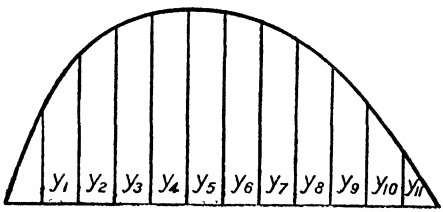

Fig. 66.—Equi-spaced ordinates of a complex wave whose highest harmonic is the eleventh.

By a suitable grouping of these terms the work of eliminating the amplitudes of the component harmonics is much simplified. A good method of grouping is that given by Professor S. P. Thompson.* It is as follows :

a. Select the highest harmonic for which the curve is to be analysed. Suppose this is the *eleventh*. Then divide the base of the curve into a number of equal parts, the number being one more than the highest harmonic. This is done in Fig. 66, and the ordinates are marked y_1, y_2, etc.

* *Electrician*, 1905, vol. lv. p. 79.

b. Make the following arrangement of the values of the ordinates, the first line beginning with y_1 and continuing until the *middle* ordinate, that is the one at 90 degrees, is reached. Then form the second line, starting from the other end. Thus

	y_1	y_2	y_3	y_4	y_5	y_6
	y_{11}	y_{10}	y_9	y_8	y_7	
Sum	a_1	a_2	a_3	a_4	a_5	a_6
Diff.	b_1	b_2	b_3	b_4	b_5	

That is, $a_2 = y_2 + y_{10}$ and $b_2 = y_2 - y_{10}$, and so on.

c. Group as in the following table, which is for the analysis up to the *fifth* harmonic:

Sines.					Cosines.			
Angles.	Sines.	1st and 5th.		3rd.	Angles.	1st and 5th.		3rd.
30	0·5	$0{\cdot}5a_1$			60	$0{\cdot}5b_2$		
60	0·86		$0{\cdot}86a_2$		30		$0{\cdot}86b_1$	
90	1·0	a_3		$a_1 - a_3$	0			$-b_2$
Tot. 1st col.		$0{\cdot}5a_1 + a_3$		$a_1 - a_3$		$0{\cdot}5b_2$		$-b_2$
Tot. 2nd col.		$0{\cdot}86a_2$		0		$0{\cdot}86b_1$		0
Sum		$3A_1$		$3A_3$		$3B_1$		$3B_3$
Difference.		$3A_5$				$3B_5$		

This table shows the necessary arrangement of the values of a_1, a_2, etc., multiplied by their corresponding sines, and of b_1, b_2, etc., multiplied by their corresponding cosines, for to determine the amplitudes A_1, A_2, etc., and B_1, B_2, etc., of the harmonics in the curve under analysis.

The equation of the curve is expressed by

$$Y = A_1 \sin \omega t + A_3 \sin 3\omega t + A_5 \sin 5\omega t$$
$$+ B_1 \cos \omega t + B_3 \cos 3\omega t + B_5 \cos 5\omega t,$$

or, by

$$Y = \sqrt{A_1^2 + B_1^2} \sin(\omega t + \theta_1) + \sqrt{A_3^2 + B_3^2} \sin(3\omega t + \theta_3)$$
$$+ \sqrt{A_5^2 + B_5^2} \sin(5\omega t + \theta_5),$$

in which

$$\tan \theta_1 = \frac{B_1}{A_1}, \quad \tan \theta_3 = \frac{B_3}{A_3}, \quad \tan \theta_5 = \frac{B_5}{A_5}.$$

Thus the amplitudes of the harmonics and their phase differences may be calculated. Y may be either voltage or current.

The grouping follows a definite plan in all cases. a_1, a_2, a_3 multiplied by the sines of the corresponding angles make up the first double column, headed 1st and 5th harmonics. As the 3rd is the only remaining harmonic it has a single column to itself. The values to be entered into the latter column are determined thus. Consider

the sequence of angles and that of the values of a_1, a_2, etc., and the third harmonic; then enter into the column of this harmonic

$$a_1 \sin 3 \times 30 + a_3 \sin 3 \times 90 = a_1 - a_3.$$

The values with even suffixes are zero, and do not appear in this column.

In the *sine* half of the table, values with *odd* suffixes are placed on the left, and those with *even* suffixes on the right side of each double column.

In the *cosine* half of the table the angles are inverted. Values with *odd* suffixes are placed on the right, and those with even ones on the left side of the double column. Begin at the lowest angle, that is the first angle above zero, and work up the column of angles. To deal with the third harmonic column, only enter values with even suffixes; the others are zero. The value to be entered is

$$b_2 \cos 3 \times 60 = -b_2.$$

The coefficient 3 before the amplitudes A_1A_2, etc., B_1B_2, etc., is found by taking one half of the number of the highest harmonic, increased by one. In this case the coefficient is $\frac{5+1}{2} = 3$.

The tabulation for analysis up to the *ninth* harmonic is as follows:

Angles.	Sines.	1st and 9th.		3rd and 7th.		5th.
18	k_1	a_1k_1		a_3k_1		
36	k_2		a_2k_2		$-a_4k_2$	
54	k_3	a_3k_3		a_1k_3		
72	k_4		a_4k_4		a_2k_4	
90	1	a_5		$-a_5$		$a_1 - a_3 + a_5$
Tot. 1st Col.		$a_1k_1 + a_3k_3 + a_5$				
Tot. 2nd Col.		$a_2k_2 + a_4k_4$				
Sum.		$5A_1$		$5A_3$		$5A_5$
Difference.		$5A_9$		$5A_7$		

Angles.	Cosines.	1st and 9th.		3rd and 7th.		5th.
72	k_1	b_4k_1		$-k_1b_2$		
54	k_2		b_3k_2		b_1k_2	
36	k_3	b_2k_3		$-k_3b_4$		
18	k_4		b_1k_4		$-b_3k_4$	
0	1					$-b_2 + b_4$
Tot. 1st Col.		$b_4k_1 + b_2k_3$				
Tot. 2nd Col.		$b_3k_2 + b_1k_4$				
Sum.		$5B_1$		$5B_3$		$5B_5$
Difference.		$5B_9$		$5B_7$		

The columns for the third and seventh harmonic in the sine half of the table are obtained thus:

$$a_1 \sin 3 \times 18 + a_3 \sin 3 \times 54 + a_5 \sin 3 \times 90, \quad \text{.................}(a)$$

$$a_2 \sin 3 \times 36 + a_4 \sin 3 \times 72. \quad \text{.......................................}(b)$$

(a) goes under the third and (b) under the seventh harmonics.

The columns for the third and seventh harmonic in the cosine half of the table are obtained thus ·

$$b_1 \cos 3 \times 18 + b_3 \cos 3 \times 54, \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$
$$b_2 \cos 3 \times 36 + b_4 \cos 3 \times 72. \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

(a) goes under the seventh and (b) under the third harmonics.

The column of the fifth harmonic for the cosine half of the table is

$$b_2 \cos 5 \times 36 + b_4 \cos 5 \times 72 = -b_2 + b_4.$$

Values with odd suffixes are zero.

The next table gives the analysis up to the *eleventh* harmonic. It will be noted that the second harmonic of each double column has its column filled as a result of using the value of the first of the two harmonics, and has consequently no function to perform in the construction of the tables.

Angles.	Sines.	1st and 11th.		3rd and 9th.		5th and 7th.	
15	k_1	a_1k_1				a_5k_1	
30	k_2		a_2k_2				a_2k_2
45	k_3	a_3k_3		$(a_1 + a_3 - a_5) \times k_3$		$-a_3k_3$	
60	k_4		a_4k_4				$-a_4k_4$
75	k_5	a_5k_5				a_1k_5	
90	1		a_6		$a_2 - a_6$		a_6
Tot. 1st col.							
Tot. 2nd col.							
Sum.		$6A_1$		$6A_3$		$6A_5$	
Difference.		$6A_{11}$		$6A_9$		$6A_7$	

Angles.	Cosines	1st and 11th.		3rd and 9th.		5th and 7th.	
75	k_1		b_5k_1				b_1k_1
60	k_2	b_4k_2				b_4k_2	
45	k_3		b_3k_3		$(b_1 - b_3 - b_5) \times k_3$		$-b_3k_3$
30	k_4	b_2k_4				$-b_2k_4$	
15	k_5		b_1k_5				b_5k_5
0	1			$-b_4$			
Tot. 1st col.							
Tot. 2nd col.							
Sum.		$6B_1$		$6B_3$		$6B_5$	
Difference.		$6B_{11}$		$6B_9$		$6B_7$	

Concrete case.—A voltage curve is to be analysed up to the eleventh harmonic. The base line of the curve is divided into 12 equal parts, and the ordinates are found to be

30, 66, 100, 200, 160, 80, 60, 50, 40, 30, and 20.

These are arranged thus :

	30,	66,	100,	200,	160,	80.
	20,	30,	40,	50,	60.	
Sum.	50,	96,	140,	250,	220,	80.
Diff.	10,	36,	60,	150,	100.	

Angles.	Sines.	1st and 11th.		3rd and 9th.		5th and 7th.	
15	0·26	13				57	
30	0·5		48				48
45	0·71	99·3		−21·3		−99·3	
60	0·87		217				−217
75	0·97	214				48·5	
90	1		80		16		80
Tot. 1st col.		326·3		−21·3		6·2	
Tot. 2nd col.		345		16		−89	
Sum.		($6A_1$) 671·3		($6A_3$) − 5·3		($6A_5$) −82·8	
Difference.		($6A_{11}$) − 18·7		($6A_9$) −37·3		($6A_7$) 95·2	

Angles.	Cosines.	1st and 11th.		3rd and 9th.		5th and 7th.	
75	0·26		26				2·6
60	0·5	75				75	
45	0·71		42·6		−106		−42·6
30	0·87	31·3				−31·3	
15	0·97		9·7				97
0	1			−150			
Tot. 1st col.		106·3		−150		43·7	
Tot. 2nd col.		78·3		−106		57	
Sum.		($6B_1$) 184·6		($6B_3$) −256		($6B_5$) 100·7	
Difference.		($6B_{11}$) 28		($6B_9$) − 44		($6B_7$) − 13·3	

The equation to this complex voltage curve is therefore given by

$$\begin{aligned} e = {} & (112 \sin \omega t + 30{\cdot}8 \cos \omega t) - (0{\cdot}9 \sin 3\omega t + 42{\cdot}7 \cos 3\omega t) \\ & - (13{\cdot}8 \sin 5\omega t - 16{\cdot}8 \cos 5\omega t) + (15{\cdot}9 \sin 7\omega t - 2{\cdot}2 \cos 7\omega t) \\ & - (6{\cdot}2 \sin 9\omega t + 7{\cdot}3 \cos 9\omega t) - (3{\cdot}1 \sin 11\omega t - 4{\cdot}7 \cos 11\omega t). \end{aligned}$$

The amplitudes are as follows :

Fundamental	$\sqrt{112^2 + 30{\cdot}8^2}$	= 116 volts.
Third harmonic	$\sqrt{0{\cdot}9^2 + (42{\cdot}7)^2}$	= 42·75 volts.
Fifth ,,	$\sqrt{13{\cdot}8^2 + (16{\cdot}8)^2}$	= 21·7 volts.
Seventh ,,	$\sqrt{15{\cdot}9^2 + 2{\cdot}2^2}$	= 16·0 volts.
Ninth ,,	$\sqrt{6{\cdot}2^2 + 7{\cdot}3^2}$	= 9·6 volts.
Eleventh ,,	$\sqrt{3{\cdot}1^2 + 4{\cdot}7^2}$	= 4·8 volts.

This shows that the third harmonic comes next to the fundamental in the value of its amplitude.

ω is equal to $2\pi f$, in which f is the frequency of the fundamental, that is, equal to $\frac{np}{2}$, if this wave is given by an alternator; p being the number of poles and n the revolutions per second.

The reading of this voltage on an instrument such as the hot wire voltmeter would be

$$\frac{1}{\sqrt{2}}\sqrt{116^2+42{\cdot}75^2+21{\cdot}7^2+16{\cdot}0^2+9{\cdot}6^2+4{\cdot}8^2}=89{\cdot}6 \text{ volts.}$$

EXAMPLES.—1. A current wave is to be analysed up to the fifth harmonic. The five equally spaced ordinates are 1·5, 6·1, 10·8, 12·1, and 8·4 amperes. Find the amplitudes of the harmonics and fundamental.

Ans. The amplitude of the third harmonic is $-2{\cdot}02$, that of the fifth is 1·0, and the fundamental has an amplitude of 10·9 amperes.

2. Analyse a given voltage wave up to the thirteenth harmonic, given that its equally spaced ordinates are of values 80, 160, 200, 240, 310, 330, 360, 300, 250, 180, 130, 90, and 50 volts.

Ans. The amplitudes of the fundamental, third harmonic, and so on, taken in order, are approximately 308·6, $-30{\cdot}2$, 17, 5·7, $-3{\cdot}2$, $-5{\cdot}8$, and 6·9 volts.

CHAPTER V.

THE MOVING-COIL MIRROR GALVANOMETER.

PRACTICALLY all sensitive galvanometers are of the moving-coil type, with or without an iron core. Without a core, the coil is generally very long and narrow and encased in a light metal or ivory case. With a core, the coil is rectangular or circular.

Such galvanometers may be used for steady deflection or zero tests; or if the coil is sufficiently heavy and the suspension suitable, they may be used for the measurement of the quantity of electricity in transient currents, such as those produced in coils due to being threaded by lines of magnetic force, or the charge and discharge currents of a condenser.

Theory of the galvanometer for steady currents.—Let H be the strength of the magnetic field in which the coil operates, l the mean length of the vertical coil-side, a_1 the mean length of the horizontal coil-side, S the number of turns, and I C.G.S. units the current in the coil.

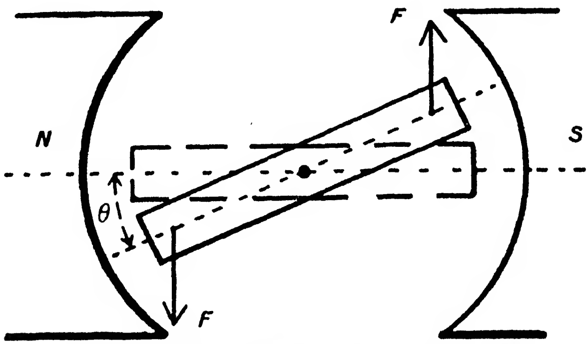

FIG. 67.—The forces acting on the coil of a galvanometer.

If θ is the deflection of the coil from its zero position (Fig. 67), a deflecting couple of value

$$\mathrm{HI}la_1\mathrm{S}\cos\theta$$

will act on the coil. This follows from equation (10), page 8.

Now, this is balanced by a torsion couple of value $k\theta$; k being the torsion coefficient of the suspension, or the torsion couple necessary to twist the coil through one radian. If the suspension is bifilar k will depend upon the torsion of the wires and the force of gravitation.

Therefore
$$I = \frac{k}{Ha_1lS} \cdot \frac{\theta}{\cos\theta},$$

$$I = K_g \cdot \frac{\theta}{\cos\theta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

K_g being the constructional constant of the galvanometer of value

$$K_g = \frac{k}{Ha_1lS} = \frac{k}{HA}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

$A = la_1S$, and is called the *induction area* of the coil.

In practice a reading d divisions is observed on a scale L *millimetres* from the mirror of the galvanometer. If a scale division is one millimetre,

$$\frac{d}{L} = \tan 2\theta.$$

When θ is very small, I is approximately given by the relation

$$I = \frac{K_g}{2L} \cdot d.$$

The coefficient $\frac{K_g}{2L}$ may be found as follows. A good steady cell whose voltage E has been accurately determined by a potentiometer method, is shunted with a small resistance R_1 in series with a very large one R_2. G, the resistance of the galvanometer, is also supplemented with a high resistance box and a resistance value R used in it. This galvanometer circuit is placed across the ends of R_1 and the scale reading D observed. Then

$$I = \frac{E}{10(G+R)} \cdot \frac{R_1}{R_1+R_2} = \frac{K_g}{2L} \cdot D.$$

EXAMPLE.—A galvanometer of resistance 400 ohms is placed in series with a resistance of 1600 ohms. A cell of voltage 1·07 is connected in series with a resistance of 4 ohms, and one of 7996 ohms. The galvanometer circuit is connected across the 4 ohms, and a deflection of 200 divisions on the galvanometer scale is read. Calculate the value in *amperes* of a scale division. *Ans.* $1{\cdot}3375 \times 10^{-9}$ ampere.

Theory of the ballistic galvanometer.—This instrument is used for measuring the quantity of electricity in transient currents of short duration. The period of undamped vibration for this type of galvanometer is usually about 5 seconds for general work, and up to as much as 15 seconds for some class of magnetic testing.

The coil must be sufficiently ballistic in order that it does not appreciably move from its zero position until all the transient current has passed through it. Thus the coil swings off from its zero position under the cumulative electromagnetic effect of the whole transient current. The beginning of this swing may take place, as it probably will, at an interval after the transient current has ceased flowing.

The retarding forces which come into play as soon as the coil starts to move from its zero position are the torsion resistance and the damping resistance. The latter is nearly wholly electromagnetic, and only a very small part, generally negligible, is due to air damping.

Electromagnetic damping occurs, because the coil forms part of a *closed* circuit, through which the transient current has passed. Before this damping begins, the transient current will have ceased An induced current is then produced in the coil, and by Lenz's rule its magnetic action tends to stop the movement of the coil.

The more resistance in the galvanometer circuit the less the damping, and when it is infinite there is only the small mechanical damping of the air. Thus, the deflections read on each side of the zero of the scale become successively smaller and smaller; the diminution being very small for open circuit, and large for short circuit with negligible resistance.

A body suspended by a metal wire and immersed in an oil, when started oscillating about the wire as axis, behaves like the moving galvanometer coil, for it has the same kind of damping resistances. If the body is a smooth metal cylinder provided with a pointer, which can move across a circular scale fitted to the rim of the vessel containing the oil, quantitative measurements may be made for different oils.

Let i C.G.S. units be the current in the coil at time t, measured from the starting of the transient current. Then, as the coil is in its zero position all the time the current is flowing, the electromagnetic couple acting on it at time t is

$$\mathrm{HA}i = \mathrm{K}i.$$

This couple changes with i; starting at zero, reaching a maximum, and then falling to zero again as soon as the transient current has ceased.

The cumulative effect of this couple is simply to drive off the coil from its zero position, with an angular velocity of ω radians; it has no further effect. This angular velocity has been produced in T seconds, the time the transient current was in operation.

Let y be the angular velocity at time t, an instant during the growth of the angular velocity up to ω. Consider a particle of mass m of the coil at a distance x from the axis of oscillation. The velocity of the particle will be xy, and its acceleration will be $x\frac{dy}{dt}$. The force producing this acceleration will be, by *the second law of motion*, of value

$$mx\frac{dy}{dt},$$

and the moment of the force producing this acceleration will be

$$mx^2\frac{dy}{dt}.$$

Therefore the moment needed to produce this *angular* acceleration for all the particles of the coil will be

$$\sum mx^2\frac{dy}{dt}=\Delta\frac{dy}{dt},$$

Δ being the moment of inertia of the coil.

The actual moment producing this acceleration is, however, equal to Ki, the electromagnetic couple acting at time t. Therefore

$$dy=\frac{\mathrm{K}}{\Delta}i\,dt,$$

and for the whole time T,

$$\int_0^\omega dy=\frac{\mathrm{K}}{\Delta}\int_0^{\mathrm{T}}i\,dt;$$

$$\therefore\quad \omega=\frac{\mathrm{K}}{\Delta}\cdot q, \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad(c)$$

q being the total quantity in C.G.S. units of the electricity in the transient current.

The coil moves off with angular velocity ω against the resistance of the torsion and the damping effect. During the outward movement the torsion opposes the motion of the coil, while it assists, in fact produces, the return motion. The damping always opposes the motion of the coil.

The value of the torsion couple is $k\theta$; θ being the angle of deflection of the coil in radians. It would be expected that the damping couple at any given instant would depend upon the angular velocity of the coil, because the induced voltage in its vertical limbs is approximately proportional to their velocity for the limited movement of the coil. Thus the damping couple may be approximately represented by

$$g\frac{d\theta}{dt},$$

g being a coefficient depending upon the constructional constants, and the value of the resistance associated with the galvanometer.

After the impulsive couple which started the coil to swing out from its zero position has ceased its action, the impressed couples acting on the *moving* coil are

$$k\theta \quad\text{and}\quad g\frac{d\theta}{dt}.$$

Let m be the mass of a particle of the coil at a distance x from its axis of oscillation. Its velocity at time t will be

$$x\frac{d\theta}{dt},$$

and its retardation
$$x\frac{d^2\theta}{dt^2}.$$

The force on the particle necessary to produce this retardation will be, by *the second law of motion*, of value

$$mx\frac{d^2\theta}{dt^2},$$

and the impressed moment necessary to produce this retardation will be

$$mx^2\frac{d^2\theta}{dt^2}.$$

Therefore, to produce this retardation in the whole coil, the impressed moment must be of value

$$\frac{d^2\theta}{dt^2}\sum mx^2 = \Delta\frac{d^2\theta}{dt^2},$$

and the actual impressed moment is

$$k\theta + g\frac{d\theta}{dt}.$$

Thus the equation of the damped motion of the coil is given by

$$\frac{d^2\theta}{dt^2} + \frac{g}{\Delta}\frac{d\theta}{dt} + \frac{k}{\Delta}\theta = 0. \quad \text{......................................}(d)$$

Let
$$a = \frac{g}{2\Delta} \quad \text{and} \quad b = \frac{k}{\Delta};$$

$$\therefore \quad \frac{d^2\theta}{dt^2} + 2a\frac{d\theta}{dt} + b\theta = 0.$$

The solving equation is

$$x^2 + 2ax + b = 0,$$

the two roots of which are $(-a+\sqrt{a^2-b})$ and $(-a-\sqrt{a^2-b})$.

In the ballistic galvanometer, except for extreme values of g, b is always greater than a^2; for otherwise, after the first kick or throw there would be no deflection on the opposite side of zero, but the coil would slowly swing back to its zero position.

Let $m=\sqrt{b-a^2}$, that is $m=j\sqrt{a^2-b}$, in which $j=\sqrt{-1}$.

The two roots of the solving equation are therefore

$$-a+jm \quad \text{and} \quad -a-jm,$$

and the solution of the differential equation (d) is given by

$$\theta = A_1 e^{(-a+jm)t} + A_2 e^{(-a-jm)t}$$
$$= e^{-at}\{A_1 e^{jmt} + A_2 e^{-jmt}\};$$
$$\therefore\ \theta = e^{-at}\{B \sin mt + C \cos mt\}.$$

If time is measured from the beginning of the first swing,

$$\theta = 0 \quad \text{when} \quad t = 0,$$

so that $C = 0$. The solution of (d) is therefore given by

$$\theta = Be^{-at} \sin mt. \qquad \text{.......... }(e)$$

If the motion were *undamped* the coil would swing under the sole effect of the torsion couple. In this case, $g=0$, $a=0$, and $m=\sqrt{b}$. From (e),

$$\theta = B \sin \sqrt{b} \,.\, t, \qquad \text{.......... }(f)$$

which is represented graphically in Fig. 68. That is, the coil would go on swinging equally each side of the zero position without stopping. This undamped oscillation is approximately realised when the coil is swinging and its circuit is open. It swings to and fro for a considerable time, but is gradually brought to rest by the slight air damping.

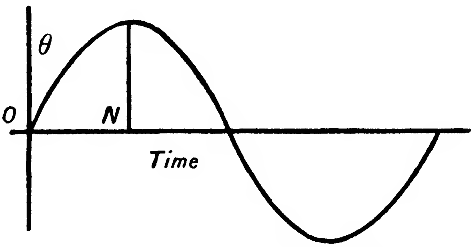

FIG. 68.—The undamped motion of a galvanometer coil.

In this case of undamped motion, the coil moves off with angular velocity ω, and reaches a deflection or throw of value B. Now ω is the value of $\frac{d\theta}{dt}$ when $t=0$, so, by differentiating (f) and putting $t=0$,

$$\omega = \sqrt{b} \,.\, B.$$

If α is the *undamped* deflection or throw of the coil,

$$\alpha = B = \frac{\omega}{\sqrt{b}}. \qquad \text{.......... }(g)$$

Also the *undamped* period of the time is given by

$$T = \frac{2\pi}{\sqrt{b}} = 2\pi\sqrt{\frac{\Delta}{k}}, \qquad \text{.......... }(h)$$

for, from Fig. 68, the time represented by ON is $\frac{\pi}{2\sqrt{b}}$ second. Therefore a period is four times this value.

The *real* motion of the coil is a *damped* one and represented by equation (*e*), and the angular velocity at time t is

$$\frac{d\theta}{dt} = \{Bm\cos mt - Ba\sin mt\}e^{-at}.$$

Now, each time the coil reaches the limit of its swing on each side of the zero, the angular velocity becomes zero, and the condition for this is that

$$\tan mt = \frac{m}{a}.$$

Also $$\tan mt = \tan m\left(t + \frac{\pi}{m}\right) = \tan m\left(t + \frac{2\pi}{m}\right),$$

and so on; so that if t_1, t_2, t_3, etc., are the respective times reckoned from the start of the coil's motion up to the end of the first deflection on the right, the end of the first deflection on the left, etc.,

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3, \text{ etc.} = \frac{\pi}{m},$$

for $$t_2 = t_1 + \frac{\pi}{m}; \quad t_3 = t_2 + \frac{\pi}{m};$$

and so on.

Thus, the period of the damped swing, reckoning from the end of the deflection on one side to the end of the next on the same side, is

$$\frac{2\pi}{m} = \frac{2\pi}{\sqrt{b - a^2}}.$$

The motion of the coil is therefore slower in this case than when undamped.

The time between a displacement on the extreme left to one on the right is constant, and is equal to $\frac{\pi}{m}$ or half the period.

Let a_1, a_2, a_3, etc., be the *actual* successive angular deflections in radians of the coil from the zero position. Then

$$a_1 = Be^{-at_1}\sin mt_1,$$

$$a_2 = Be^{-at_2}\sin mt_2,$$

and so on. Now

$$t_2 - t_1 = \frac{\pi}{m}, \quad \text{or} \quad t_2 = t_1 + \frac{\pi}{m};$$

$$\therefore \sin mt_2 = \sin m\left(t_1 + \frac{\pi}{m}\right) = \sin(mt_1 + \pi),$$

or $$\sin mt_2 = -\sin mt_1.$$

That is, $\sin mt_1$ is numerically the same as $\sin mt_2$; the minus

sign appearing because the second deflection is on the opposite side of zero.

It follows that $$\frac{a_1}{a_2}=\frac{a_2}{a_3}=\frac{a_3}{a_4}=e^{\frac{\pi a}{m}}.$$

From which, $$\log_e a_1 - \log_e a_2 = \frac{\pi a}{m} = \lambda.$$

Thus the difference due to damping between the logarithms of two succeeding swings, one to the right and the other to the left, is constant. From this it is assumed that

$$\log_e \alpha - \log_e \alpha_1 = \frac{\lambda}{2},$$

α being the first throw if there were no damping and α_1 the actual first throw. One half of λ is taken, because the damping occurs only in going from zero to the extreme limit on the right-hand side, and not from one side to the other. This assumption becomes more justifiable, the smaller the difference is between α_1 and α_2. Thus

$$a = a_1 e^{\frac{\lambda}{2}}.$$

Also, since $$\frac{a_1}{a_2}=\frac{a_2}{a_3}, \text{ etc.} = e^{\lambda}$$

$$\sqrt{\frac{a_1}{a_2}}=\sqrt{\frac{a_2}{a_3}}=e^{\frac{\lambda}{2}}.$$

Therefore $$a=\sqrt{\frac{a_1}{a_2}}\cdot a_1.$$

Let $$Z=\sqrt{\frac{a_1}{a_2}}=\sqrt{\frac{a_2}{a_3}}, \text{ etc.},$$

Then $$\alpha = Z\alpha_1. \quad \ldots\ldots\ldots\ldots (i)$$

Z is called the *correction factor* for *damping*, and may readily be found by experiment for the particular circuit through which the transient current has flowed. Thus

$$Z=\sqrt{\frac{D_1}{D_2}},$$

D_1 being the scale reading on one side to zero, and D_2 the following one on the other side.

Finally, using equations (*c*), (*g*), (*h*), (*i*); and also that

$$b=\frac{k}{\Delta} \quad \text{and} \quad K=HA,$$

$$q=\frac{T}{2\pi}Z\alpha_1\frac{k}{HA} \text{ C.G.S. units.}$$

Then, from equation (*b*),

$$q = K_g \cdot \frac{T}{2\pi} \cdot Z\alpha_1 \text{ C.G.S. units. } \qquad (j)$$

The right side will be multiplied by 10 for the quantity to be in *coulombs*.

If D is the throw in scale divisions and L millimetres the distance of the mirror from the scale, and the value of a scale division is one millimetre, this equation becomes

$$q = \frac{K_g}{2L} \cdot \frac{T}{2\pi} \cdot ZD \text{ C.G.S. units.} \qquad (k)$$

K_g is the constructional coefficient of the instrument; T the *undamped* period in seconds, obtained for the galvanometer on open circuit; Z the damping factor, which must be obtained for the particular circuit, and the associated circuits through which the transient current has passed; and D the actual first throw or kick in scale divisions.

In the formula, T is the *undamped period*, which in most cases is approximately equal to T_0, the observed period on open circuit. If the air damping is greater than usual, T may be determined as follows:

$$T_0 = \frac{2\pi}{m_0}, \qquad (1)$$

$$T = \frac{2\pi}{\sqrt{b}}, \qquad (2)$$

$$\log_e \frac{\alpha_1}{\alpha_2} = \frac{\pi a_0}{m_0}, \qquad (3)$$

$$m_0 = \sqrt{b - a_0^2}. \qquad (4)$$

T_0, α_1, and α_2 are determined from experiment, the coil moving only against air damping.

From (1) find m_0; using this value in (3) get a_0, and from (4) calculate b. Finally, from (2) calculate T, the undamped period.

The value of a scale division in micro-coulombs.—For condenser work, in which Z is always of open circuit value, the value of the scale division is constant; in other cases it depends upon the resistance of the circuits associated with the galvanometer.

A condenser of known capacity C *farads* is charged by E volts, and then discharged through the galvanometer whose deflection or throw D_0 is observed. Then

$$\frac{EC \times 10^6}{D_0}$$

is the value of a scale division in micro-coulombs.

The constructional constant of the instrument K_g may be found

by measuring L and D in the same units, T and Z_0, and using equation (*k*). Thus

$$EC = 10 \cdot \frac{K_g}{2L} \cdot \frac{T}{2\pi} \cdot Z_0 \cdot D_0,$$

from which K_g may be calculated.

A *second* method of finding the value of a scale division, is to find the value of $\frac{K_g}{2L}$ by the method on page 90, and then measure T. From equation (*k*), putting D the throw, equal to unity,

$$q_1 = 10 \cdot \frac{K_g}{2L} \cdot \frac{T}{2\pi} \cdot Z \text{ coulombs,}$$

q_1 being the value of a scale division in coulombs when any particular value of Z is included in the formula.

Example.—Find the value of a scale division of a ballistic galvanometer in micro-coulombs, given that its constructional constant is $7{\cdot}32 \times 10^{-6}$, its undamped period 5 seconds, the distance of its scale from the mirror 1000 millimetres, its scale divisions in millimetres, and that the particular circuit it has to be calibrated for has a damping factor of 1·1. *Ans.* 0·032 micro-coulomb.

A *third* method is to use a long solenoid with a secondary coil wound over its central part. The solenoid is connected by a reversing key to a storage battery, an adjustable resistance, and an ammeter. The secondary coil is connected in series with the galvanometer and a resistance box if necessary.

The current I amperes in the solenoid produces a flux

$$\phi = \frac{4\pi SI}{10l} \cdot \pi r^2$$

through the secondary coil which has s turns. S is the number of turns on the solenoid, l its length, and r the mean radius of the turns.

When I is quickly reversed a change of $2\phi s$ magnetic linkages is produced in the secondary coil, and by equation (4), page 4, the quantity of electricity passing through the galvanometer is

$$\frac{2\phi s}{10^8 R} \text{ coulombs,}$$

R being the resistance of the *circuit* of the galvanometer. If D is the throw on the scale,

$$\frac{2\phi s}{10^2 RD} \text{ micro-coulombs}$$

is the value of a scale division for the particular circuit used in carrying the transient current, or for any other which has the same value of Z.

Example.—The secondary coil on a solenoid is connected to a ballistic galvanometer, and this circuit has a resistance of 300 ohms.

The solenoid has 2000 turns, a length 80 cms., a mean radius 2 cms., and carries 0·4 ampere. This current is quickly reversed and the throw on the scale is 200 divisions. Calculate the value of a scale division in micro-coulombs; the number of turns on the secondary coil being 400. *Ans.* 0·021 micro-coulomb.

In the *comparison* method of measuring the strengths of magnetic fields, the damping factor is eliminated in the two operations. In this case a test coil of S_1 turns and mean radius r_1 is joined in series with the secondary coil of the standard solenoid and the ballistic galvanometer.

The test coil is made to suddenly cut the lines of the unknown field of strength H, thereby producing a throw D, and then the solenoid current is reversed and the throw D_0 due to the secondary coil is read.

The quantity of electricity produced in the galvanometer circuit in the first case is

$$\frac{HS_1\pi r_1^2}{10^8R},$$

and in the second case

$$\frac{2\phi s}{10^8R},$$

R being the total resistance of the galvanometer circuit, s the turns on the secondary coil, and ϕ the total solenoid flux threading these s turns.

The damping factor is unchanged, so that H is found from the relation

$$\frac{HS_1\pi r_1^2}{2\phi s}=\frac{D}{D_0}.$$

The ballistic galvanometer is used for deflection tests of capacity and inductance; and also for the magnetic testing of iron in the ring form.

Duddell's sensitive thermo-electric ammeter or galvanometer.—This is useful for measuring the R.M.S. value of very small or, if necessary, large alternating currents. The principle of the instrument is roughly indicated in Fig. 69. A thermo-couple of bismuth and antimony A, is connected to the narrow coil placed as shown between two near poles of a magnet. The coil is suspended by a quartz fibre.

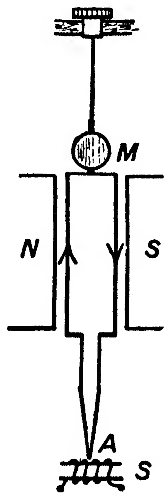

FIG. 69.—The principle of Duddell's sensitive thermoelectric galvanometer.

The small alternating current to be measured is passed through a metallic strip S fixed on a block of insulation. S becomes heated by the current and communicates heat to the junction A. A thermoelectric current passes round the coil, which is thereby deflected, and the spot of light reflected from the mirror M, travels across the scale until it reaches a certain value.

The instrument may be calibrated by passing known small values of direct current through the strip. By using strips of less heating capacity for a given current, larger alternating currents may be measured.

This instrument has negligible self inductance, and is independent of frequency and wave form. In addition to being useful for measuring small alternating currents, it is capable of measuring the small periodic currents used in telephony. The latter have a high frequency and are of complex wave form. The frequency of the highest harmonic, according to *B. S. Cohen*, lies between 4000 and 5000, and the fundamental between about 100 and 300.*

* *Phil. Mag.* (6), vol. 16, 1908, p. 480.

CHAPTER VI.

MEASUREMENT OF RESISTANCE AND THE TESTING OF INSULATION.

RESISTANCES may be classed as low, medium, high, and those which contain a polarisation voltage. In the case of alternating currents, the product of self inductance and frequency, and the reciprocal of the product of capacity and frequency have the characteristic, among others, of behaving as a simple resistance in diminishing the value of the current. The method of measuring a resistance will therefore generally depend upon the class to which it belongs.

Ordinary resistance of low value may be measured by using the slide-metre bridge in a special way; by the fall of potential method; or by a potentiometer method. Those of medium value, by a bridge arrangement such as the post office or dial box; or by a substitution method. Resistance of high value, by a sensitive galvanometer and high resistance of known value; by the leakage method; or by special testing sets.

Resistances containing a polarising voltage may be measured by a bridge method, using alternating currents and a telephone receiver; by a direct current compensating method; or by an alternating-current method, using ammeter and voltmeter. The resistance or impedance due to self inductance and capacity has already been considered in Chapter IV.

Measurement of low resistance.—Carey-Foster's method of using the slide-metre bridge for measuring small resistances is illustrated in Fig. 70. R is a box containing a set of known resistances of small value and X is the small resistance to be measured. P and Q are pieces of german silver wire of about the same resistance as X.

A balance is first found by moving C along the wire AB, when R and X are in the positions shown. Then, if r_1 and r_2 are the resistances of the connecting pieces of the bridge between A and R, and between B and X, and l_1 is equal to AC,

$$\frac{P}{Q}=\frac{R+r_1+l_1\sigma}{X+r_2+(L-l_1)\sigma};$$

$$\therefore\ \frac{P+Q}{Q}=\frac{R+X+r_1+r_2+L\sigma}{X+r_2+(L-l_1)\sigma}, \quad \ldots\ldots\ldots\ldots\ldots\ldots(a)$$

in which σ is the resistance per centimetre of the slide wire and L its length.

Now X and R are interchanged; then, if l_2 is the new value of AC for balance,

$$\frac{P+Q}{Q}=\frac{R+X+(r_1+r_2)+L\sigma}{R+r_2+(L-l_2)\sigma}. \quad \text{.....................}(b)$$

Therefore, from (*a*) and (*b*),

$$X=R+(l_1-l_2)\sigma. \quad \text{..............................}(c)$$

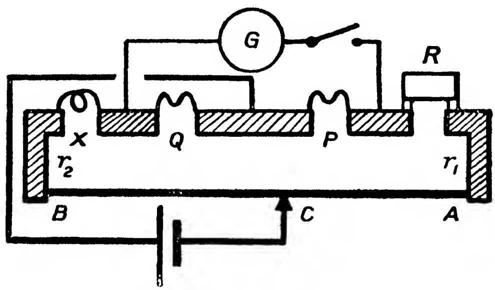

FIG. 70.—The measurement of low resistance by the slide-metre bridge; Carey-Foster's method.

Next, σ is found by repeating the preceding two operations, but using a *thick* copper plate instead of X and a smaller value of R, namely R_0. Then, if L_1 is the length of AC in the first and L_2 in the second of the operations,

$$O=R_0+(L_1-L_2)\sigma,$$

$$\sigma=\frac{R_0}{L_2-L_1}. \quad \text{................................}(d)$$

From (*c*) and (*d*) X may be obtained.

For very accurate determinations the slide wire of the metre bridge should be carefully calibrated. This may be done by using a second length of wire CD (Fig. 71), a thick copper plate, and a known small resistance r_0, of value one-tenth to one-twentieth of the resistance of AB.

P is first placed at B, the resistance r_0 at *a*, and S the copper plate at *b*. Q is then moved along CD until a balance is obtained. If CQ is equal to *x*, QD to *y*, and the length of BA is L,

$$\frac{x}{y}=\frac{r_0+r_2}{L\sigma+r_1},$$

$$\frac{x+y}{y}=\frac{r_0+r_1+r_2+L\sigma}{L\sigma+r_1}. \quad \text{........................}(a)$$

Now, leaving Q at its present position, S and r_0 are interchanged to occupy the positions shown, and P is moved along the wire from B until a balance is found. If BP is equal to l_1,

$$\frac{x+y}{y}=\frac{l_1\sigma+r_2+r_0+r_1+(L-l_1)\sigma}{(L-l_1)\sigma+r_1+r_0}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(b)$$

From (a) and (b), $\qquad BP=l_1\sigma=r_0$.
CQ is also equal to r_0.

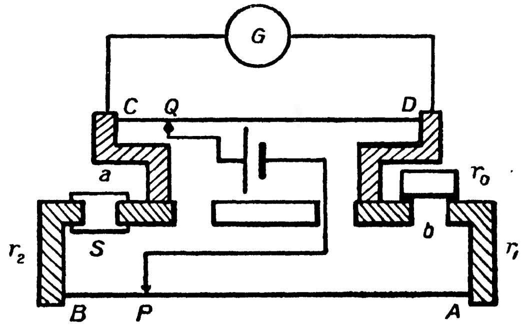

FIG. 71—The calibration of the wire of the slide-metre bridge.

Again, leaving P at its new position, r_0 and S are interchanged, and a new position of Q for balance is found. CQ will then be of resistance $2r_0$.

With Q left in this position, r_0 and S are interchanged, and the balancing position of P is found. BP will then be of resistance $2r_0$.

By continuing this process, BA and also CD may be divided into lengths having the same resistance as r_0. A graph may then be plotted relating distance from B along the slide wire, and its resistance.

Measuring small resistances by the fall of potential method.—If the resistance to be measured can carry a large current without appreciable heating, an ammeter and voltmeter may be used. The former gives the current passing through the resistance and the latter the voltage driving the current through it. Then

$$R=\frac{E}{I}.$$

In the case of smaller currents, X the unknown resistance is placed in series with R_0, a known resistance, an adjustable resistance, and a battery. A sensitive galvanometer in series with a high-resistance box is used as a voltmeter across the ends of X, and R_0 in turn, and the deflections read. Then

$$\frac{X}{R_0}=\frac{D_x}{D_0},$$

or X is directly proportional to its galvanometer scale deflection.

This is a good method of *comparing the conductivities* of samples of cables or thick conductors with that of a standard copper rod. The arrangement is shown in Fig. 72. A and B are the rods whose conductivities are to be compared. PP are placed on each rod in turn with sufficient span to give a large deflection. The diameters of the rod are then measured by a micrometer screw.

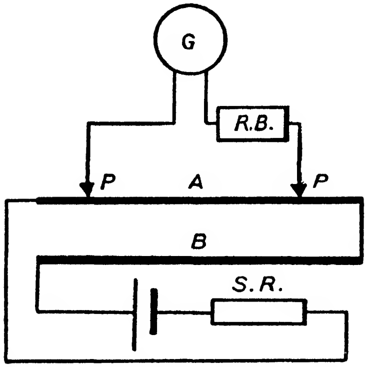

FIG. 72.—The arrangement for the comparison of conductivities of metal rods, by the fall of potential method.

If ALDK and *aldk* are the respective cross-sectional area, length of span PP used, scale deflection, and conductivity, of the standard and test rods,

$$k = K \cdot \frac{DAl}{daL}.$$

The potentiometer method of measuring small resistance.—A known resistance R is placed in series with X, the unknown resistance, an adjustable resistance, and a cell or battery of steady voltage. The balancing length L on the potentiometer wire is found for the voltage across R, and l for that across X. Then

$$\frac{X}{R} = \frac{l}{L}.$$

This method may be used for finding the very small *resistance of a storage cell.* The apparatus and connections for this test are shown in Fig. 73. C is the cell whose resistance b is to be determined, and B is a standard cell or a steady cell of known voltage E_0.

With S_1 and S_2 on the lower contacts, B is balanced on the potentiometer wire and $OP = L_0$ measured.

With S_1 and S_2 on the top contacts and K *out*, the cell C of voltage E is balanced and $OP = L_1$ measured.

With the switches unchanged and K *closed*, a third balancing length $OP = L_2$ is obtained and the current I read on the ammeter.

These three operations should be performed as quickly as possible after each other, especially the last two. Then

$$E_0 = kL_0,$$
$$E = kL_1,$$
$$E + I \cdot b = kL_2;$$
$$\therefore \; b = \frac{E_0}{I} \cdot \frac{L_2 - L_1}{L_0}.$$

In the case of a battery of storage cells, the resistance may be found by reading E, the voltage across the battery when it carries

a current of I amperes, and again reading E_1, the voltage across the same points, immediately when I is switched off. Then

$$\frac{E - E_1}{I}$$

is the resistance of the battery.

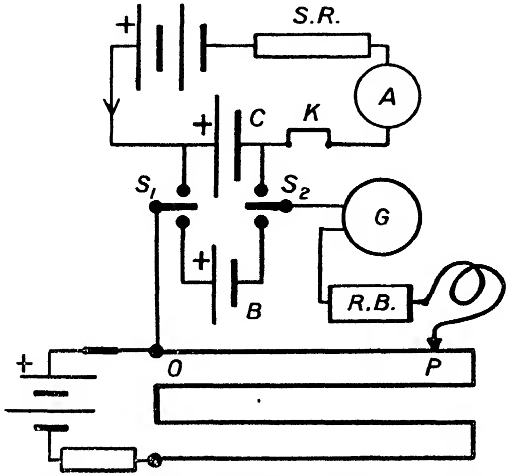

FIG. 73.—The arrangement for determining the resistance of a storage cell, by the potentiometer method.

Potentiometers.—These are made to measure from 0·001 volt or lower to 300 volts or higher. A good type is that of *Crompton*. This potentiometer is illustrated in Fig. 74, which shows the main features

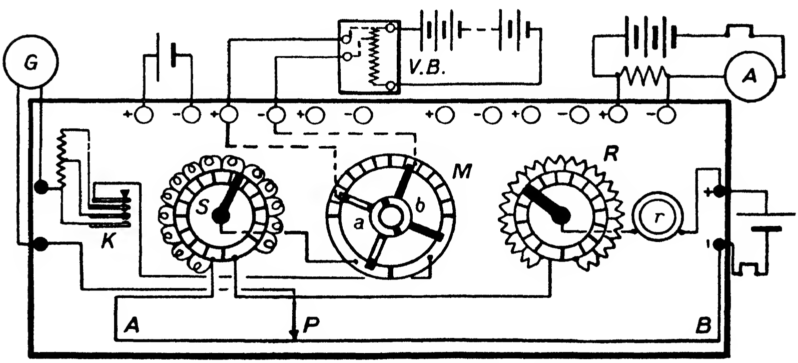

FIG. 74.—The arrangement of Crompton's potentiometer.

of its construction. AB is the slide wire and P its contact piece, which can be moved along a scale fixed above the wire.

In series with this wire are fourteen coils, each of resistance equal to that of one hundred divisions of the slide wire. By means of switch S a different number of these coils may be included between it and P. The total equivalent length of wire between the farthest positions apart of S and P, when at the hundredth scale division, is 1500 divisions.

Across the ends of the slide wire and coils are two rheostats and terminals for a single storage cell. One rheostat R has a coarse, and the other *r* a fine, adjustment.

There are also terminals for G the galvanometer, and a special tap key K, which, if lightly pressed, switches in the galvanometer in series with a high resistance. A further pressure cuts out some of this resistance, and a still harder pressure cuts it all out. The first pressure should be used until balance is almost obtained.

Pairs of terminals, one marked positive, the other negative, are provided on the potentiometer, and these are controlled by a main switch M, which connects any given pair to the potentiometer wire. The second pair from the left is connected by the dotted connections down to a pair of segments of M. The other pairs are similarly connected. Also one half *a*, of the switch contact piece is insulated from the other *b*.

A Clark standard cell fitted with a thermometer is used to calibrate the potentiometer. A short scale of temperatures is provided on the potentiometer and runs close to the wire. If the temperature is 15° C. and P is brought opposite to 15° C. on the temperature scale, the reading of P on the slide wire scale will be 34 divisions.

If S is on the fourteenth segment, the total length of wire up to P is 1434, and 1·434 is the voltage of a Clark's standard cell at 15° C.

P is left at the point 15° C. and the standard cell balanced along the wire by *only* adjusting the battery rheostats R and *r*. Then each scale division represents 0·001 volt, and each of the 14 coils of S, 0·1 volt. The instrument is now arranged for testing any required voltage within the limits of its range.

In this test, care must be taken to connect the terminals to those of right polarity. R and *r* must not be changed, and the only adjustments needed are made by moving P and S. If a balance is obtained for S at 9, and P at 54 scale divisions, the voltage required is 0·954.

A *volt box* is used when higher voltages are to be measured. It consists of a high resistance with tappings from it; one pair including a tenth of it, and another a hundredth. The test voltage is applied to the high resistance, and a tenth or a hundredth is applied to the potentiometer. This is shown at the second pair of terminals.

Current is measured by sending it through suitable known resistances and measuring the voltage across the latter by the potentiometer. This is shown at the last pair of terminals.

Power is measured by measuring current and voltage, and taking the product.

A Clark standard cell is not absolutely necessary, for any good type of standard cell, such as a cadmium cell, may be used if its voltage for different temperature is known. In the absence of a special calibration, the following values may be taken for the voltage of this type of cell.

Temp. 0° C.	10	15	20	25
Voltage	1·01892	1·01877	1·0186	1·0184

Measurement of medium values of resistance.—These are usually measured by the dial or a post-office box. Clean plugs, clean contacts, and good keys are essential for accurate work. The plugs may be cleaned by rubbing them on smooth paper or cloth.

The sensitiveness of the bridge arrangement depends upon the values used in the ratio arms. Thus, 100 to 10 may not give as much sensitiveness as 1000 to 100. For maximum sensitiveness the galvanometer should also be placed across the junction of the two arms having the greater resistances, and that of the two having the lesser resistances, as shown in Fig. 75. If the battery has a larger

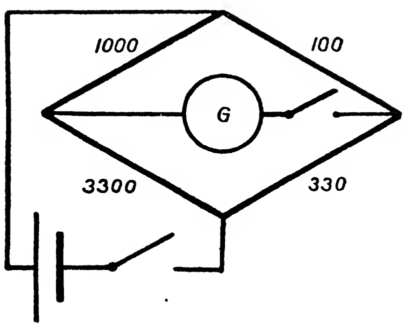

FIG. 75.—The best relative positions of galvanometer and battery in bridge testing.

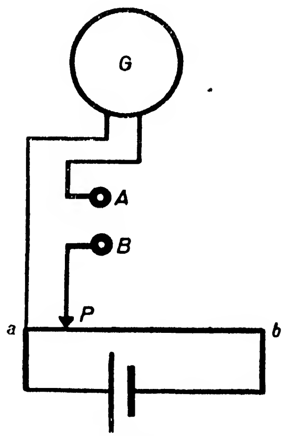

FIG. 76.—A substitution method of measuring resistance.

resistance than G, which is rarely the case, then the former should occupy the position of G shown in the diagram.

A *substitution method*, such as that illustrated in Fig. 76, is sometimes useful. A cell of steady voltage is connected to, a piece of german silver wire *ab*, and if necessary an adjustable resistance. The unknown resistance X is placed across AB, and a small voltage, represented by the length *a*P of the wire, produces a current through G. P is adjusted until the deflection on the galvanometer scale is as large

as possible. X is removed and a resistance box substituted for it, and adjusted until the same deflection as before is obtained without altering the position of P. X is equal to the resistance used in the resistance box.

A simple and useful method of finding the *resistance of a galvanometer*, either by itself or coupled up to some resistance, follows from the preceding method. Thus, with AB short circuited, P is moved along *ab* until a large deflection D is obtained. Leaving P in the same position, a resistance box is placed across AB and a value R selected which reduces the deflection to a value *d*, about one half the value of D.

If e is the voltage across aP, then

$$e = i_1 G = k DG,$$

and

$$e = i_2(G+R) = kd(G+R);$$

$$\therefore \ G = \frac{d}{D-d} \cdot R.$$

Sometimes the value of the resistance of G in series with a coil is needed. In this case the resistance of the coil is considered part of the galvanometer resistance, and the total may be found by this method.

Measurement of resistance which contains a polarising voltage.— This polarising effect is either eliminated as far as possible by using alternating current, or its effect is neutralised in the galvanometer circuit. The latter is adopted in *Stroud and Henderson's direct-current method* of measuring the resistance of a liquid. This is illustrated in Fig. 77. Equal ratio arms PP are used, and R the third arm is supple-

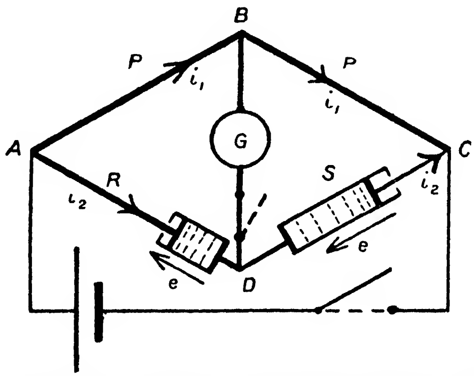

FIG. 77.—A direct-current method of measuring liquid resistance; Stroud and Henderson's method.

mented with a short column of the liquid under test, while a longer column makes the fourth arm S of the bridge.

Balance is obtained by adjusting R, and, if necessary, the difference between the lengths of the two liquid columns. When both keys are down and the deflection of G is zero, the voltage between A and B is the same as between A and D, so that

$$Pi_1 = e + (R + r_1)i_2.$$

Similarly the voltage between B and C equals that between D and C, so that

$$Pi_1 = e + r_2 i_2,$$

r_1 and r_2 being the resistances of the short and long columns of liquid. Therefore

$$r_2 - r_1 = R.$$

If σ is the specific resistance of the liquid at the temperature of testing and l_1 and l_2 the respective lengths in cms. of the short and long columns, then, assuming the cross-sectional area of the latter to be the same and of value a sq. cms.,

$$\sigma = \frac{Ra}{l_2 - l_1}.$$

The connections for this test, using a post-office box, are shown in Fig. 78.

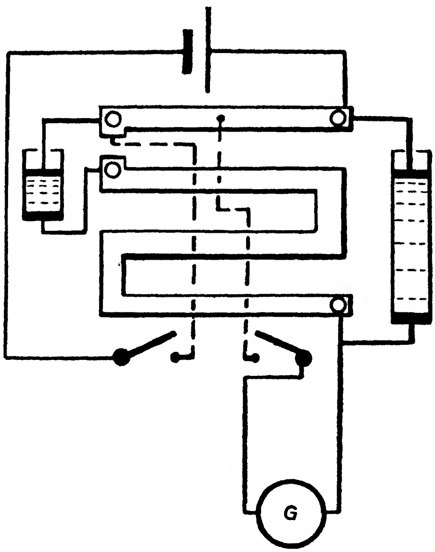

FIG. 78.—Stroud and Henderson's arrangement; using a post-office box.

Alternating-current and telephone-receiver method of measuring resistance of liquids: Kohlrausch's method.—The apparatus and

connections for this method are shown in Fig. 79. A telephone receiver is used in place of a galvanometer, and a buzzer or a source

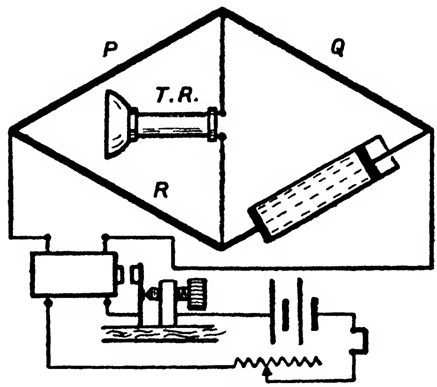

FIG. 79.—Kohlrauch's arrangement for measuring liquid resistance.

of weak alternating current is used instead of the battery. By adjusting R, and if necessary the length of liquid, a balance is obtained, that is, silence or minimum sound is indicated by T.R. The resistance of the liquid is then given by

$$\frac{QR}{P}.$$

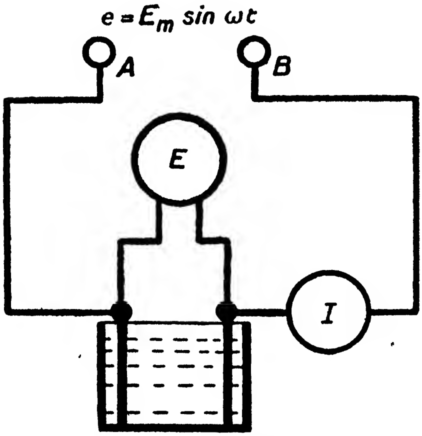

FIG. 80.—The alternating current arrangement for measuring liquid resistance; using an ammeter and voltmeter.

In testing liquids such as copper sulphate, the electrodes may be of copper well cleaned with nitric acid and afterwards thoroughly washed with water. Platinum electrodes may be generally used, and it has been found that a coating of platinum black upon them increases the sensitiveness of the telephone receiver.*

The alternating-current method; using an ammeter and voltmeter.—For liquids in bulk this is a simple and sufficiently accurate method for many purposes. The arrangement of connections is shown in Fig. 80, and the value of the resistance is found by dividing the reading of the voltmeter by that of the ammeter.

* The frequency of the current used should preferably be about 1000 per second, and the telephone receiver tuned for this frequency.

The measurement of temperature by change of resistance ; Callendar's pyrometer.—Platinum wire is used in this pyrometer, and temperatures to a small fraction of a degree and high temperatures exceeding 1000° C. may be measured by the instrument. The latter and its connections to the bridge are shown in Fig. 81.

P, the pyrometer, consists of,—a coil of fine platinum wire wound non-inductively on a mica frame ; two thick platinum leads from the

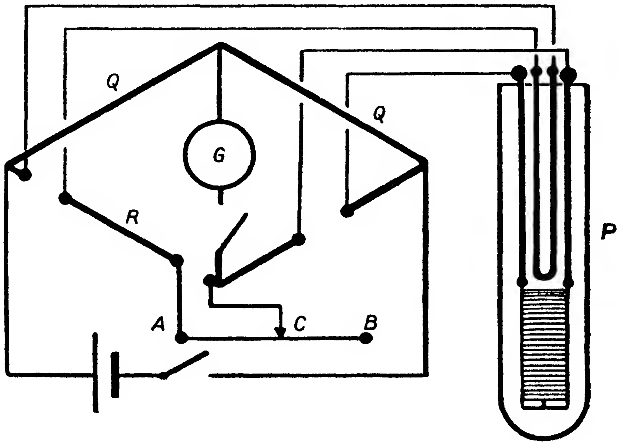

Fig. 81.—The arrangement of Callendar's pyrometer.

ends of the coil ; a compensating loop of platinum of the same resistance as the two leads of the coil ; and a porcelain tube, to protect the coil, fitted with four terminals at its top.

Equal ratio arms QQ are used in this test, and the compensating loop of P is included in the third arm, which also contains a sliding wire AB of total resistance equal to the smallest resistance of R. The fourth arm contains the coil of P and its leads.

In testing an unknown temperature, the end of the tube containing the coil is well immersed in the heated locality, and a balance obtained. The resistance of the coil part of the pyrometer at this temperature is the value of R, plus the resistance of AC, the part of the slide wire used. The uncertain effect due to the leads of the coil being at different temperatures, is eliminated by the use of the compensating loop.

Charts giving the relation between the resistance and temperature are generally supplied with the pyrometer. If necessary it may be calibrated by measuring its resistance when immersed in ice, boiling water, and in the vapour of boiling sulphur, which at 760 mm. of mercury pressure has a temperature of 444·5° C. These values may then be used in the equation

$$R = R_0(1 + at + bt^2),$$

which represents the variation of resistance of pure platinum wire

with temperature, and the constants a and b obtained. A curve relating R the resistance of the coil and temperature in degrees centigrade may then be drawn.

Testing insulation.—Insulation has in general to be tested for the determination of its electrical resistance, dielectric strength, capability of withstanding high or moderate temperatures, absorption power for moisture, the condensing action of its surface, its permittance of the creeping of moisture along its surface, and its mechanical strength under one or more of the stresses, which materials may be subjected to.

The rating of an insulation will therefore depend upon the temperature and the degree of moisture present in the locality in which it is to be used; upon the nature of its surface and also upon the physical characteristics of the materials with which it is more or less directly in contact with.

The conditions of the test of a sample of insulation should be clearly stated, and these should as far as possible be the same as those under which the material is to be used.

Insulation resistance or insulativity.—The resistance in megohms of a cylinder one cm. in length and one sq. cm. in cross section of an insulating material, measured from one end to the other, is termed its insulation resistance or insulativity. If R is the resistance of a length l cms. of insulation, and its cross-sectional area is a sq. cms., σ the insulativity is given by

$$\sigma = \frac{\mathrm{R}a}{l} \text{ megohms.}$$

Insulation resistance is generally determined by ordinary laboratory apparatus or by special testing sets. In this determination the temperature should always be noted, as, in general, σ rapidly decreases with increase of temperature. For instance, flint glass at 60° C. has about *one-twelfth* the insulativity that it has at 20° C.

For ordinary temperatures ebonite or vulcanite of good quality, protected from the influence of too intense light, is one of the best insulators. Rubber, flint glass, gutta-percha, porcelain are also good; their insulation resistance being more than 10^8 megohms per cm. cube. Mica is probably the best insulator for temperatures ranging from 100° to 700° C., and is also good at ordinary temperature.

Insulation or high resistances by the mirror-galvanometer method.—The apparatus and connections for this test are shown in Fig. 82. The galvanometer is placed in series with a known resistance R, as large as possible, consistent with sensitiveness. Adjust R_1, R_2, and R if necessary until a sufficiently large deflection D_0 is observed on the galvanometer scale.

Then, if E_0 is the voltage of the steady cell C,

$$E_0 \cdot \frac{R_1}{R_1+R_2} \cdot \frac{1}{R+G} = kD_0, \qquad\text{..............................}(a)$$

k being the constant of the galvanometer.

Now substitute x, the unknown resistance, for R, or better leave R in the galvanometer circuit and add x. If the deflection is then very small, as it will be for high values of x, arrange the latter as shown in the lower diagram, using a higher voltage E. Then

$$\frac{E}{x+b}=kD, \quad \text{...}(b)$$

b being the resistance of the battery, which may be neglected, and D the deflection. Therefore, from (a) and (b),

$$x=\frac{E}{E_0}\cdot\frac{D_0}{D}\cdot\frac{R_1+R_2}{R_1}\cdot(R+G).$$

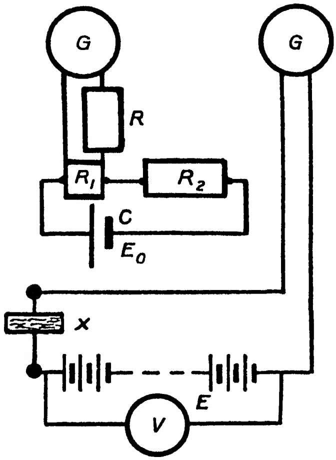

FIG. 82.—A galvanometer arrangement for determining high resistances.

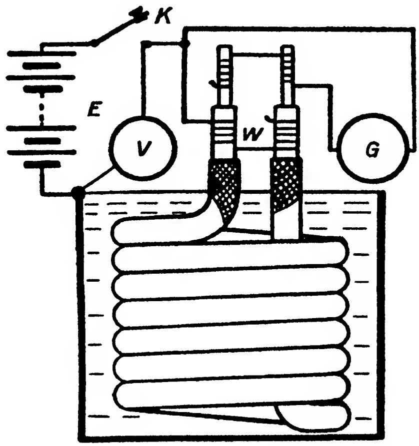

FIG 83.—Arrangement for testing the insulation resistance of a cable.

EXAMPLE.—E_0 is 1·07 volts, R_1 1000 ohms, R_2 9000 ohms, R 200,000 ohms, G of negligible resistance compared with R, E 200 volts, D_0 300, and D 50 divisions. Find x the insulation resistance. *Ans.* 2243 megohms.

By using suitable values of R, R_1, and R_2, this method may be used for a large range of values of x; from a few hundred ohms or lower up to a considerable number of megohms, depending upon the sensitiveness of G.

In Fig. 83 this method is shown applied to the case of a cable. This diagram represents the second part of the method, the first part being to find the constant of the galvanometer. The coil of cable is placed in a metal tank containing water in which some common salt is dissolved, and is generally immersed 24 hours before making the test. The two extreme ends of the cable are bared and the outside protective covering is stripped off for a length of about 8 to 12 inches,

as shown. Connections are then made to galvanometer, voltmeter, and battery, as in the diagram.

An outside leakage current may flow from the water by way of the surface of the cable ends. G would thus receive the dielectric and leakage currents, and the deflection would be too large; the insulation resistance would work out too small. By means of *Price's guard wire W*, this leakage current is shunted from G, and the latter only indicates the dielectric current which passes from the water through the insulation to the metal core of the cable.

The testing voltage should be about 50 per cent. higher than the rated value of the cable; the instruments and battery should be placed on blocks of insulation; G should be provided with a short circuiting key, as on closing K the battery switch, there may be a sudden rush of current through the circuit, lasting but a short time, and more violent the longer the length of cable.

Owing to the absorption effect of the insulating material of the cable the deflection of the galvanometer does not remain constant, but decreases as time goes on, and therefore it is necessary to state the time which elapses between closing K and reading G. Also insulation resistance decreases considerably with a moderate rise of temperature, and the latter should be recorded.

Thus, in recording the insulation resistance of a cable, the time of immersion in water, the voltage, temperature, and the time of applying the voltage should be stated. A graph relating apparent insulation resistance and time of applying the test voltage for a given temperature should also be obtained.

In this and other deflection methods of measuring insulation resistances, if x is so large that no deflection is indicated on the scale, instead of stating that the insulation resistance is practically infinite, it is better to assume that D, the deflection, is about *one-tenth* of a scale division or of value as small as could just be detected. Then x is calculated on this assumption, and the statement that x is at least greater than the worked-out value, is its best possible representation with the apparatus employed.

When an insulation resistance has to be tested at a *high* voltage, usually 50 per cent. higher than its working voltage, a large resistance should be added to x in the second part of the test, and then gradually reduced if the deflection is small. An adjustable known resistance may also be used with the galvanometer.

A set of known high resistances ranging from one quarter to one or two megohms is necessary in this and other tests. High resistances may be constructed from clay-pipe stems or strips of ground glass by marking on them a line of graphite from a pencil. The resistance of the line depends upon the length, width, and depth of deposit. A larger deposit at the ends of the line may be electro-plated with copper and then covered with thin copper wire. This resistance may be placed in a small wooden trough, which is then filled with paraffin wax, and the ends of the resistance connected to two terminals fixed on the trough.

The resistance of the insulation of a cable.—Consider one centimetre length of the cable, and let r_1 be the outer radius of the conductor, and r_2 the outer radius of the insulating covering. Also let x be the internal and $x+dx$ the external radius of an elemental tube (Fig. 84), co-axial with the axis of the cable. Then, if R_0 is the

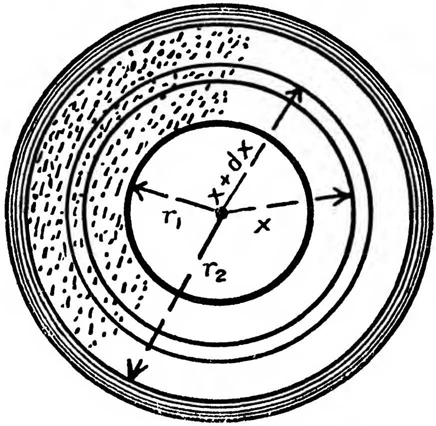

FIG. 84.—A cross section of an insulated conductor.

insulation resistance of one centimetre length of the cable in *megohms*, and σ the insulativity of its insulation,

$$R_0 = \int_{r_1}^{r_2} \frac{\sigma \, . \, dx}{2\pi x} = \frac{\sigma}{2\pi} \log_e \frac{r_2}{r_1},$$

and if R is the resistance of the insulation when the length of cable is l cms.,

$$R = \frac{\sigma}{2\pi l} \log_e \frac{r_2}{r_1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

When R is the insulation resistance of the cable in megohms per mile,

$$\sigma = 4{\cdot}39 \times 10^5 \times \frac{R}{\log_{10} \frac{r_2}{r_1}} \text{ megohms.}$$

If there are layers of different insulating materials around the conductor and r_2, r_3, etc., are the bounding radii of these layers,

$$R = \frac{1}{2\pi l} \left\{ \sigma_1 \log_e \frac{r_2}{r_1} + \sigma_2 \log_e \frac{r_3}{r_2} + \ldots \right\}.$$

By finding R from the preceding experiment and measuring l, r_1 and r_2, the value of σ may be obtained from formula (a).

EXAMPLE.—The insulation resistance of two miles of a cable is 500 megohms and the radius of the conductor is 0·5 inch. If the thickness of the insulating covering is 0·6 inch, find its insulativity. *Ans.* 1280×10^6 megohms.

In determining the value of σ for samples of insulation in the form of thin sheets it is difficult to obtain consistent results by placing the sample between metal plates and pressing the latter together by weights. A much better arrangement is that of *R. Appleyard,** the principle of which is illustrated in Fig. 85. The sample is placed vertically

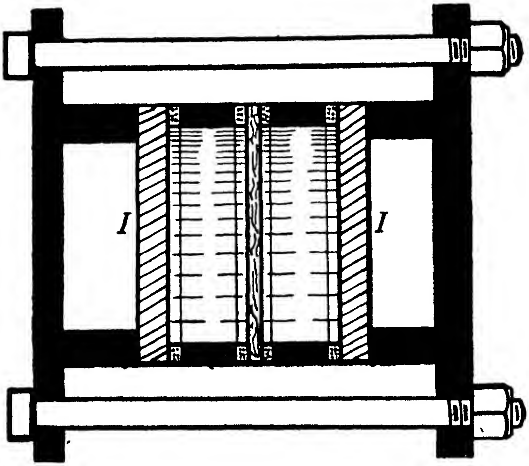

FIG. 85.—Arrangement for holding thin sheets of insulating material when testing their resistances; Appleyard's method.

between two flat rings of ebonite faced on each side with india-rubber. A disc of iron I is placed against each ring and clamped together as shown by an ebonite vice. Mercury is poured into the spaces between the iron discs and the sample by means of a hole at the top of each disc. A thermometer placed in the mercury gives the temperature. The insulation resistance of the sample may then be tested by a good method and the value of σ determined.

Another good arrangement is to use a sample of tabular form closed at one end, with mercury electrodes inside and outside the tube; several centimetres of the latter projecting above the surface of the mercury.

Kelvin's testing set.—This is a compact set for determining insulation resistances with considerable accuracy, from a fraction of a megohm up to about a thousand megohms. The instrument is illustrated in Fig. 86.

Its galvanometer circuit has a resistance G of value 50,000 ohms, whatever the shunt used. This is arranged for by the provision of compensation resistances (C.R.), which are thrown in this circuit simultaneously with throwing in the shunts.

* *Phil. Mag.* (6), vol. 10, 1905, p. 485.

For the range $t=0$ and $t=10\times 60=600$ seconds, the value of x rom the formula is

$$\frac{10\times 60\times 10^6}{2\cdot 3\times 1\times \log_{10}\frac{100}{67}}=1490 \text{ megohms.}$$

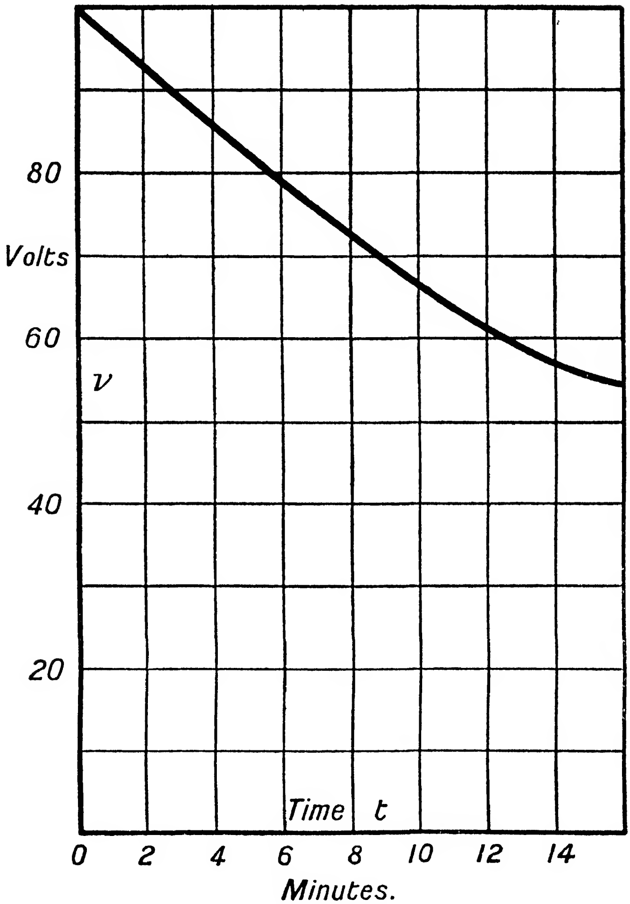

FIG. 89.—The volt-time curve for the leakage test of insulation resistance.

For the range $t=2$ and $t=8$ minutes, x is similarly found to be 1500 megohms.

Before making this test, the insulation of the voltmeter and condenser should first be tested by charging the condenser, with x disconnected, and noting whether the reading remains constant. If there is leakage, the connections and surfaces of the condenser and

voltmeter between their terminals should be examined for the cause. With a good mica condenser and a dry room, there is generally very little leakage in the arrangement itself.

The preceding methods may be used for testing the insulation resistance of samples of insulation under different conditions of temperature, moisture, and mechanical stress; for the insulation in stationary electrical machines; and for *dead* wiring circuits or networks of conductors.

Determination of dielectric strength.—The dielectric strength of a material or its ability to withstand rupture when exposed to electric stress, depends upon the physical conditions and the form of the material, and also upon the manner of applying the stress.

A material of specified size, shape, physical condition, exposed to a uniform potential gradient throughout its mass, has a definite dielectric strength represented by the fall of potential per centimetre of thickness which is needed to rupture or break down the insulation.

The shape of the metal or conductive surfaces across which the voltage operates is one of the most important factors in the production of rupture. The breakdown will begin at the place of maximum potential gradient, that is, where

$$\frac{dV}{dx}$$

is a maximum. In other words, where the electric stress is a maximum.

In the case of a charged sphere the potential gradient at outside points x cms. from its centre is given by

$$-\frac{dV}{dx}=\frac{Q}{x^2},$$

Q being the quantity on the sphere.

The electric stress is therefore a maximum at the surface, and falls off rapidly as x increases.

In the case of a long charged cylinder the external maximum stress is at its ends; at middle parts its value is given by

$$-\frac{dV}{dx}=\frac{2q}{x},$$

according to page 30.

If the surface potential gradient is higher than that necessary to rupture air, the sphere, for instance, will be surrounded by a layer of air whose insulation has broken down and which has become a gaseous conductor, exhibiting brush discharge. Outside of this envelope will be layers of air of unimpaired insulation, so that this type of breakdown may be termed a partial one.

Practically it is impossible to subject an insulating material to a uniform potential gradient across its thickness. Thus, if a sheet of insulation is placed between two metal plates, there will be a maximum gradient at the corners, and that along the edges will be greater than

that for parts well covered by the plates. A spherical condenser with thin dielectric is the nearest approach to the case of uniform potential gradient, but even here there must be a hole in the outer envelope to allow the metal connection of the inner sphere to be brought out.

It follows, then, that the division of the applied voltage by the thickness of the sample across which it acts gives no exact representation of the dielectric strength of the material.

Whatever the method adopted for determining dielectric strength, no test is completely represented unless the physical conditions and, as far as possible, the past history of the sample are given ; and also the size, shape, nature of surface, and distance apart of electrodes.

In practice, most of the testing of insulation for dielectric strength is done with alternating voltages, as very high values of direct voltage are difficult to produce. The R.M.S. value of the disruptive voltage as read on the electrostatic voltmeter must be multiplied by the *amplitude factor* of the voltage wave, to obtain the maximum value of the break-down voltage.

A suitable step-up transformer may be used to supply the disruptive voltage. The secondary coil will consist of sections of fine wire of many turns ; each section insulated from the others, and the whole coil well insulated from the primary coil and iron core.

If the primary is fed by a single-phase alternator, driven by a direct-current shunt motor with adjustable speed, then, by adjusting the field current and the speed of the alternator, the primary, and therefore the secondary volts may be varied through a large range.

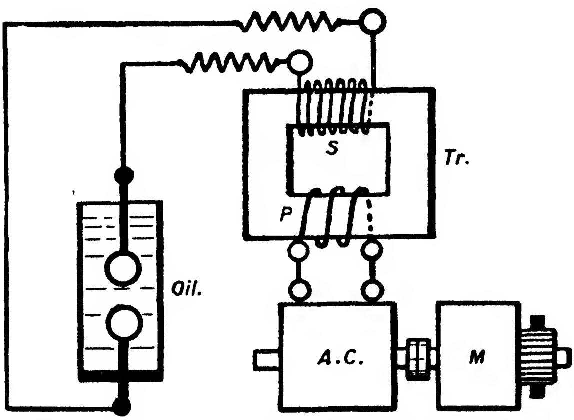

FIG. 90.—Arrangement for testing the dielectrie strength of an oil.

The sudden break-down of the insulation is accompanied by surging of the voltage and current, which is very violent at high voltages. This surging is considerably damped out by using a non-inductive resistance at each of the secondary terminals, as in Fig. 90.

A suitable value of each resistance, according to A. Russell, is about one-half ohm per volt of the secondary coil.

The sample connected to the secondary terminals may thus be subjected to a gradually increasing voltage until rupture occurs. For exact determinations, the wave form of the testing voltage would be necessary in order to obtain the amplitude factor.

A. Russell's method of determining dielectric strength of insulation.* —In this method an accurate determination of dielectric strength may be made. Two *spherical* electrodes are used, and in the case of solid dielectrics should preferably be embedded in the material. Russell has shown that when these electrodes are separated a distance apart not greater than twice the diameter, a disruptive discharge will occur the instant the maximum electric stress between them is equal to the dielectric strength of the material. He has also shown that the maximum electric stress between the two spheres, each of radius r, one at potential $+\frac{V}{2}$, and the other at $-\frac{V}{2}$, is given by

$$\frac{V}{x} \cdot f,$$

V being the voltage between the spheres, x the *minimum* distance between the surfaces of the spheres, and f a coefficient depending upon the value of $\frac{x}{r}$. Some of the values of f given by Russell are here tabulated:

$\frac{x}{r}$	0·2	0·3	0·4	0·6	0·8	1·0	1·5	2·0
f	1·068	1·102	1·137	1·208	1·283	1·359	1·559	1·77

An example of this method is as follows. The breaking down R.M.S. voltage of an insulation was 80 kilovolts, and the amplitude factor of the voltage wave was 1·45. The spherical electrodes were each of one cm. diameter and separated by a minimum distance from surface to surface of 0·4 cm. It is required to find the dielectric strength of the material in which the spheres are embedded.

In this case $$\frac{x}{r} = \frac{0{\cdot}4}{0{\cdot}5} = 0{\cdot}8,$$

and from the preceding table $f = 1{\cdot}283$.

The *dielectric strength* is therefore

$$\frac{1{\cdot}45 \times 80 \times 1{\cdot}283}{0{\cdot}4} = 373 \text{ kilovolts per centimetre.}$$

The spherical electrodes, though they should be at a *less* distance apart, from centre to centre, than twice the diameter, should not be

* *Phil. Mag.* (6), vol. 11, 1906, p. 258.

too close together; they should also be smooth and well polished. Also, it is desirable to earth the middle point of the secondary coil of the transformer, so as to have the potentials of the electrodes equal and opposite at the discharge through the sample.

The dielectric strength of air under normal conditions determined by the preceding method has been found to be 38 kilovolts per centimetre, and for the range of the method is very nearly constant.

The maximum value for the disruptive voltage of the air between two spherical electrodes, each of radius 2·5 cms., and separated by a minimum distance from surface to surface of 3 cms., was found to be 80 kilovolts. To calculate the dielectric strength of air,

$$\frac{x}{r}=\frac{3}{2\cdot 5}=1\cdot 2, \quad f=1\cdot 438.$$

Therefore the dielectric strength of the air is

$$\tfrac{80}{3}\times 1\cdot 438=38\cdot 3 \text{ kilovolts per centimetre.}$$

Example.—The dielectric strength of air at normal conditions is 38 kilovolts per centimetre. Calculate the R.M.S. voltage, of amplitude factor 1·52, necessary to break down the air between two spherical electrodes, each of radius 3 cms., and separated by a minimum distance from surface to surface of 2·4 cms. *Ans.* Nearly 47 kilovolts.

The constancy of the dielectric strength of air under ordinary conditions renders this method valuable for determining the voltage generated in circuits containing spark gaps.

In practice, what is generally needed much more than individual dielectric strengths of isolated insulating materials, is the dielectric strength of composite insulation assembled in a certain way between special metal surfaces such as the insulation of the slot of an electric machine or that of a cable.

In the case of the slot insulation, its dielectric strength will depend among other things upon whether the conductors are round or rectangular, large or small. The greatest stress will occur at metal corners and edges, and a film of air or *air pocket* in the slot insulation will in general constitute the weakest part.

If such an air film is ruptured, ozone and oxides of nitrogen are formed, and these oxidise nearly all the insulating materials used in the slots of electrical machines. This oxidisation impairs the insulating property and makes the insulation brittle, so that it easily powders under vibration.

The liability of an air film to break down, when present in composite insulation, between metal surfaces is illustrated by the following experiment. Two metal plates or spheres are separated by an air space of one cm. for instance, and are subjected to a high voltage not quite sufficient to rupture the air between the plates. A slab of glass or a number of sheets of mica of thickness 0·8 cm. is then placed between the plates, and the layer of air, 0·2 cm. thick, is at once

ruptured. This is accounted for by the fact that the potential gradient of a dielectric is represented by

$$\frac{1}{k}\frac{dV}{dx},$$

k being the specific inductive capacity of the dielectric, which in the case of glass may be about 6, and is unity for air.

The presence of the glass, therefore, lowers the potential gradient across its own width much less than the same width of air would do, thus leaving a considerably higher gradient across the 0·2 cm. layer of air than in the original case when the glass was away.

For this and other reasons, a thorough impregnation of armature coils with an insulating compound of higher specific inductive capacity than air is necessary for high-voltage machines.

An investigation of dielectric strength of composite insulation for slots may be made by using experimental slots in which conductors of different shape and size, and insulated with different grades of insulation, may be tested.

In the case of a single-core cable the potential gradient is a maximum at the surface of the conductor, and therefore, in order to diminish this gradient as much as possible, a suitable insulation of large specific inductive capacity should be used as the first covering of the cable. The second covering may have a smaller specific inductive capacity, and so on. This *grading* of the insulation of a cable is of great importance in the case of high-voltage cables.

CHAPTER VII.

TESTING LIVE CIRCUITS AND THE LOCATION OF FAULTS ON THEM.

THE determination of the insulation resistance of a circuit is a means of ascertaining whether it is free from serious faults or not. In practice it is often necessary to make this determination when the circuit is *live*, and special methods of doing this are required. Some of these will now be given for simple cases of the usual types of circuits.

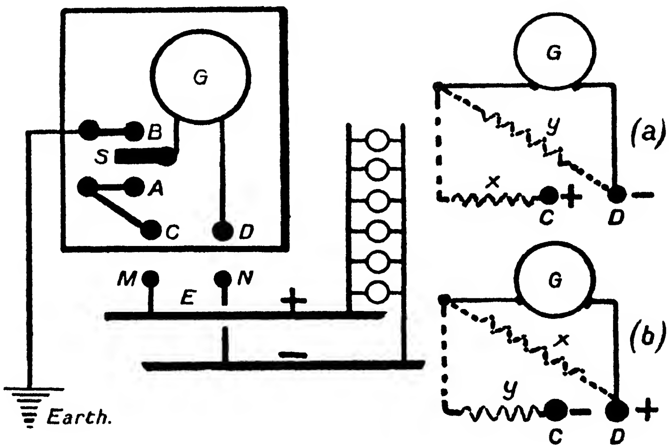

FIG. 91.—An arrangement for testing a live cable; galvanometer method.

In most of the following tests an earthing point is employed. This may be a connection to a system of water pipes. Gas piping should not be used. Masses of old iron, or other metal, buried about ten feet below the ground in moist earth, and preferably surrounded by coke, may be used for earthing.

The galvanometer or Kelvin's testing-set method.—For voltages not exceeding about 400 the Kelvin set is a convenient instrument to make this test with, provided care is taken in dealing with such voltages. A diagram of the connections necessary for making a test on a *live* two-wire system is shown in Fig. 91. The shunts and

compensating resistances of the set are not shown. The terminal C is *first joined* to M and D to N.

Let x be the insulation resistance of the positive main and all the *positive* connections to the consuming devices of the system, and y that for the *negative* side of the system.

S is placed on A, and the deflection multiplied by the shunt power is obtained, namely D_0S_0. In this case the voltage E of the mains acts only through the resistance G equal to 50,000 ohms. Thus

$$\frac{E}{G}=k\,.\,D_0S_0. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(a)$$

Next S is placed on B and D_1S_1 obtained. E acts through a circuit illustrated in diagram (*a*); first through x and then through a multiple arc of value

$$\frac{Gy}{G+y}.$$

If I is the main current, that is, the current which passes through x, the value of the current in the galvanometer producing deflection D_1 is

$$\frac{y}{G+y}\,.\,I,$$

and I is given by, $$I=\frac{E}{x+\frac{Gy}{G+y}}=\frac{E(G+y)}{Gx+xy+Gy}.$$

Therefore $$\frac{Ey}{Gx+xy+Gy}=kD_1S_1. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(b)$$

Finally, terminal M is joined to D and N to C; also S is placed on B. The voltage E thus acts through the circuit illustrated by diagram (*b*), and the deflection multiplied by the power of the shunt, namely D_2S_2, is obtained. The following equation is then similarly derived as the last:

$$\frac{Ex}{Gx+xy+Gy}=kD_2S_2. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c)$$

From (*b*) and (*c*), $$\frac{y}{x}=\frac{D_1S_1}{D_2S_2}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(d)$$

and from (*a*), (*b*), and (*c*),

$$x=G\,.\,\frac{D_0S_0-D_1S_1-D_2S_2}{D_1S_1},$$

$$y=G\,.\,\frac{D_0S_0-D_1S_1-D_2S_2}{D_2S_2}.$$

If the positive main is badly insulated from earth, the ratio of (*d*) will be very large, unless the negative main is also faulty.

In the case of the positive side of the system being faulty and the negative side well insulated, D_2 will be of small value compared with D_1.

This fault on the positive side may be in one of the consuming devices or in a particular section of the supply mains. By disconnecting these devices in turn, the faulty one may be discovered by observing each time the values of D_1 and D_2. If the fault is in the positive mains, the sections of the latter should be cut out in turn and the insulation tested each time. Similarly for the case of the negative side of the system being faulty and the positive side well insulated.

If both sides are bad and the fuses are still intact, the faulty part may be discovered by the same process of elimination.

The following results were obtained for a *live* two-wire system of 110 volts :

$$D_0S_0\ 2200, \quad D_1S_1\ 37, \quad D_2S_2\ 55\ ;$$

$$\therefore\ x = 50{,}000 \times \frac{2200 - 37 - 55}{37} = 2{\cdot}85 \text{ megohms,}$$

and
$$y = x\,.\,\frac{D_1S_1}{D_2S_2} = 2{\cdot}85 \times \frac{37}{55} = 1{\cdot}92 \text{ megohms.}$$

If the two sides of the two-wire system *when dead* are metallically joined, that is, short circuited, and the insulation resistance is measured between any point on the system and the earth by means of the megger or any other good method, the value F found is termed the insulation resistance of the system. It follows that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{F},$$

x and y being the respective insulation resistances of the positive and negative sides of the system when dead. F calculated from the live system may be somewhat different to that obtained on the dead system on account of increase of temperature and electrolytic action in the earth paths.

For the two-wire 110-volt system, previously referred to,

$$F = \frac{2{\cdot}85 \times 1{\cdot}92}{4{\cdot}77} = 1{\cdot}15 \text{ megohm,}$$

when it is live.

A calibrated sensitive galvanometer in series with a large adjustable resistance R of known value may be used instead of the testing set in the preceding method. This arrangement is connected across the positive side of the system and earth. R is then adjusted to give a suitable current I_1 through the galvanometer. As before,

$$I_1 = \frac{xE}{Rx + Ry + xy}.$$

The same operation is performed for the negative side of the system, and

$$I_2 = \frac{yE}{Rx + Ry + xy}.$$

From these two equations,

$$x = \frac{E - R(I_1 + I_2)}{I_2},$$

$$y = \frac{E - R(I_1 + I_2)}{I_1}.$$

Raphael's method.—A milliammeter A and a resistance R in series with it, are placed across the negative side of the system and earth, as in Fig. 92. The voltage E across the mains and I_1, the current through A, are read. An equal resistance R is then shunted across the ammeter circuit as shown, and I_2 is read.

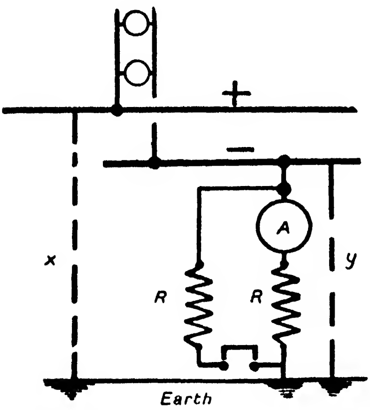

FIG. 92.—An arrangement for testing a live cable: Raphael's method.

From the first operation,

$$\frac{E - RI_1}{x} = I_1 + \frac{RI_1}{y},$$

or $$\frac{E}{x} = I_1 + \frac{RI_1}{F}, \quad \text{......}(a)$$

$\frac{1}{F}$ being equal to $\frac{1}{x} + \frac{1}{y}$.

From the second operation,

$$\frac{E - RI_2}{x} = 2I_2 + \frac{RI_2}{y},$$

that is, $$\frac{E}{x} = 2I_2 + \frac{RI_2}{F}. \quad \text{........}(b)$$

Subtracting (*b*) from (*a*),

$$F = R \cdot \frac{I_1 - I_2}{2I_2 - I_1}.$$

Putting this value of F in (*a*),

$$x = E \cdot \frac{I_1 - I_2}{I_1 I_2}.$$

The value of y may then be found from

$$\frac{1}{F} = \frac{1}{x} + \frac{1}{y}.$$

For accurate determination of F, x, and y, the difference between I_1 and I_2 should not be too small. This will be the case if R is taken roughly equal to F, which may be approximately determined by a

preliminary test. The milliammeter should always be protected by a fuse

Raphael's method for a three-wire system.—A simple illustration of a three-wire system is shown in Fig. 93. The three mains are fed

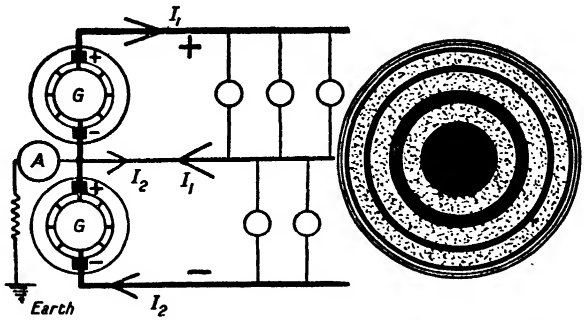

FIG. 93.—A simple three-wire system, and the cross section of a feeder cable.

by two generators in series. As the middle or *neutral* wire only carries the difference of the currents I_1 and I_2 in the two outer mains, its cross-sectional area may be made smaller than that of the outers; usually it is made equal to one-half of their cross section.

These three mains constitute a single cable of cross section, either as shown in Fig. 93, the neutral being nearest the lead sheath, or more commonly of cross section shown in Fig. 94. In both cases the conductors are stranded, that is, made up of wire of small diameter.

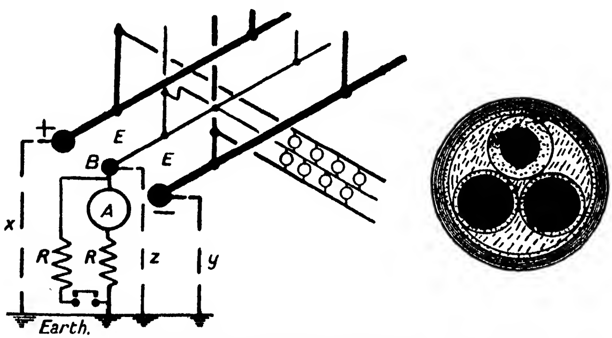

FIG. 94.—Raphael's arrangement for testing a *live* three-wire system. Cross section of a feeder cable.

At the power station, according to the Board of Trade regulations, the neutral must be earth connected through a recording ammeter, fitted with protective devices against any sudden rushes of current due to faults on the outer mains.

The object of this device is to ascertain, whether any excessive earth currents are flowing, due to faults on the system. Such currents

act electrolytically on gas or water piping in the vicinity of the system, and must therefore be reduced to a minimum. If the recording ammeter indicates an excessive earth current, the insulation of the system has to be tested, the fault located and removed.

In Raphael's test the earthing device is temporarily disconnected, and the neutral is connected to earth through an ammeter and resistance as shown in Fig. 94. R is the resistance of this ammeter circuit. The current I_1 is read on the ammeter. A shunt of value R is then placed across the ammeter and resistance, and I_2 is read. Let x, z, and y be the respective insulation resistances of the positive, neutral, and negative sides of the system.

Suppose the potential of B, the neutral main, to be above that of the earth. Then the first operation gives

$$\frac{E + I_1 R}{x} + I_1 + \frac{RI_1}{z} = \frac{E - RI_1}{y},$$

or

$$\frac{E}{x} + \frac{RI_1}{F} + I_1 = \frac{E}{y}, \quad \ldots\ldots\ldots\ldots (a)$$

in which

$$\frac{1}{F} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}. \quad \ldots\ldots\ldots\ldots (b)$$

The second operation gives

$$\frac{E + I_2 R}{x} + 2 I_2 + \frac{RI_2}{z} = \frac{E - RI_2}{y},$$

or

$$\frac{E}{x} + \frac{RI_2}{F} + 2 I_2 = \frac{E}{y}. \quad \ldots\ldots\ldots\ldots (c)$$

Subtracting (c) from (a),

$$F = \frac{R(I_1 - I_2)}{2 I_2 - I_1}.$$

For accurate determination of F, the difference between I_1 and I_2 should be sufficiently large; this will be the case when R and F are roughly of the same value.

Putting this value of F in (a) and (c), the values of x and y may be obtained. Then from (b) the value of z also may be found. The same result follows by taking the potential of B to be below that of the earth.

A more detailed representation of a three-wire system is shown in Fig. 95. B_1, B_2, B_3 are the three bus bars connected to the machines at the station. S is the main switch, F_1, F_2, F_3 the main fuses of *one* of the feeder cables connected to the bus bars. The type shown is called a *ring* feeder, and its chief advantages over the straight type, which does not return to the bus bars, are that it has a lower voltage drop along its length, and one of its sections may be cut out without greatly affecting the working of all the other sections. F.J.B. are

junction boxes on the feeder, to which the distributing cables (D.C.) are connected. These boxes are bridged by copper or fuse links. Similar boxes (D.J.B.) are used on the distribution mains. S.M. are the service

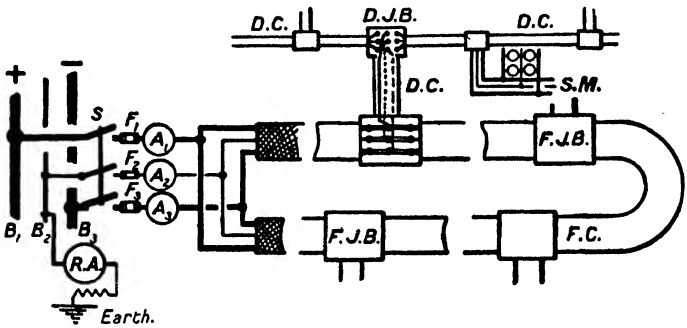

FIG. 95.—A ring feeder; three-wire system.

or consumer's mains, which are connected to the distribution mains as shown. The indoor wiring to the service mains is shown in Fig. 106. A_1, A_2, and A_3 are ammeters, and R.A. a recording ammeter.

The use of lamps for indicating the condition of the insulation of a system of cables.—These lamps are connected from the system to earth, as shown in Fig. 96. Each lamp may be of the same voltage

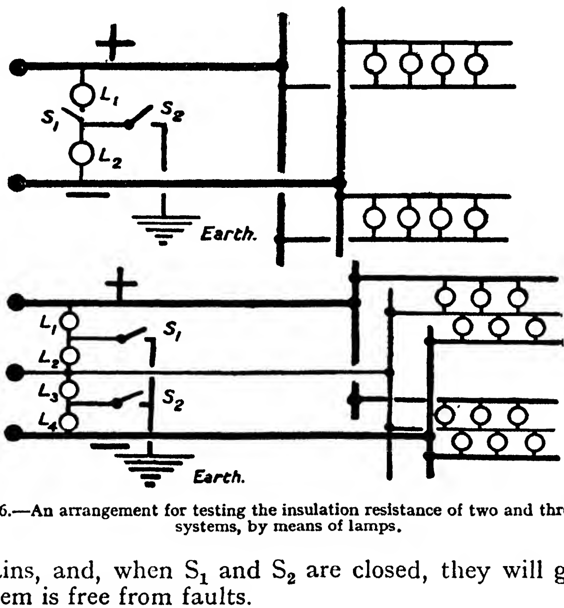

FIG. 96.—An arrangement for testing the insulation resistance of two and three-wire systems, by means of lamps.

as the mains, and, when S_1 and S_2 are closed, they will glow dimly if the system is free from faults.

The upper diagram of the figure illustrates a two-wire system; the lamps* being at the station end of the system. If there is an earth fault on the positive side of the system, L_1 will be darker and L_2 brighter when S_1 and S_2 are closed. L_2 will glow brightly and L_1 be quite dark, if the fault is very bad. Similarly for a fault on the negative side of the system.

By cutting out in turn each section of the system, the faulty section will be discovered by the lamps returning to a state of equal illumination.

If both sides of the system are faulty, L_1 and L_2 may be equally bright, and thus give no indication of the existence of these faults.

If S_1 is open and S_2 is closed, and L_2 glows brightly, then it follows that the fault on the positive side of the system is of much smaller resistance than the lamp itself.

In the three-wire system illustrated in the lower diagram, if the B.O.T. earthing device is removed from the neutral, and the earth lamps are switched in by closing S_1 *or* S_2, L_1, L_2, L_3, and L_4 will be in a state of equal illumination when the system is free from faults.

L_1 will be dark and L_2 bright for a serious fault on the positive side of the system; L_4 dark and L_3 bright for a similar fault on the negative side; L_2 and L_3 dark, and L_1 and L_4 bright for a fault on the neutral.

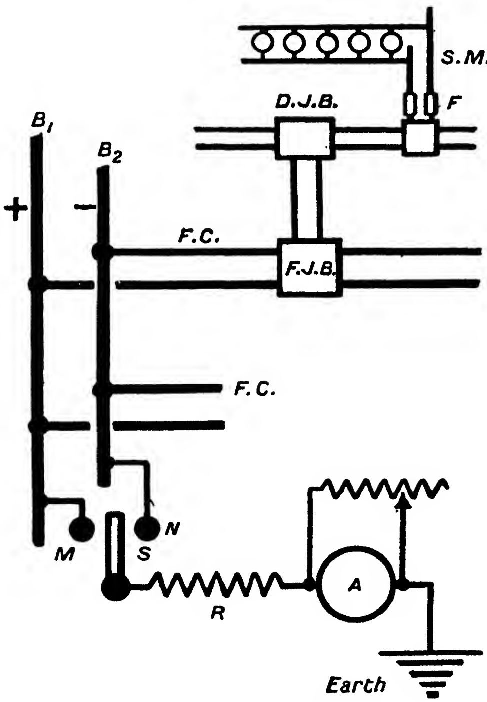

FIG. 97.—The arrangement for the flash test; two-wire system.

Testing a two-wire system by the flash test.—Part of a two-wire system, with an earthing arrangement for making the test, is shown in Fig. 97. R is a high resistance in series with a shunted ammeter A and earth. A fuse may be used instead of the shunted ammeter for rougher indications of the presence of faults.

If a fault occurs on the positive side of the system, suppose on the service mains (S.M.) of a consumer, and switch S is placed on N, the ammeter will give a reading whose value will

* Voltmeters are sometimes used instead of lamps.

depend upon that of R, the resistance of the fault, and the value of the shunt. When the fault is bad, as soon as S is placed on N, the reading of A will suddenly rise to a high value, and on the blowing of the consumer's fuse will immediately fall to a very low value.

Similarly, if there is a bad fault on the consumer's negative main, by placing S on M his negative fuse will blow.

If the fault is on a distribution main or a feeder, the test should be made at a time when the load is lightest. In this case a feeder cable is cut out and the earthing test applied. If the fault is still indicated, the feeder is switched on to the bus bars again and another feeder tested. This will be repeated until the feeder with the faulty distributor is located.

The distributors branching from the feeder are disconnected or transferred in turn to another feeder at the junction boxes in the street pillars or manholes, and the test repeated after each transfer. After one transfer the ammeter will cease to respond, thus indicating the faulty distributor.

When the fault is on the feeder itself, its distributors may be transferred to other feeders and its different sections be disconnected one at a time, and the flash test repeated after each change.

Testing a three-wire system by the flash test.—In this case the B.O.T. earthing device is disconnected from the middle or neutral wire. The connections for this test are shown in Fig. 98. In the

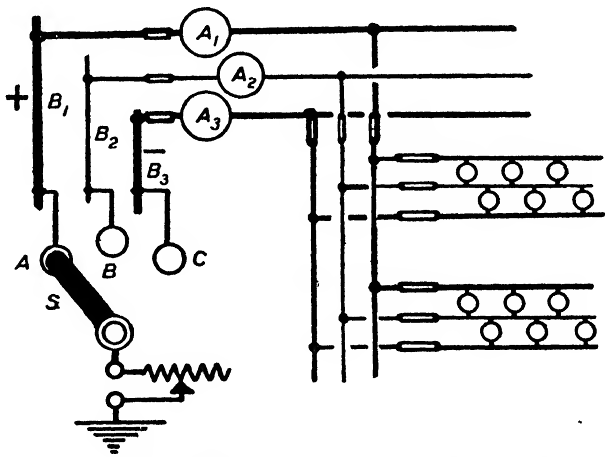

FIG. 98.—The arrangement for the flash test; three-wire system.

case of a fault on the neutral side of the system, by watching the ammeter A_2 when S is suddenly placed on A, a sudden change will be observed in the reading. This is due to the leakage current passing in succession from the positive main through the earth connection, the fault on the neutral, and the ammeter A_2.

This leakage current is also indicated by ammeter A_1, but as the latter carries a much larger load current than A_2, which only carries the out-of-balance current, the indication on A_1 is not nearly so marked.

As the out-of-balance current is subject to frequent and sudden fluctuations, especially at certain hours, this test should be made as quickly as possible when the reading of A_2 is steady, and be repeated a number of times. A similar indication is produced on A_2 by flashing the negative, that is, with S on C.

If the fault is on the positive side of the system, then, when S is on A, there will be no indication of its presence, provided the rest of the system is well insulated from earth. In this case S is placed on B, and a sudden increase in the reading of A_1 will be noted, provided the fault is bad enough.

This movement of S on B will produce an increase in the reading of A_3 if the fault is on the negative side.

Also, when the fault is on the positive side and S is placed on C, an increase will be observed in the reading of A_1, while, if it is on the negative side, and S is placed on A, an increase will be observed in the reading of A_3.

In normal operation, with the B.O.T. device used at the neutral, if a serious fault occurs on the positive or negative main of a consumer, his positive or negative fuse will at once blow. When this occurs, the station engineer should be at once informed. A serious fault on his neutral wire may not blow the neutral fuse, but if the fault provides a sufficiently good shunt path through the earth for the currents in the neutral conductors of the distributors and feeder cable, an excess current will be indicated by the station's recording ammeter, or the consumer's neutral fuse * will blow. The latter fuse will at once blow during the application of the flash test, that is, whenever the positive or negative sides of the system are flashed or earthed.

The earthing arrangement in these tests should be suitably fused to prevent any excessive leakage currents passing through the ammeters which indicate them, and through the resistance and switch gear used in the test.

Simple alternating-current networks.—It is difficult to determine the insulation resistance of such a network chiefly on account of the capacity effect between adjacent conductors, and between them and earth.

A simple single-phase network similar to that of Fig. 97, with its two sides insulated from earth, may be tested by means of the flash test or by earth lamps, and the fault located by the process of elimination, that is, if the blowing of a fuse does not locate it.

If one of the two wires is permanently earthed, the fuse of the other will blow when a serious fault occurs on it. The consumer's fuse will blow if the fault is on the unearthed service main. If on a distribution main, a fuse in the junction box will blow.

* In general this fuse is made much heavier than the other fuse.

In a three-phase system of distribution the power is generally generated at a high voltage, and then transformed down to working voltage. Fig. 99 shows the distribution mains of one of the three-phase step-down transformers of a substation. Three separate

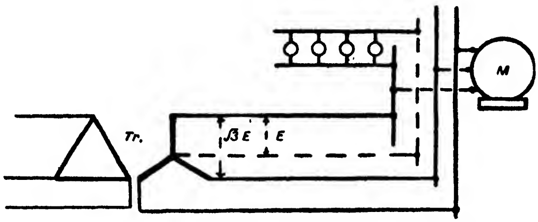

FIG. 99.—Three-phase distribution mains; four-wire system.

transformers, one for each phase, are often used instead of a single three-phase transformer.

The phases of the primary are mesh connected, and those of the secondary are star connected. The neutral wire on the distribution side of the transformer is earthed, and the lamp load is fed by an outer and the neutral. Power for motors is taken from the outers.

Thus a consumer with only lighting on his premises has an outer main, and a neutral wire which is earth connected. If a serious fault occurs on the outer, its fuse will blow; if on the outer of a distribution main, the fuse of this outer in the junction box will blow.

The growth of leaks on an alternating current network may be observed by *Sahulka's method* of connecting an electrostatic voltmeter across an outer and earth, and connecting the same outer with a sufficiently large adjustable non-inductive resistance R to earth, as shown in Fig. 100.

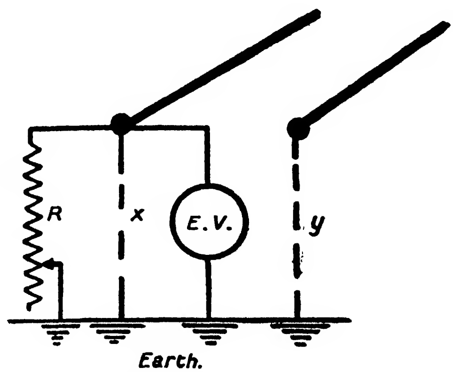

FIG. 100.—Sahulka's arrangement for indicating leaks on alternating-current circuits.

By adjusting R, the reading of the voltmeter may be reduced in value by a certain percentage. The smaller the insulation resistance of this outer, the smaller will be the value of R necessary to reduce the voltage to the specified extent.

This may be shown as follows. Let x be the insulation resistance of one outer, and y that of the other in the two-wire system shown. Let E_1 be the reading of the electrostatic voltmeter before R is switched to

earth, and E_2 the value after S is closed. Then, if E is the voltage across the two outers, and capacity effect is neglected,

$$E_1 = E\frac{x}{x+y}.$$

Also,
$$E_2 = E\,.\,\frac{Rx}{(x+R)y+xR} = E\,.\,\frac{Rx}{R(x+y)+xy}.$$

Therefore,
$$\frac{E_1}{E_2} = \frac{R(x+y)+xy}{R(x+y)} = 1 + \frac{1}{R\left(\frac{1}{x}+\frac{1}{y}\right)},$$

or
$$\frac{E_1}{E_2} = 1 + \frac{F}{R},$$

$\frac{1}{F}$ being equal to $\frac{1}{x}+\frac{1}{y}$.

Thus, for $\frac{E_1}{E_2}$ to remain constant, R must be diminished if F decreases. The latter will happen when a fault develops on either of the outers or on both.

A periodic test by this method will therefore indicate any changes which may occur in the insulation resistance of the system. To make a comparison between two such tests the same consuming devices must be used in each case; otherwise the comparison is of no value.

An important feature of this test is the fact that if F has a satisfactory value, *both* x and y will have satisfactory values, for F is always smaller than either x or y.

This test should be made at light load and at full load. If R has to be very large for light load and very small for full load, the insulation resistance of one or more of the consuming devices is probably low. By the process of elimination the faulty ones may be located.

The location of faults.—The location of a fault in a network of conductors is sometimes arrived at by using the process of elimination already described until the faulty section is found. This is then isolated from the rest of the system, and tested as a dead circuit by means of the megger, loop test, or other methods.

This sequence of locating a fault is not always necessary owing to the use of fuses throughout the system; these, if the faults are serious, blowing on the faulty sections. The latter are then disconnected and tested.

In a three-wire system with its neutral earthed through a recording ammeter at the power station, if a serious earth fault occurs on the positive or negative side of the service mains of a consumer, his positive or negative fuse will at once blow and indicate that his wiring is faulty. If the fault is on the distribution or feeder mains, the fuse links in the junction box will blow, or the fuses may blow at the station for a fault between the latter and the first feeder junction box.

There are various methods for determining the position of a fault in an isolated section or length of cable. Some of these will now be considered.

A completely earthed fault in a long cable.—This case is rare; one well-known illustration is that of a telegraph cable which has completely snapped off in sea-water. The position P of the fault (Fig. 101) is found by means of a post-office box. A in the upper diagram is joined to one terminal of the box, and C, another terminal, to the earth by a thick connection. The resistance between A and C, that is, the resistance of AP, constitutes the fourth arm of the bridge and may be measured.

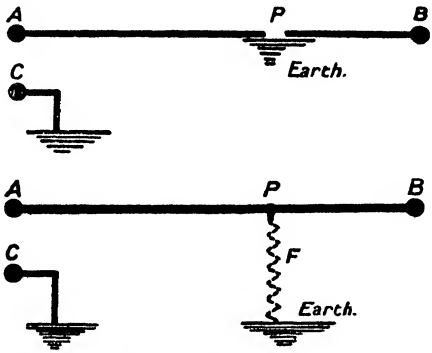

FIG. 101.—Cases of complete and partial faults.

If the resistance of a mile of the cable is known, the distance AP may at once be calculated. When both ends of the cable are accessible or under control, the value of BP may be similarly found, and the ratio of AP to AB obtained without knowing the total resistance of the cable or a sample of it.

An imperfectly earthed fault in a long cable.—This is illustrated in the lower diagram of Fig. 101. F is the resistance of the fault. When both ends of the cable are accessible or under control, B is insulated from earth, and the post-office box used across A and C. A is next insulated from earth, and the resistance between B and earth measured.

If x is the resistance of AP, y the resistance of BP, R_1 the resistance measured in the first case, R_2 that in the second case, and R is the total resistance of the cable conductor, supposed known,

$$R_1 = x + F,$$

$$R_2 = y + F,$$

$$x + y = R.$$

Therefore $$x = \frac{R + R_1 - R_2}{2},$$

and the distance of the fault from A is

$$\frac{L}{2R}\{R + R_1 - R_2\},$$

L being the total length of the cable.

If the test is made from one end of the cable, B is first earthed

and the resistance between A and C measured; B is then insulated and the resistance between A and C again measured. Therefore

$$R_1 = x + F \cdot \frac{R - x}{F + R - x},$$

$$R_2 = x + F.$$

From these two equations x may be obtained, and the length of AP calculated from

$$AP = \frac{x}{R} \cdot L.$$

Unless F is small or comparable with x, this method cannot be very accurate, as F is measured with x.

A fault between two cables both insulated from earth.—This case, illustrated by Fig. 102, may happen with the two conductors of a

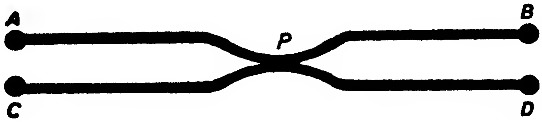

FIG. 102.—A fault with both cables perfectly insulated from earth.

double-core cable or to two adjacent cables due to some external pressure. In this case the resistance R_1 between A and C is measured, and then R_2 between B and D.

If x is the resistance of the length AP of the single conductor and y that for BP,

$$R_1 = 2x + F,$$

$$R_2 = 2y + F.$$

Next, *earthing* B and measuring R_3 between A and earth,

$$R_3 = x + y.$$

From the first two equations,

$$R_1 - R_2 = 2x - 2y.$$

Therefore
$$x = \frac{2R_3 + R_1 - R_2}{4}.$$

So that, if L is the length of AB,

$$AP = AB \frac{x}{x+y} = \frac{2R_3 + R_1 - R_2}{4R_3} . AB.$$

In the case of the conductors being of unequal cross section, let x_1, y_1, x_2, and y_2 be the respective resistances of AP, PB, CP, and PD. Then, repeating the preceding test, with the addition of finding

the resistance of CD by earthing D, and measuring the resistance R_4 between C and earth, the following equations are derived :

$$R_1 = x_1 + x_2 + F, \quad \text{.......................} (a)$$
$$R_2 = y_1 + y_2 + F, \quad \text{.......................} (b)$$
$$R_3 = x_1 + y_1, \quad \text{.......................} (c)$$
$$R_4 = x_2 + y_2. \quad \text{.......................} (d)$$

Also
$$\frac{x_1}{y_1} = \frac{x_2}{y_2}. \quad \text{.......................} (e)$$

Subtracting (*b*) from (*a*),

$$R_1 - R_2 = (x_1 + x_2) - (y_1 + y_2), \quad \text{.......................} (f)$$

and adding (*c*) and (*d*),

$$R_3 + R_4 = (x_1 + x_2) + (y_1 + y_2). \quad \text{.......................} (g)$$

From (*f*) and (*g*),

$$x_1 + x_2 = \tfrac{1}{2}\{R_3 + R_4 + R_1 - R_2\},$$
$$y_1 + y_2 = \tfrac{1}{2}\{R_3 + R_4 - R_1 + R_2\}.$$

Also, from (*e*),
$$\frac{x_2}{x_1} = \frac{y_2}{y_1} \quad \text{and} \quad \frac{x_1 + x_2}{x_1} = \frac{y_1 + y_2}{y_1}.$$

Therefore
$$\frac{x_1}{y_1} = \frac{R_3 + R_4 + R_1 - R_2}{R_3 + R_4 - R_1 + R_2}$$

and
$$\frac{x_1}{y_1} = \frac{AP}{PB}.$$

Thus knowing the length of AB, the position of the fault may be determined.

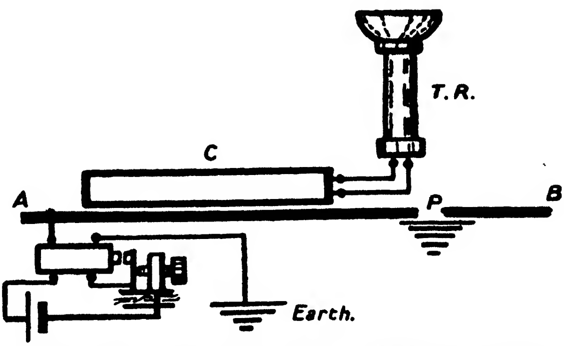

FIG. 103.—An arrangement for locating faults by the telephone receiver method.

The telephone-receiver method.—This is an excellent method of locating faults in the cases where it is possible to use it. The arrangement for this test is shown in Fig. 103. AB is part of the cable, which

s faulty at P. C is a long shuttle coil wound with many turns of fine wire in series with a telephone receiver, or preferably a pair of sensitive head telephone receivers. The secondary coil of a buzzer is connected to one end or another part of the cable conductor and the earth as shown.

C is moved along the position of the cable, and as close as possible to it, until a place is found at which the sound heard in T.R. abruptly ceases or becomes less intense. The position P is thus obtained.

If the cable has a lead sheath, the earth connection of the buzzer or source of alternating or interrupted current should be connected to it. The advantage of this method is that there can be no errors due to bad contacts or calculation. Its disadvantage is that close proximity of C to certain parts of the cable near the fault is necessary for exact location. This method is useless if the cables are laid in iron troughs.

The loop test.—This is generally the best and most used method of locating a fault. It has the advantage of not combining the fault resistance with those needed for calculation. This is done by putting the fault resistance in the battery circuit of a bridge arrangement. In Fig. 104 is shown *Murray's* arrangement for the loop test.

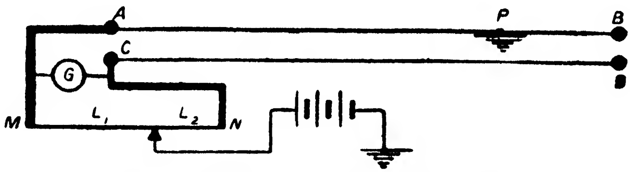

FIG. 104.—Murray's loop test for locating a fault.

AB is the faulty cable, and CD is a second cable or conductor in its vicinity. CD may be the return cable of AB or another cable of different copper cross-sectional area. MN is a slide wire with thick end connections which are joined to A and C. B and D are joined by a piece of copper wire.

A balance is obtained by moving the contact piece along MN, and if the cables have the same copper cross-sectional area,

$$\frac{L_1}{L_2}=\frac{AP}{2AB-AP},$$

from which AP may be calculated.

If the cross-sectional areas of the conductors are a_1 and a_2,

$$\frac{L_1}{L_2}=\frac{AP}{AB\left(1+\frac{a_1}{a_2}\right)-AP}.$$

Varley's loop test is somewhat different to the preceding. The arrangement for this test is shown in Fig. 105. B and D are short

The fault may be indicated after all the switches on M.D.B. have been opened and all the fuses removed. In this case it is on the board itself. By using the megger between each isolated piece of metal or terminal on the board, and earth, the faulty part or parts may be discovered.

After the meter M is connected to T_2 and T_1, and T_3 joined to T, the company's representative may test the insulation of T_1T_2 and TT_3 from earth. In this case M.S. is opened and the fuses removed from C.M.F. The megger is then used between T_1 and earth, and afterwards between T and earth. If a fault is indicated, the fixing screws of the terminals and the wiring to the meter should be examined for its cause.

In such a wiring system as the preceding, the insulation resistance between it and earth may be very high, but leakage may occur from the positive to the negative side across some of the surfaces associated with the consuming devices. A leakage may take place across an open switch which is well insulated from earth. This leakage chiefly concerns the consumer, as it is registered on the meter.

The insulation resistance between the two sides of the system may be determined by using the megger across T_4 and T_5 after opening M.S.; closing all the wiring switches; and disconnecting all the consuming devices from their terminals, or, in the case of lamps, removing them from their holders.

If the reading is low, the switches on M.D.B. should be opened one by one, and the megger read each time, until the faulty section is indicated. The circuits of this section are then similarly tested. The fault may finally be discovered on some part of the wiring well insulated from earth; for instance, the insulation between the two parts of a flexible lead connected to the terminals of a consuming device may be bad.

Let x be the insulation resistance of the positive side of the system from earth, y that of the negative side from earth, and z that between the two sides of the system. Let these values relate to the case when all the switches of the wiring are closed, and all the consuming devices are disconnected from their terminals; lamps being removed from their holders.

With M.S. open, and T_4 and T_5 short circuited with wire, the megger is used across T_4 and earth. Let R_1 be its reading. Then

$$\frac{1}{R_1}=\frac{1}{x}+\frac{1}{y}.$$

Next, the short circuit wire is removed and T_4 joined by wire to earth. The reading of the megger used across T_4 and T_5 is R_2, and

$$\frac{1}{R_2}=\frac{1}{y}+\frac{1}{z}.$$

Finally, T_5 is joined by wire to earth instead of T_4, and the reading of the megger used across T_4 and T_5 is R_3. Then

$$\frac{1}{R_3}=\frac{1}{x}+\frac{1}{z}.$$

From these three equations the values of x, y, and z may be found.

In a certain test the values of R_1, R_2, and R_3 were found to be respectively 5, 2·6, and 3 megohms. It is required to find x, y, and z. Let $\frac{1}{x}$, $\frac{1}{y}$, and $\frac{1}{z}$ be replaced by x_1, y_1, and z_1. Then

$$x_1+y_1=0{\cdot}200, \qquad \text{(1)}$$

$$z_1+y_1=0{\cdot}385, \qquad \text{(2)}$$

$$z_1+x_1=0{\cdot}334. \qquad \text{(3)}$$

From (1) and (2), $x_1-z_1=-0{\cdot}185$. ......(4)

From (4) and (3), $x_1=0{\cdot}0745$

and $z_1=0{\cdot}2595$.

Then, from (1), $y_1=0{\cdot}1255$.

Therefore $x=13{\cdot}4$, $y=8{\cdot}0$, and $z=3{\cdot}9$ megohms.

This shows that the positive side of the system is better insulated from earth than the negative side. If one or more consuming devices are switched on, the negative side may have a higher insulation resistance than the positive side. That is, the insulation resistances may fluctuate with the load.

The insulation resistance of a network of cables, even with the same consuming devices, has no fixed value. It varies with temperature and the degree of humidity of the air. In the case of direct-current systems, the value of the insulation resistance for a given temperature improves more or less as the time of working goes on. For, the conducting water of certain of the insulating materials is largely driven by the operating voltage, from the positive surface of the insulation towards the negative surface, by a process known as electric endosmosis.

A system, therefore, which may be faulty for alternating currents, may after some use be quite satisfactory for direct-current work. From this, it follows that, the insulating of an alternating-current system should receive greater care than that of a direct-current system.

CHAPTER VIII.

TESTING THE MAGNETIC QUALITIES OF IRON AND STEEL.

THE magnetic qualities of iron or steel practically comprise the relation between B, the flux-density, and H, the magnetising field which produces this density; the value of the hysteresis loop; the magnetic retentivity; and the hysteresis and eddy-current loss in the material when it is assembled together in thin plates.

Thompson's permeameter.—A ready and simple method of obtaining the relation between B and H in the form of a short rod, is by means of the permeameter, shown in Fig. 107. A is a massive iron block with a rectangular hole cut through it, and in which is inserted a solenoid. R, the test rod, is placed as shown, with its lower end in contact with block A.

The solenoid is excited, and P, the pull, just necessary to separate the end of the rod from A is read on the spring balance. If a is the cross-sectional area of the rod in sq. cms.,

$$P = \frac{B^2}{8\pi} \cdot a \text{ dynes},$$

B being the flux density in lines per sq. cm.

Also, the magnetising field H due to the solenoid is given by

$$H = \frac{4\pi SI}{10l} \text{ C.G.S. units},$$

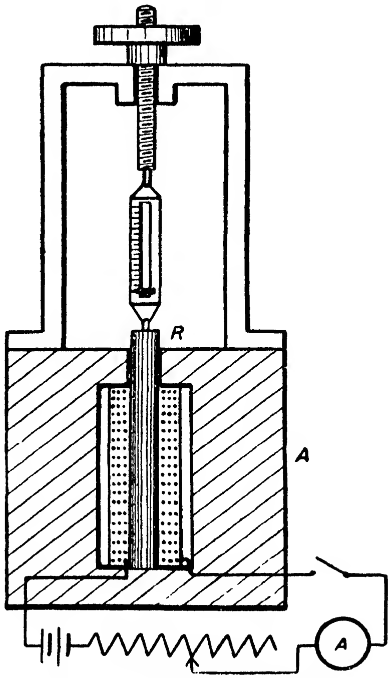

FIG. 107.—Thompson's permeameter.

in which S is the number of turns, I the current in amperes, and l the length of the solenoid in cms.

The graphs for B and H; and μ, the magnetic permeability, and H, may then be drawn.

In the formula for P, B – H should be used instead of B in cases of iron alloys of very inferior magnetic quality, but for ordinary brands of iron, H is practically negligible compared with B.

If P is in *lbs.* and *a* in sq. cms., the value of B in lines per sq. cm. is given by

$$B = 3340\sqrt{\frac{P}{a}}.$$

In a test, the cross-sectional area of the sample rod was 0·7 sq. cm., S was 120, *l* 9·4 cms., and the pull 8·5 lbs. for an exciting current 0·5 ampere. To find B, H, and μ.

$$H = \frac{4\pi \times 120 \times 0{\cdot}5}{10 \times 9{\cdot}4} = 8 \text{ C.G.S. units},$$

$$B = 3340\sqrt{\frac{8{\cdot}5}{0{\cdot}7}} = 11{,}620 \text{ C.G.S. units},$$

$$\mu = \frac{B}{H} = \frac{11{,}620}{8} = 1453.$$

The magnetometer method of obtaining the B-H curve for a sample of iron.—In practice, the whole hysteresis loop is generally required, and this may be obtained by the magnetometer method. The sample in the form of a long thin rod is placed well inside a vertical solenoid longer than itself, and is magnetised by a current sent through the solenoid. The experiment is simply to find *m* the strength of one of the poles of the sample for different values of H, the magnetising field in the solenoid.

The value of *m* is found by means of a mirror magnetometer, or one with a delicately pivoted magnet of small dimensions.

Since $4\pi m$ lines of magnetic force emanate from a pole of strength *m* units, and this pole is near the end in a long magnet, all these lines must pass through the cross section of the rod throughout its length, except close at the ends. The number, therefore, passing through one sq. cm. is $\frac{4\pi m}{a}$; the ratio $\frac{m}{a}$ being defined as the intensity of magnetisation of the material.

Now, in addition to this system of lines, which are really the lines of magnetic induction due to the iron being magnetised, there is the system of H lines per sq. cm. which magnetised the iron in the solenoid; these are in the same direction as the other lines.

There are also two minor systems of lines present, namely, that due to the poles of the rod in opposition to H, and the other system due to the vertical component of the earth's magnetism.

The first of these is practically negligible if the diameter of the rod is less than about $\frac{1}{200}$ of its length, and the effect of the second may be neutralised by using a few turns round the solenoid, and supplying them with sufficient current to produce an equal and

opposite field to that of the earth's vertical component. Except for very good quality iron and very accurate determinations, this latter arrangement is not necessary.

Neglecting these minor systems of lines, the total flux passing through one sq. cm. of the cross section of the rod, that is, the flux density B, is given by

$$B=\frac{4\pi m}{a}+H,$$

and H is given by $$H=\frac{4\pi SI}{10l}.$$

Thus, by reading the current I, knowing $\frac{S}{l}$, the turns per cm., and determining m by experiment, the values of B and H may be calculated.

To find m.—The arrangement for doing this is shown in Fig. 108.

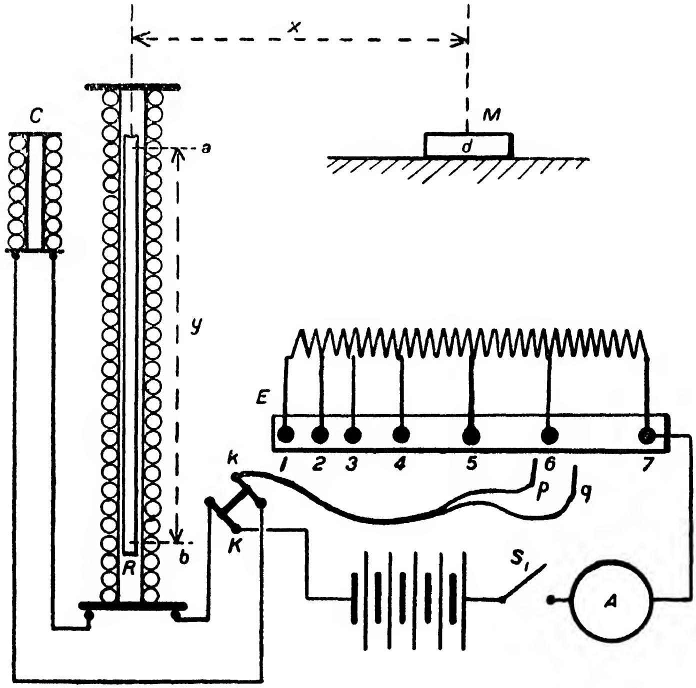

FIG. 108.—The arrangement for the magnetometer method of determining the hysteresis cycle of a sample of iron.

C is a compensating coil placed so that its magnetic effect on the magnetometer needle, exactly neutralises that due to the solenoid without the iron rod; the effect of the rod alone on the needle being required in this experiment.

The turns on the solenoid for neutralising the effect of the earth's vertical component on the rod are not shown, but these should be used if B is required for small fields, or when specially good samples are to be tested.

A resistance adjustable through certain steps may be used, or a set of mercury cups as shown connected to resistance coils. By this means the current may be increased or decreased by definite steps without breaking the current. Electrodes p and q form a double lead to the same point k of the reversing key K.

The first three or four steps should give *small* changes of current, as the change of B for small changes in H, when the latter is small, is large.

The circuits are connected up as shown, and the vertical plane *abd* adjusted to be at right angles to the magnetic meridian.

With the rod removed, C is moved until its effect on M neutralises that of the solenoid for a current roughly equal to the maximum value to be used. The switch S_1 is then opened and the rod inserted. If the latter is magnetised, it is demagnetised by producing a demagnetising flux in the solenoid. This may be done by dipping p into one of the mercury cups or brushing the resistance wire with it, near the end E of the rheostat. If the rod then becomes more magnetised, K is reversed and the operation repeated until the deflection is zero.

A better method is to use a second coil on the solenoid, though the same coil would do, and send through it an alternating current, which, by means of a rheostat, is then gradually reduced to zero. This operation demagnetises the rod. For ordinary purposes an alternating current of the usual frequencies may be used, and its value at the commencement of demagnetisation may be as large or larger than that required to produce a field of 20 units in the solenoid.

For special work in which it is important to have the material as completely demagnetised as possible, it is necessary to take greater precautions. C. W. Burrows has found that demagnetisation is best accomplished by a current which alternates at the low rate of approximately *one* per *second*; that the rate of decrease of the induction due to the decreasing alternating current should be as uniform as possible; and that for most specimens of soft iron the initial value of the demagnetising current need not be larger than that necessary to produce a magnetising field of 15 C.G.S. units, while its final value need not be carried down to a much lower value than that of the smallest magnetising current to be used in the test. Complete demagnetisation by this method may be accomplished in about 90 seconds.*

The contact rod p is now placed in mercury cup 1, and both I the current, and θ the deflection on M, are read. Next, the rod q is placed in cup 2 and p removed from cup 1, and I and θ again observed. This process is continued to cup 7, then backwards down to break, and values of current and magnetometer deflections taken at each step, including the deflection of M at break.

K is then reversed, and the preceding repeated. Once more K is

* *Bulletin of the American Bureau of Standards*, 1915, vol. 4, p. 205.

reversed, and readings taken for the different steps up to cup 7. This ends the complete magnetic cycle.

The distances *ad* from the top magnetic pole of the rod to the centre of the magnet needle of M, and *ab* between the poles are measured. Then, if $x = ad$ and $y = ab$, the strength of the horizontal field acting on the needle of M due to the upper pole of the sample is $\frac{m}{x^2}$, and that due to the lower one

$$\frac{m}{x^2+y^2} \cdot \frac{x}{\sqrt{x^2+y^2}}.$$

The difference between them will give F the strength of the deflecting force on the poles of the needle due to the sample of iron, and

$$F = H_e \tan \theta,$$

H_e being the strength of the earth's horizontal field, or the value of the control field of M.

Thus, if H_e is known, m may be first calculated, and then, afterwards, the value of B.

The shape of the B-H curve is roughly shown in Fig. 109. OC is termed the retentivity, PM the maximum induction or flux density of the cycle, and OG the coercive force of the sample when subjected to a magnetising field of OM. The area of the loop PCDQFGP in C.G.S. units divided by 4π is the work done in ergs, in subjecting the molecular magnets in one cubic centimetre of the sample to one complete cycle of magnetisation. A proof of this follows from equation (5), page 5.

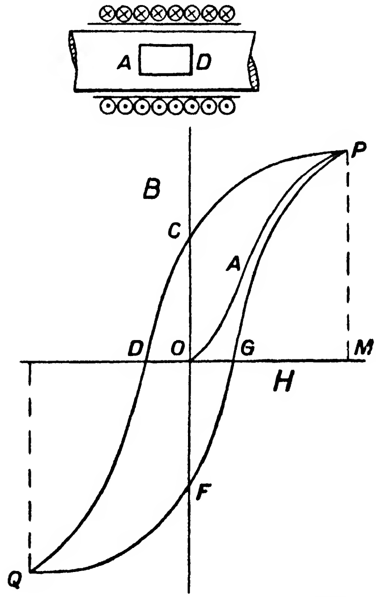

FIG. 109.—The shape of the B-H or hysteresis curve of a magnetic material.

Thus, part of the solenoid is shown in Fig. 109, and AD is a cylinder of the sample, one cm. long and one sq. cm. in cross-sectional area. Then $\frac{S}{l}$ is the number of turns between A and D.

Now, during a cycle, a certain value of H produces a certain number of lines B through this cylinder, and for an increment of H, a corresponding increment dB of B is produced. That is, these $\frac{S}{l}$ turns are threaded during the change by dB lines, and the work done on the circuit is

$$dB \cdot \frac{S}{l} \cdot I \text{ ergs},$$

I being in C.G.S. units. From $H = \frac{4\pi SI}{l}$, the work done is equal to

$$\frac{H \, . \, dB}{4\pi} \text{ ergs},$$

and $\Sigma H \, . \, dB$ is the area of the loop (Fig. 110).

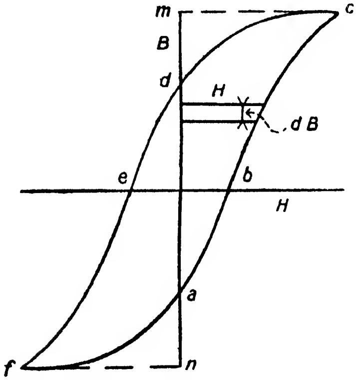

FIG. 110.—The area of the hysteresis loop.

In going from a to c through b, work is done on the molecular magnets, which by the preceding is $\frac{1}{4}\pi$ times the area $abcm$. From c to d work is done by the molecular magnets. From d to f, through e, work is done on, and from f to a work is done by, the molecular magnets.

Thus the work done per cubic cm. per cycle is A, the area of the loop in absolute units, divided by 4π.

If the iron has a mass M grammes, a density ρ, and is subjected to a cycle of frequency f, the hysteresis loss in the iron is

$$\frac{A}{4\pi} \, . \, f \, . \, \frac{M}{\rho} \, 10^{-7} \text{ watts.}$$

Steinmetz's law.—It was found by Steinmetz that the area of the hysteresis loop was approximately a simple function of the maximum flux density of the cycle, namely,

$$A = kB^x,$$

in which k was a coefficient, called the *hysteretic constant*, depending upon the magnetic quality of the iron, and x a coefficient of nearly constant value 1·6 for a certain range of maximum flux densities.

This law is generally expressed by

$$W = \eta B^{1 \cdot 6},$$

in which W is the work done in ergs per cycle per cubic cm., B is the maximum flux density in lines per sq. cm., and η is termed the hysteretic constant of the sample.

The value of η for ordinary temperatures usually ranges from 0·0012 to 0·002 for good quality iron. Approximate values are given in the following table.

x is usually taken as 1·6, which is generally representative for flux densities between about 8,000 to 16,000 lines per sq. cm. Below 8,000 x may be taken as 1·5. Above 16,000 the formula will only give very rough results.*

* By finding A of a sample for pairs of values of B as (3000 and 4000), (7000 and 9000), (13,000 and 15,000), x and η, and their variation for different ranges of B, may be obtained. Thus for the first range, $\frac{A_1}{4\pi} = \eta \; 3000^x$ and $\frac{A_2}{4\pi} = \eta \; 4000^x$; from these x and η may be calculated.

The values of η and x depend upon temperature, magnetic density, physical condition, composition, and former magnetic treatment of the sample.

EXAMPLE.—The hysteretic constant of a sample of iron is 0·0018. A mass of one ton of this material is subject to an alternating magnetic flux of maximum value 8,000 lines per sq. cm. and frequency 50 per second. Find an approximate value of the power lost through hysteresis and the rise of temperature of the iron, assuming that no heat is lost, through radiation, for one hour's application of this flux. The specific heat of iron may be taken as 0·11, and the weight of one cubic inch as 0·28 lb. *Ans.* 2·1 kilowatts; 16 degrees centigrade rise of temperature.

MATERIAL.	APPROXIMATE VALUE OF η.
Stalloy - -	0·0012
Lohys iron -	0·0013
Very soft iron -	0·0020
Annealed steel -	0·0060
Hard steel -	0·0200
Cast iron -	0·0150

Stalloy is an alloy of iron containing about 3½ per cent. of silicon. It has a very low hysteresis loss, and is very suitable for transformers, especially if the frequency is high. This material does not appreciably deteriorate magnetically, that is, *age*. *Lohys iron* is an iron subjected to special treatment resulting in an excellent material for magnetic purposes. Its hysteresis loss is somewhat larger than that of stalloy, but for flux densities higher than the moderate ones used in most alternating-current machines, Lohys iron gives for a given magnetising field a greater magnetic density than stalloy. Hence its use in direct-current armature cores. This iron is also practically free from ageing.

Inferior iron is likely to age after a time, especially if worked at a temperature above 60° C. Through this cause, its hysteresis loss may become much increased.

Wiping out hysteresis.*—A B-H curve may be obtained for a sample of iron in which the hysteresis is wiped out for each step of the magnetic cycle. To do this for the case of the rod in the magnetometer method of obtaining the B-H curve, a second coil is placed on the solenoid and fed by an alternating current preferably of low frequency.

* The following method was used by W. Steinhaus and E. Gumlich. See *Deutsch. Phys. Gesell. Verh.* 17. 21, 1915, p. 369.

The magnetic cycle is passed through step by step, as in the ordinary case, but before the reading of the magnetometer is taken, the alternating current is sent through the second coil, and then uniformly diminished down to a sufficiently small value before switching it off. In this operation the method of *Burrows*, described on page 150, should preferably be used. The reading of the magnetometer and magnetising current are then taken.

This is repeated for all the steps of the cycle, and the result should be a practically *loopless* curve. In Fig. 111 is shown the two curves

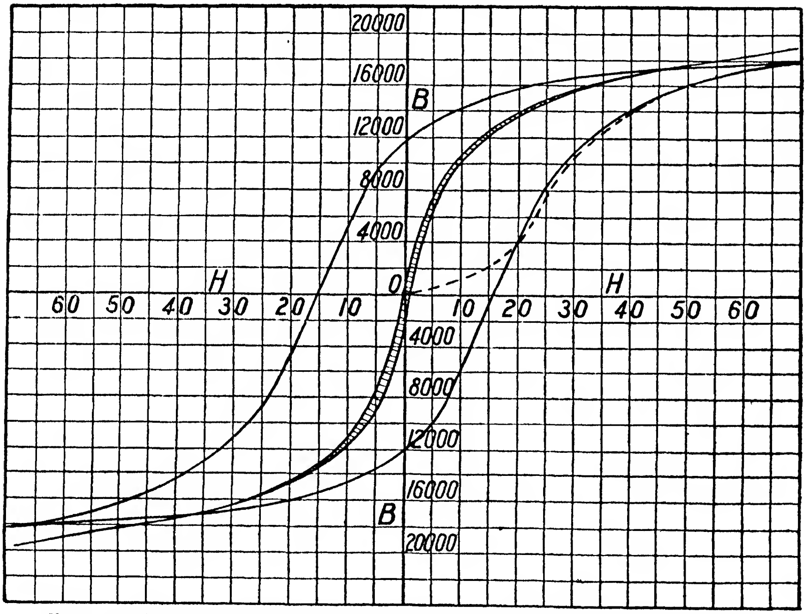

FIG. 111.—The ordinary hysteresis curve of a sample of iron, and the curve when nearly all its hysteresis effect is wiped out.

of a sample of iron, one obtained in the ordinary way, and the other by the method just described. The latter is the shaded curve which exhibits but little hysteresis, and the little shown may be due to the fact that an alternating current of frequency 20 was used instead of about one per second.

This curve, so obtained, is more representative of the magnetic quality of the material than the ordinary one, as it is independent of the past history of the sample.

By applying this method, a sample of iron may be made to give the same flux density for the same current. With hysteresis present, the same current may give different flux densities every time it is used.

Thus a certain exciting current may be passed through a magnetising coil when the iron has residual magnetism, and produce either

a larger or smaller flux density, according to the direction of the current, than that produced if the iron was demagnetised at the start.

In certain investigations it is a great advantage to be able to free the iron part of a circuit from the effect of hysteresis, and thus obtain in succeeding operations the same flux density for the same exciting current.

The ballistic galvanometer method.—The advantage of this method of obtaining the B-H curve is that there are no poles to weaken the magnetising effect of the solenoid, and thereby introduce an error; but the experimental part is more troublesome, and a ballistic galvanometer is needed instead of the simple magnetometer.

In this case the sample has to be in the form of a ring, and, in order to have a nearly uniform flux density through its cross section, the diameter of the ring should be sufficiently large and the cross section an elongated rectangle, as shown at M (Fig. 112).

This figure shows the arrangement for making the test. P is a secondary coil on the circular solenoid containing the sample ring, S a secondary coil on a standard solenoid, and R.B. a resistance box.

The connections to the ring solenoid are similar to those in the magnetometer method, and the ring should be first demagnetised before passing it through a magnetic cycle. The method is as follows.

The tap key T is closed, and the throw D read on the B.G. scale, and I_1 on A_1. By this action, coil S has been threaded with ϕ_1 lines of magnetic force, and

$$\phi_1 = \frac{4\pi S_1 I_1 a_1}{10 l_1},$$

in which S_1 is the number of turns, l_1 the length in cms., I_1 the current in amperes, and a_1 the mean cross-sectional area of the turns of the standard solenoid.

Therefore the quantity of electricity produced in the galvanometer circuit, according to equation (4), page 4, will be

$$\frac{S\phi_1}{10^8 R} \text{ coulombs},$$

S being the number of turns of coil S, and R the resistance of the whole of the galvanometer circuit. R.B., after its first adjustment, must not be changed throughout the test. Hence

$$S\phi_1 = kD,$$

k being a constant.

K is now closed, and p placed in cup 1; then d_1 the throw of B.G. and I the magnetising current are read. If B_1 is the flux density in the sample, and a the cross-sectional area of the sample in sq. cms., $B_1 a$ is the number of lines threading P. Hence the quantity of electricity produced in the galvanometer circuit is

$$\frac{PB_1 a}{10^8 R} \text{ coulombs},$$

P being the number of turns on coil P.

Therefore $$PB_1a = kd_1$$

and $$B_1 = \frac{S\phi_1}{PaD} \cdot d_1.$$

That is, $$B_1 = K \cdot d_1,$$

in which K the constant may be calculated.

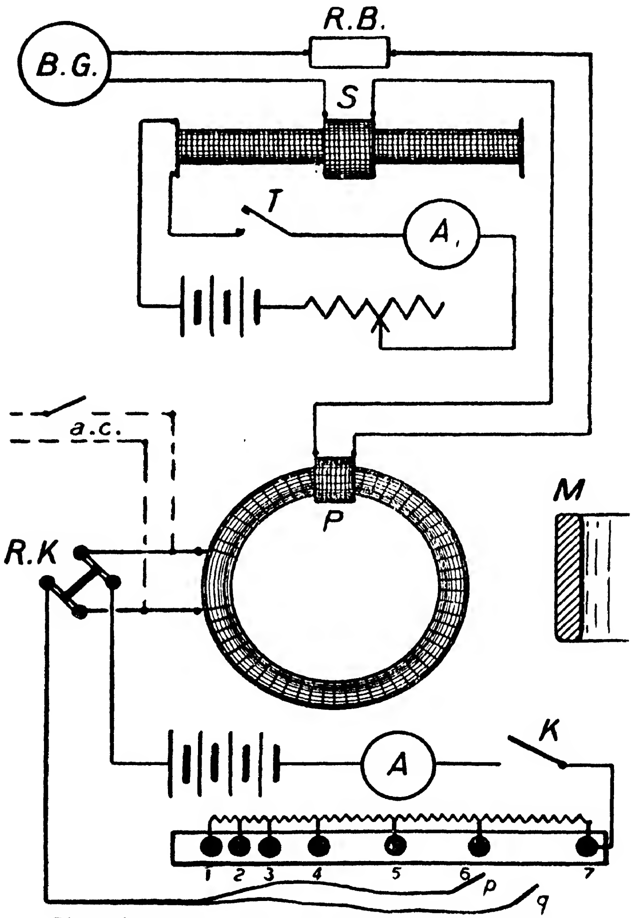

FIG. 112.—The ballistic galvanometer or ring arrangement for obtaining the hysteresis or B-H curve of a sample of iron.

Next, place q in cup 2, and note the kick d_2,

$$B_2 = K \cdot d_2,$$

and the flux density in the core is $B_1 + B_2 = K(d_1 + d_2)$.

The method of entry is illustrated by the following table :

Steps.	Current Amps.	H.	Throws.	Sum of Throws.	B.
0	0	0	0	0	0
1	0·008	0·24	29	29	370
2	0·015	0·45	22	51	650
3	0·024	0·72	58	109	1400
4	0·031	0·93	58	167	2150
5	0·039	1·17	90	257	3300
6	0·051	1·53	179	436	5600
7	0·081	2·43	288	724	9300
8	0·137	4·11	202	926	11900
7	0·081	2·43	−23	903	11600
6	0·051	1·53	−32	871	11200
5	0·039	1·17	−23	848	10900
4	0·031	0·93	−15	833	10700
3	0·024	0·72	−16	817	10500
2	0·015	0·45	−39	778	10000
1	0·008	0·24	−23	755	9700
Break.	0	0	−39	716	9200
1	0·008	0·24	−55	661	8500
2	0·015	0·45	−54	607	7800
3	0·024	0·72	−140	467	6000
4	0·031	0·93	−234	233	3000
5	0·039	1·17	−321	−88	−1130
6	0·051	1·53	−302	−390	−5000
7	0·081	2·43	−312	−702	−9000
8	0·137	4·11	−224	−926	−11900
7	0·081	2·43	26	−900	−11600
6	0·051	1·53	25	−875	−11250
5	0·039	1·17	17	−858	−11000
4	0·031	0·93	18	−840	−10800
3	0·024	0·72	23	−817	−10500
2	0·015	0·45	37	−780	−10000
1	0·008	0·24	25	−755	−9700
Break.	0	0	39	−716	−9200
1	0·008	0·24	62	−654	−8400
2	0·015	0·45	55	−599	−7700
3	0·024	0·72	241	−358	−4600
4	0·031	0·93	226	−132	−1700
5	0·039	1·17	257	125	1600
6	0·051	1·53	265	390	5000
7	0·081	2·43	312	702	9000
8	0·137	4·11	224	926	11900

In the same way the complete cycle is performed by going first up the steps, then down, changing the direction by the reversing key, again up and down, changing the direction by the reversing key, and finally going up the steps.

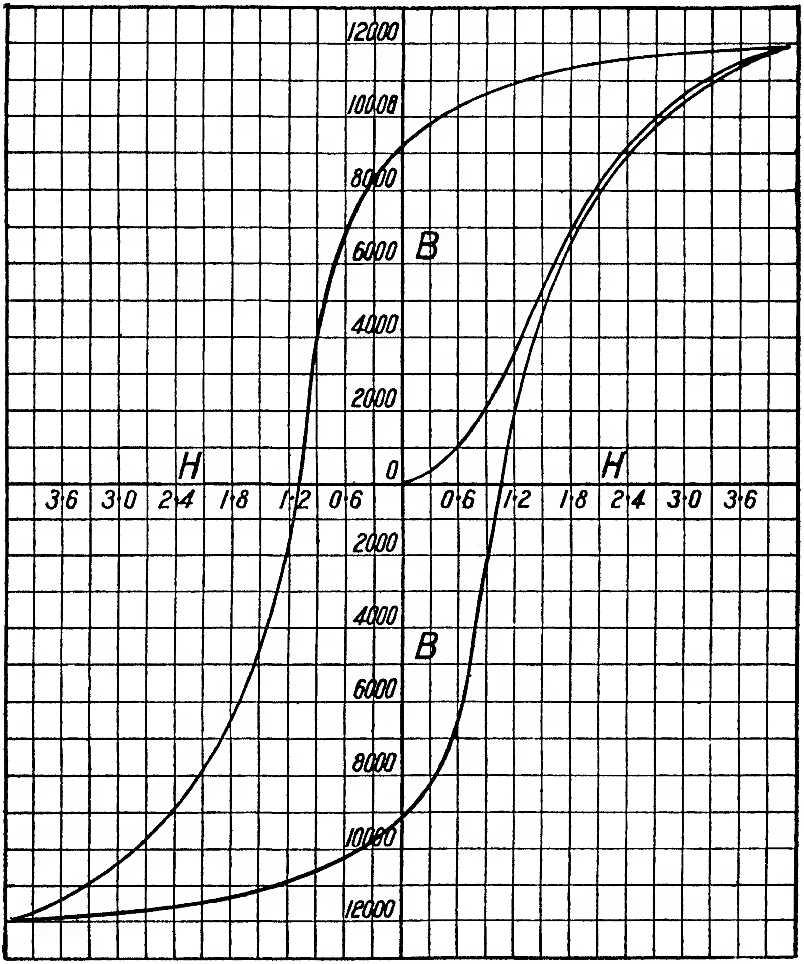

FIG. 113.—The hysteresis curve of a sample of iron, tested by the ballistic galvanometer or ring method.

At any step the induction is the algebraical sum of all the deflections reckoned from the start to its own deflection, multiplied by K. As soon as a deflection changes its sign, it must be tabulated according to its direction.

The curve relating B and H is shown in Fig. 113.

The area of the loop in small squares was found to sum up to 180. The vertical or B side of one small square is 1000 C.G.S. units, and the horizontal or H side 0·3 C.G.S. unit.

Therefore the area of the loop is

$$180 \times 0{\cdot}3 \times 1000 = 54000 \text{ units,}$$

and the number of ergs per cycle per cubic cm. of the sample is

$$\frac{54000}{4\pi} = 4300.$$

The value of the hysteretic constant is thus given by

$$4300 = \eta (11900)^{1{\cdot}6},$$

$$\eta = 0{\cdot}0013.$$

Wattmeter method of determining iron loss.—For the practical determination of iron loss due to hysteresis and eddy currents, either in samples of iron in the form of laminations, or in certain types of electrical machines, the wattmeter method is one of the best.

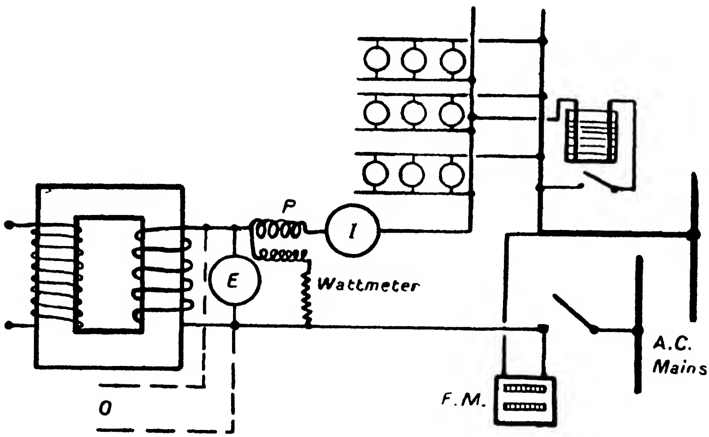

FIG. 114.—The arrangement for the wattmeter method of determining the iron loss of a sample of laminated iron.

The wattmeter employed for this test should be able to carry a fairly large current through its current coil and also indicate low values of power, that is, it should be constructed for very small power factors, as the power factor of the circuit in this test is always small.

The iron to be tested is in the form of laminations, which are built up like the core of a transformer. A coil of S turns is placed round the core, and is fed from mains connected to a single-phase alternator coupled, if possible, to a direct-current shunt motor of large speed variation. By this means the iron may be tested for different frequencies, and also for different magnetic densities.

The arrangement for this test is given in Fig. 114, and shows a

lamp and liquid rheostat, F.M. a frequency meter, wattmeter, ammeter, and voltmeter. For very accurate determinations, the voltage wave across the coil on the laminations should be obtained by an oscillograph joined to the dotted connections shown.

If P is the reading of the wattmeter, then the iron loss in the core is equal to P minus the small copper loss in the coil, which is equal to I^2r. To allow for skin and eddy action, r may be increased from 5 to 10 per cent. according to the frequency and size of the conductor. Let $W = P - I^2r$, W being the iron loss in the core.

E, the reading of the voltmeter, is related to the maximum value of the magnetic flux density in the core by the formula,

$$E = 4kSAB_m f 10^{-8},$$

given on page 49; k being the form factor of the voltage wave, which, if of sine form, equals 1·111, A the cross-sectional area of the core, f the frequency, and B_m the maximum value of the flux density in the core.

Therefore, by measuring E, f, and W, the iron loss for a certain frequency and flux density may be calculated.

The experimental part of the method is to obtain for a *given* frequency a set of different values of W, and corresponding values of E. This is done by varying the current I by means of the rheostats. Other sets for different frequencies are afterwards obtained. These results are then plotted, as roughly indicated in Fig. 115.

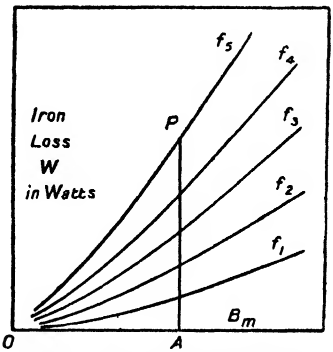

FIG. 115.—Graphs of iron-loss plotted from the results of the tests.

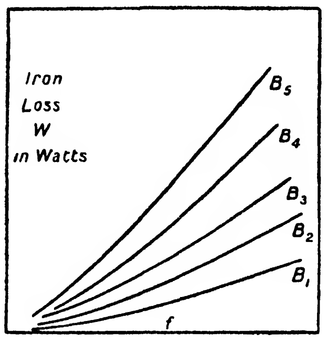

FIG. 116.—The iron-loss and frequency for a given flux density.

From these curves, a set of derived curves relating W and f for given values of B may be drawn, as shown in Fig. 116; the vertical line in Fig. 115 giving the values of W, and also f, for the value of $B_m = OA$.

Also, for a *given* B_m, the iron loss may be represented approximately by

$$W = k_1 f + k_2 f^2,$$

the first term on the right representing the hysteresis loss, and the second the eddy-current loss in the core.

This relation is only approximately true, as the clamping of the core plates together increases the hysteresis component, and the first term would be relatively greater with respect to the second term than that given in the equation. At the same time, if the laminations were badly burred at their edges, the eddy loss might be larger than the hysteresis loss, especially for high frequencies.

Another set of derived curves may now be drawn relating $\frac{W}{f}$ and f for any given induction. These are shown in Fig. 117, and are approximately straight lines, as would be expected from the equation

$$\frac{W}{f}=k_1+k_2f.$$

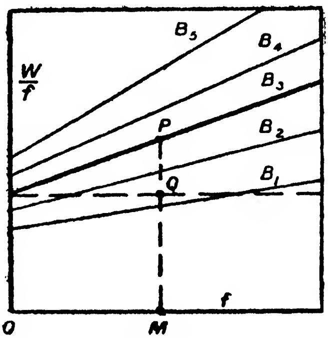

FIG. 117.—Curves for separating the eddy and hysteresis losses of the iron, in the wattmeter method.

The ordinate PM, multiplied by the frequency OM, gives the *total* iron loss for that frequency and the value of B_m affixed to the curve. $PQ \times f$ gives the *eddy* component, and $QM \times f$ the *hysteresis* component of this total loss; OM being equal to f.

A test was made on the core of a 4 K.W. transformer; its secondary coil of 96 turns and resistance 0·019 ohm, being used as the test coil, while the primary coil was left open. The area of the cross section of the iron of the core was 43·5 sq. cms.

A curve of the secondary coil's voltage was obtained by the oscillograph for several different frequencies, and gave a form factor of value 1·13.

The value of B_m is therefore given by

$$B_m=\frac{10^8}{4\times 1{\cdot}13\times 96\times 43{\cdot}5}\times\frac{E}{f}=5{\cdot}3\,\frac{E}{f}\,10^3.$$

The resistance of the coil was taken as 0·02, and the copper loss $\frac{I^2}{50}$ subtracted from the reading of the wattmeter to give W the total iron loss in watts.

The following results were obtained:

$$f=23.$$

W	4·1	9·0	14·5	25	32·5	39	46·5	71
E	9	14·1	18	24·5	28·8	31·8	34·9	43·4
B_m	2075	3250	4150	5640	6640	7350	8020	10000

$f = 30.$

W	8·5	23·5	44·8	53	75	102
E	14·9	26·2	37·6	41·1	49·8	58·5
B_m	2630	4630	6630	7280	8800	10340

$f = 35·5.$

W	14·5	25·5	46	72·5	92·5
E	20	29·5	40·2	52·2	59·4
B_m	2990	4400	6000	7800	8900

$f = 40.$

W	8·5	29·5	51·5	88·8	118·5
E	14	31	44·5	60	70
B_m	1860	4100	5900	7940	9280

$f = 49·7.$

W	21·5	46	70	120	129
E	27·5	44	56	75	78
B_m	2930	4700	5980	8000	8320

The graphs relating W, the iron loss in watts, and maximum flux density B_m in the core for the different frequencies used, are shown in Fig. 118.

From them the iron loss for a given B_m and f may be obtained.

In Fig. 119 are drawn the graphs relating W and frequency f for the different values of B_m. Thus, for a given flux density, the iron loss may be read off for a given frequency.

The graphs of W/f against f are given in Fig. 120. These are derived from Fig. 119, and by them the division of the total iron loss into their hysteresis and eddy-current components may be approximately determined.

For example, to find the eddy-current loss in the core for $B_m = 7000$ lines per sq. cm. and a frequency $f = 50$. From the graph for this density, the eddy-current loss is

$$(1·9 - 1·42)50 = 24 \text{ watts},$$

and the hysteresis loss is $1·42 \times 50 = 71$ watts.

The reading of a wattmeter on an unloaded transformer gives the value of the iron loss plus the very small no-load copper loss which

may be deducted. This iron loss is approximately constant for all loads, including the case of no load, so that the one simple determination at no load gives the value of the iron loss for any given load.

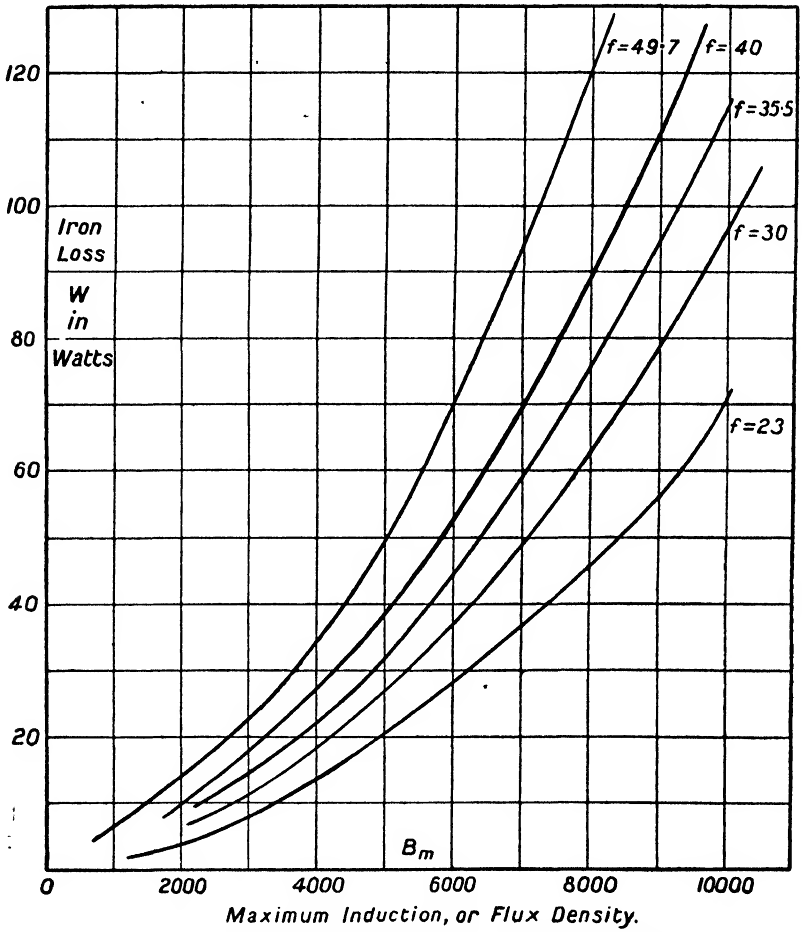

FIG. 118.—The iron-loss curves for a given sample of iron; tested by the wattmeter method.

Other commercial instruments for determining magnetic quality.—Such instruments are constructed for a quick test of a sample of iron, usually in the form of a rod, or a bundle of rectangular pieces of laminations of the sample.

Ewing's magnetic permeability bridge.—This was one of the earlier instruments for determining magnetic permeability of a sample of

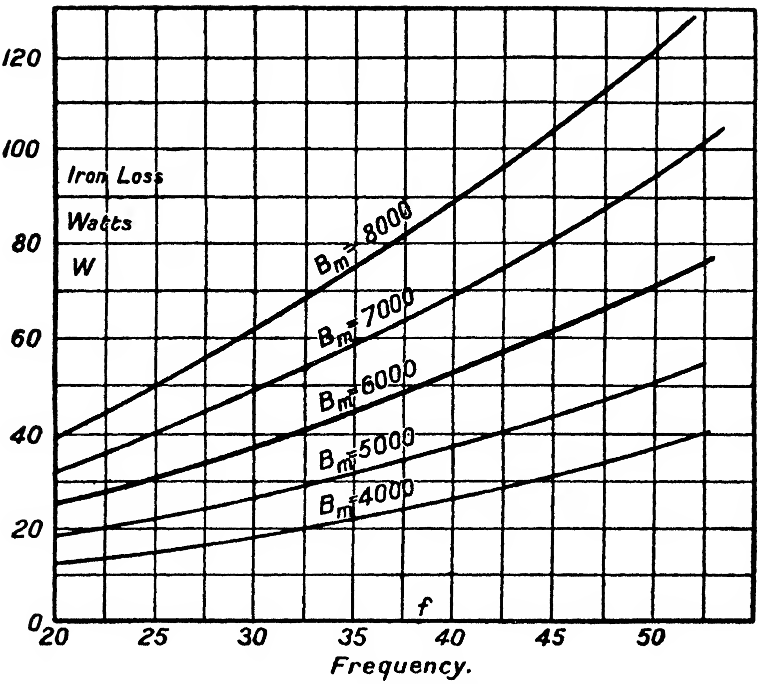

FIG. 119.—Curves showing the iron-loss and frequency, for a given flux density.

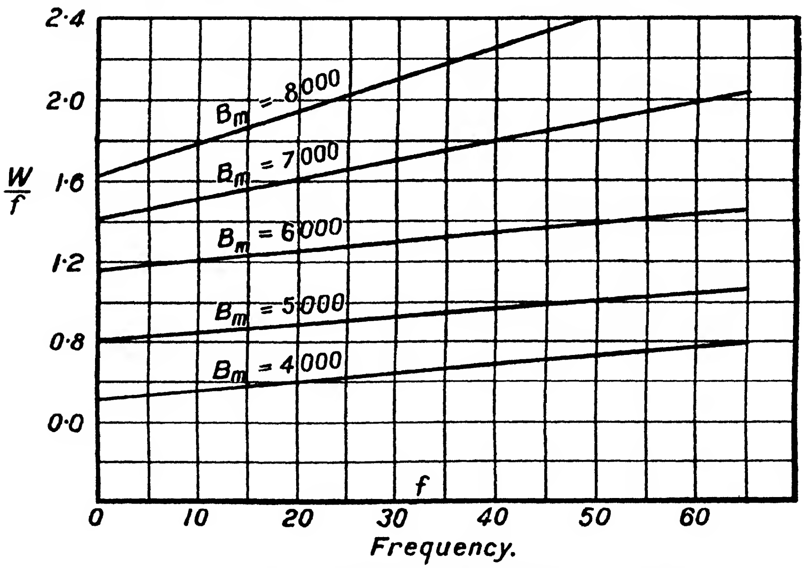

FIG. 120.—Curves from which the division of the iron-loss into its hysteresis and eddy-current components may be made.

iron in the form of a rod. By it the permeability of a material for a given magnetising field, or flux density, may be obtained in a few minutes.

The construction of the instrument is simply indicated in Fig. 121. A and B are two solenoids, each of length 4π cms. These are in series. A may supply either 50 or 100 turns; B may supply, by moving switches across dials of contacts, from 1 to 210 turns. So that in A the magnetising field may be either

5 or 10 C.G.S. units per ampere,

and that in B ranges from zero up to 21 C.G.S. units per ampere.

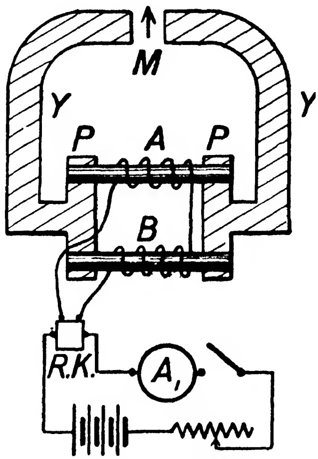

FIG. 121.—Ewing's permeability bridge.

The standard rod, whose B-H curve has been found by some other method, is usually placed in A and the test rod in B; if the test rod is of poor magnetic quality, the rods may be interchanged.

PP are soft iron blocks, YY a soft iron yoke, and M a magnetometer.

A current I amperes is passed through the solenoids with S_A turns on A and S_B turns on B, and the reversing key is operated repeatedly as S_B is increased or diminished; while doing this, the needle of M is watched.

When the needle takes up its zero position, that is, when no flux passes through the yoke YY, the magnetic induction of the one rod balances that of the other.

The induction or flux density of the standard is produced by the magnetising field

$$\frac{S_A I}{10},$$

and from its B-H curve the value B_1 of the flux density is read off.

Now, B_1 is also the flux density in the test rod, and it is produced by the magnetising field H_1, which is equal to

$$\frac{S_B I}{10}.$$

Therefore the permeability for the magnetising field H_1 is

$$\frac{B_1}{H_1}.$$

By obtaining the balancing values for other currents, and using the curve for the standard rod, the B-H curve of the sample may be obtained.

A balance was obtained when there were 100 turns on the standard rod and 120 turns on the test rod; the current being 2 amperes. The magnetising field in the standard solenoid is therefore $\frac{2 \times 100}{10} = 20$, and the flux density from the curve was found to be 16,000. This is also the density in the test rod due to a magnetising field of 24, so that this gives one point on the B-H curve of the test.

Ewing's hysteresis tester.—By this instrument a quick determination may be made of the hysteresis loss in a sample of iron. The latter is in the form of a block of thin laminations of about seven pieces, each 3″ by ⅝″.

This block is placed between the two poles of a permanent magnet balanced on knife edges. The principle of the instrument is illustrated in Fig. 122. B, the sample, is rotated rapidly between the poles and becomes magnetised; passing through a complete cycle per revolution.

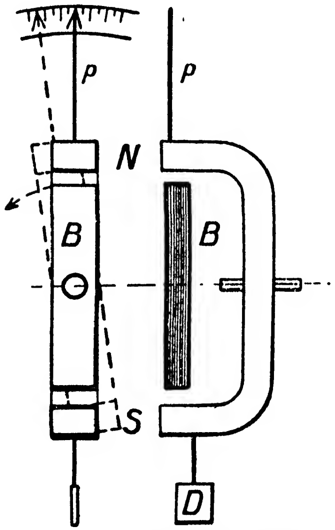

Fig. 122.—Ewing's hysteresis tester.

Owing to the lag in its polarity, B drags, by attraction, the respective poles adjacent to its ends through a certain angle, whose value depends upon the hysteresis of the sample. This angle is given by the pointer *p*.

The control force is gravity, and the oscillations are damped by D. Above a certain speed the deflection is, as would be expected, independent of the speed of B.

The deflection is proportional to the area of the hysteresis loop, and the tester is calibrated by means of a specimen sample whose hysteresis curve has been determined by other methods.

Sometimes deflections corresponding to two standards are plotted against their hysteresis losses, and a straight line is drawn through the two points. The hysteresis loss of the sample under test may then be read off as soon as its deflection is obtained.

The fluxmeter invented by *Grassot* is an instrument designed for direct measurements of magnetic flux; the latter being threaded by a search or test coil used in conjunction with the fluxmeter.

Its construction is practically that of a moving-coil galvanometer, the instrument having a suspended coil, enclosing a fixed iron cylinder, which can move freely like a galvanometer coil in the interspaces between the cylinder and the poles of a permanent magnet. In this case, however, the suspension is silk, which exerts only an extremely

small control, and causes the coil to take a considerable time to return to zero. An arrangement is provided to bring the coil quickly back to zero.

This suspended coil is in series with the search coil, which, in testing, is made to thread the unknown flux. As a result, the former coil is deflected, and a pointer on it gives the angle of displacement. A mirror on the coil and a scale may be used, and the sensitiveness of the instrument greatly increased.

The principle of the fluxmeter is as follows. If a coil A (Fig. 123) is freely suspended in the position shown, and then displaced through

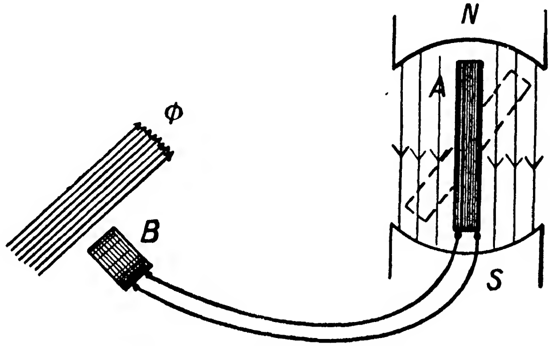

FIG. 123.—Diagram illustrating the principle of the fluxmeter.

a *small* angle θ into the dotted position, it will become threaded by a number of lines of force proportional to θ. By equation (4), page 4, a quantity q of electricity will therefore flow through the circuit of A proportional to $\frac{\theta}{R}$; the resistance of the circuit being R.

Now, the reverse of this is also true. That is, if coil A is in its zero position, and its suspension is practically torsionless, and the same quantity q of electricity is passed through it by any means, the displacement of A will be θ as before.

Thus, supposing B, the search coil, is moved across a magnetic field of strength b, and coil A was at its zero position before this motion began, a quantity of electricity will flow through the circuit of the two coils proportional to

$$\frac{bna}{R},$$

and, as a result, coil A will be deflected through an angle θ of such value that

$$\theta \propto \frac{bna}{R},$$

n being the number of turns of B, a the mean cross-sectional area of the turns of B, and R the resistance of the circuit containing A and

B. Therefore, for a given search coil, the strength of the field under test is proportional to the deflection.

The pointer, or spot of light, will stay at the limit of its deflection an appreciable time, some ten or more seconds, and θ may thus be read very accurately.

Search coils having different numbers of turns may be used, so that a large range of magnetic fields may be measured. The readings of the fluxmeter, multiplied by the constant for a given search coil, will thus give the value of the flux density under test.

The fluxmeter may be used instead of the ballistic galvanometer and its calibrating solenoid, in the ring method of obtaining the B-H curve. Also, as its deflections are practically independent of the time taken by the change of magnetic flux, the B-H curve for the core of a transformer may be obtained by the fluxmeter. The latter test is impossible with the ordinary ballistic galvanometer, as, owing to the large self inductance of the transformer coils, a comparatively long time is taken for the magnetic induction to attain its final value for each step of the operation.

In this test of the transformer, one of its coils is used to magnetise the core step by step, as in the ring method, while another is wound around the core to act as the search coil of the fluxmeter. Then, step by step, the whole hysteresis curve of the transformer core may be obtained.

CHAPTER IX.

TEMPERATURE RISE IN ELECTRICAL APPARATUS AND MACHINES.

TEMPERATURE rise in electrical apparatus and machines is mainly due to the transformation of electrical energy into heat energy. The causes of the production of this heat energy are mainly current flowing through resistance, hysteresis in iron, eddy-current action in iron and copper. While in electrical machines there are the additional causes, namely, brush and axle friction.

The value of this rise for a given amount of heat energy produced, depends upon the degree of enclosure, the temperature of the locality, the amount of radiative surface, the speed of the rotating parts, and the means provided for ventilation or cooling.

It has been experimentally shown by *Lord Kelvin* that the temperature rise in an insulated conductor exposed to air is less when the insulating covering has reached a certain thickness than when it is much thinner. The chief reason for this is the increased radiative surface of the outside covering.

Temperature rise is an important matter for the electrical designer, as most of the usual insulating materials are unable to withstand very high temperatures, and cotton or similar material may not be exposed to temperatures greatly exceeding about 100° C.

In some cases cotton insulation is specially treated to withstand higher temperatures, while enamel-coated wire is sometimes used instead of cotton-covered wire.

The heating of resistance coils.—In the case of a coil (Fig. 124) of resistance R ohms, across which a voltage of E is applied, the total

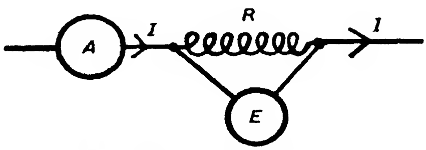

FIG. 124.—A resistance coil heated by current.

electrical power supplied to the coil is EI or I^2R watts, that is, $10^7 I^2R$ ergs of energy every second. If all this energy is converted into heat,

H the number of heat units or calories produced in t seconds will be given by

$$H \, . \, 4{\cdot}2 \times 10^7 = 10^7 I^2 R t \, ;$$

$$\therefore \; H = 0{\cdot}238 \, I^2 R t = 0{\cdot}238 E I t.$$

In the case of the coil becoming heated to incandescence, a small portion of the electrical energy will be converted into light energy.

This heat energy will cause the coil to rise in temperature until the heat energy radiated from it, balances the electrical energy received by it. The rise of temperature will, therefore, depend upon the rapidity of the processes of conduction, convection, and radiation in dissipating the heat through the surrounding medium.

To attain high temperatures with maximum economy and efficiency, it is necessary to lag the coil with insulating material of heat conductivity as low as possible, and the enveloping surface or surfaces should be of minimum radiative power.

To prevent excess temperature rise, the processes of conduction, convection, and radiation are utilised as far as possible to dissipate the heat produced. Overhead conductors can carry for the same temperature rise a considerably greater current per sq. cm. of their cross section than those under ground, especially if the latter are embedded in troughs filled with some insulating compound.

Temperature rise in electrical apparatus and machines.—In electrical heating devices a definite final temperature must be reached in a limited time. An electric kettle, of given capacity when filled, must reach boiling point in a certain number of minutes.

The case of electrical machines is different, the attaining of the final rise of temperature generally taking a long time, and in ordinary operation seldom reached.

In the former case, the apparatus is designed to reach its final temperature in a specified time with a maximum efficiency and minimum cost; this involves the right use of lagging and the minimising of radiation loss.

In the case of electrical machines, the parts are designed so that for maximum efficiency, good operation, and minimum cost, a specified final temperature may not be exceeded. Here good conduction, convection, and radiation are economic advantages, as they lower the final temperature rise, and, as a result, smaller machine parts may be used.

All the losses, excepting windage, appear in the form of heat in electrical machines, and further heating is produced by the proximity of hotter bodies. The temperature will depend upon these sources of heat, and also on the facilities by which this heat is dissipated either by natural or forced ventilation.

The *time-temperature* or *heating* curve of a certain part of an electrical machine, or piece of apparatus, is one of the most important curves representing its performance or operation, because from it the limiting capacity of the machine may often be deduced.

The most representative curve for a machine, is that of the part which will be burnt out first under extreme working conditions, and it is not necessarily the part which has the highest temperature rise. For instance, a very high temperature in the iron parts of a machine may have no injurious effect, while a temperature a little above 100° C. would destroy the cotton insulation of the armature or field coils.

In a machine the field coils may break down before the armature coils, and the primary before the secondary coil of a transformer. A knowledge of its heating behaviour would tell one which would burn out first.

The *cooling* curve is also important, especially in intermittent operation, such as in the case of crane motors, or transformers on lighting loads.

Temperature rise may be measured by thermometer, coils of fine platinum wire wound non-inductively, thermo-junctions, such as constantan and copper or iron and eureka, and in the case of coils by the change of resistance in the coils themselves.

For heating devices, the mercury thermometer and *Callendar's* platinum resistance pyrometer are generally the simplest and handiest methods of obtaining temperature rise. In the latter instrument, a range from ordinary temperature to well over 1000° C. may be conveniently and accurately measured.

In a built machine the only determinations of temperature which may be made are the surface temperatures by a mercury thermometer, and the average temperature of the coils by the resistance method. The latter is a valuable determination for field magnet bobbins, as the ratio of the maximum temperature inside the bobbin to the average temperature seldom exceeds 1·2. The reading of the surface temperature is generally unreliable, even with the bulb covered with a pad of cotton waste, and unless the ratio of the maximum inside temperature to the surface value is known, the method is useless.

Experimental machines have been built with thermo-electric junctions embedded in the vital parts, and investigations of temperature rise have been made under various conditions of operation.*

In determining average temperature by the resistance method, the following formula may be used,

$$t^\circ \text{ C.} = \frac{r - r_0}{r_0 \times 0{\cdot}00426},$$

in which r_0 may be found by measuring the resistance of the coils when at atmospheric temperature, which is read by a mercury thermometer.

The following results were obtained for a bobbin of length 40 cms., having 14 layers of No. 18 S.W.G., D.C.C. wire. It was divided into

* More recently, some manufacturers, to their credit, have inserted such junctions in their machines, so that the temperature rise, under various conditions of loading, may be studied by the user.

seven sections, each consisting of two layers in series, and the ends of each section were joined to two terminals. The coil was wound on a vulcanised fibre tube of internal bore 4·4 cms. and outer diameter 5·3 cms. Each layer was insulated from the next by a sheet of tracing cloth. The outside diameter of the coil was 9·5 cms. Sec. 1 is the innermost section, consisting of layers 1 and 2, and the temperatures given are the average temperatures in the section. This average temperature was determined by the resistance method, using ammeter and voltmeter.

The first table is for an exciting current of 3 *amperes*, and the second for one of 4 *amperes*. The time is given in minutes.

I = 3 amperes.

TIME.	0	9	17	25	32	40	49	56	70	85	100
Sec. 1	20	32	42	51	58	66	73	66	58	50	48
Sec. 2	20	33	42	50	57	64	71	63	57	51	48
Sec. 3	20	30	39	49	55	63	69	64	53	47	43
Sec. 4	20	33	42	52	60	68	75	65	55	50	46
Sec. 5	20	33	43	52	57	63	69	64	54	48	44
Sec. 6	20	32	41	50	56	61	65	59	51	46	43
Sec. 7	20	28	36	43	48	53	57	51	46	41	38
A	20	32	41	50	56	63	68	62	53	48	44

I = 4 amperes.

TIME.	0	9	17	31	40	50	70	100
Sec. 1	22	47	59	77	70	63	54	44
Sec. 2	22	47	59	77	70	64	56	47
Sec. 3	22	38	54	76	66	60	53	43
Sec. 4	22	41	58	88	67	60	51	43
Sec. 5	22	41	56	79	65	59	51	42
Sec. 6	22	40	55	75	60	54	47	42
Sec. 7	22	36	50	69	55	50	45	39
A	22	42	56	77	65	59	51	43

For the case of 3 amperes, the time of heating was 49 minutes, and for 4 amperes 31 minutes; this was followed by cooling.

The temperature-time curves for the hottest and coolest sections for an exciting current of 3 amperes are shown in Fig. 125. They are denoted by C and A.

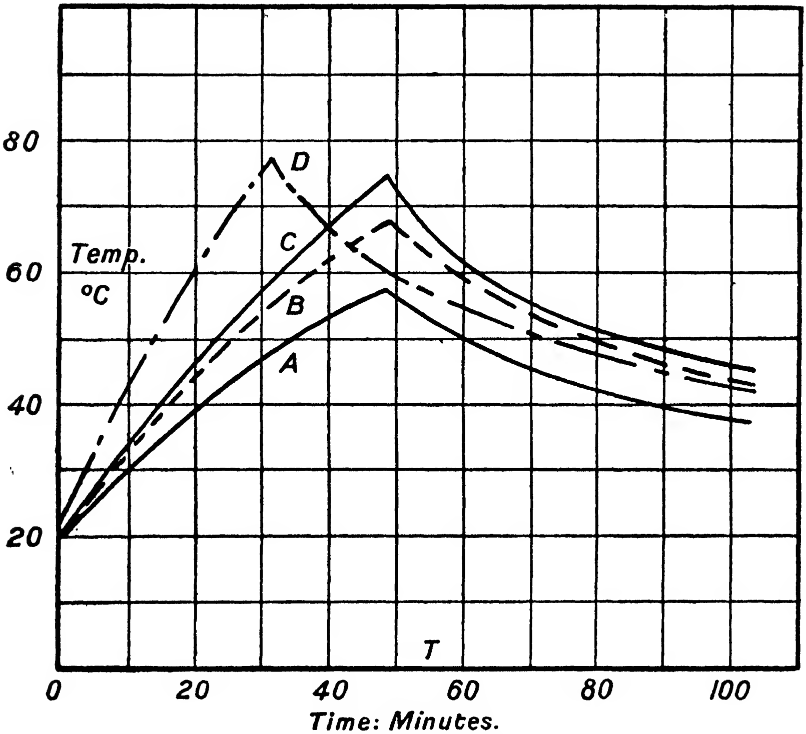

FIG. 125.—The heating and cooling curves for a bobbin of fourteen layers.

Curve B gives the *average* temperature of the bobbin for an exciting current of 3 amperes, and D for a current of 4 amperes.

By using thermo-junctions, it has been found by a number of investigators that the temperature distribution inside a field magnet bobbin is approximately represented by the dotted curves T_1 and T_2 shown in Fig. 126. The black dots denote the position of the junctions. The ordinates of T_1 are measured from line *cd*, and those of T_2 from *ab*.

The form of the curves, especially T_1, will depend on the facility with which the heat can escape from the outer and inner surfaces of the bobbin. Owing to the fan action of the rotating armature, the end *b* of the bobbin will in general be kept cooler than the end *a*.

In the case of thermo-junctions embedded in the slots of parts which rotate, it is necessary to connect them to slip rings on the shaft; the rings, fitted with brushes, being insulated from the shaft and each other.

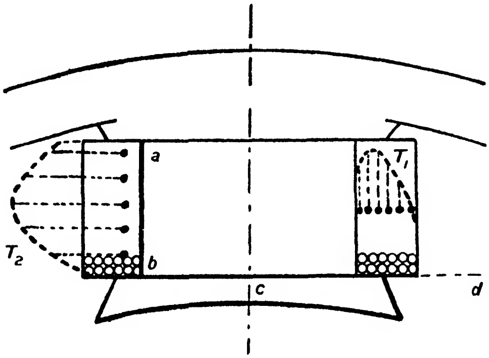

FIG. 126.—The temperature gradients in a field magnet bobbin.

Equations of temperature-time curves.—The following theory relates to the case of some substance or substances of constant specific heat, heated by means of a uniform supply of energy at constant value, and subjected to a radiation loss obeying Newton's law of cooling. The supply energy may be direct heat energy or electrical energy transformed into heat.

An electrical kettle satisfies most completely the preceding conditions, a stationary piece of apparatus or a static transformer is also a representative case, but a machine with rotating parts, or with artificial cooling, will not be quite so representative; for, in the latter case, in addition to the radiation according to Newton's law, there would be a conduction loss due to the currents of cool air rushing along the exposed surfaces and carrying off some of their heat.

In the case of a substance having a variable specific heat, the same theory will enable one to build up the temperature-time curve for the whole range of temperature, by dividing this range into a sufficient number of divisions, through each of which the specific heat of the substance is approximately constant, and then building the whole curve up part by part.

Let θ be the *excess* of temperature of the substances above that of the surrounding medium, M the sum of the different masses each multiplied by its specific heat, and P the constant power in *watts* supplied to these substances and transformed into heat. In the electric kettle, the two chief substances are the vessel and water.

The heat energy supplied, has two functions to perform, it heats up the substances, and by them radiates the remainder of its energy through the surrounding medium to neighbouring substances. The number of heat units radiated out per second, is given by Newton, as

proportional to the excess temperature of the radiating surface above that of the surrounding medium. Thus, if k is the radiation coefficient of the substances receiving heat, the number of heat units radiated out from their surfaces at time t, when the excess temperature is θ, is

$$k\theta \, dt.$$

For electrical heating, the equation relating power and *excess* temperature is therefore

$$10^7 \mathrm{EI} \, dt = \mathrm{J} \, . \, \mathrm{M} \, d\theta + \mathrm{J} \, . \, k\theta \, dt,$$

$$\mathrm{J} = 4{\cdot}2 \times 10^7.$$

Hence $$\frac{d\theta}{dt} + a\theta = \mathrm{A}, \qquad \text{(a)}$$

in which $$a = \frac{k}{\mathrm{M}} \quad \text{and} \quad \mathrm{A} = \frac{0{\cdot}238\mathrm{P}}{\mathrm{M}}.$$

The solution of this equation is as follows:

$$\frac{d\theta}{dt} \Big/ (\mathrm{A} - a\theta) = 1.$$

Integrating, $$-\frac{1}{a} \log (\mathrm{A} - a\theta) = t + \mathrm{K},$$

K being an arbitrary constant.

Therefore $$\theta = \frac{\mathrm{A}}{a} + \mathrm{B}e^{-at},$$

in which B is an arbitrary constant.

If, when the power is switched on, the temperature is that of the surrounding medium, $\theta = 0$ when $t = 0$; which gives $\mathrm{B} = -\frac{\mathrm{A}}{a}$.

If the excess temperature is α when the power is switched on,

$$\mathrm{B} = \alpha - \frac{\mathrm{A}}{a}.$$

In the first case, $$\theta = \frac{\mathrm{A}}{a}\left\{1 - \frac{1}{e^{at}}\right\}. \qquad \text{(b)}$$

In the second case, $$\theta = \frac{\mathrm{A}}{a}\left\{1 - \frac{1}{e^{at}}\right\} + \frac{\alpha}{e^{at}}. \qquad \text{(c)}$$

When t becomes very large, θ_m, the final or maximum or balancing temperature for the particular power used, is for both cases given by

$$\theta_m = \frac{\mathrm{A}}{a} = \frac{0{\cdot}238\mathrm{P}}{k},$$

that is, the balancing temperature θ_m is proportional to the power supplied.

The equation for cooling is, similarly,

$$M\,d\theta + k\theta\,dt = 0,$$

$$\frac{d\theta}{dt}\Big/\theta = -a,$$

$$\therefore\ \log\theta = -at + K_1.$$

If time t is measured from the instant when the temperature of the arrangement is θ_1, which may be any selected value after cooling has started, then $K_1 = \log\theta_1$. Therefore

$$\theta = \frac{\theta_1}{e^{at}}, \qquad \dots\dots\dots\dots\dots\dots (d)$$

in which $$a = \frac{k}{M}.$$

The forms of the heating and cooling curves, as derived from these equations, are shown in Fig. 127, the excess temperature θ being plotted against the time of heating and cooling.

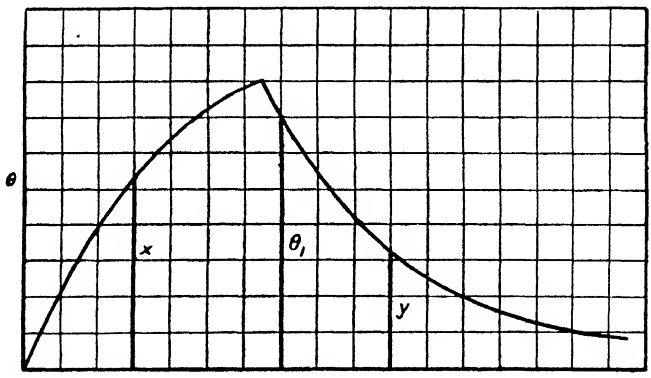

FIG. 127.—The heating and cooling curves derived from equations.

If x is the excess temperature at time t for the case of heating, and y the value for the *same* time for cooling from temperature θ_1,

$$x = \theta_m\left\{1 - \frac{1}{e^{at}}\right\},$$

$$y = \frac{\theta_1}{e^{at}}.$$

Hence $$\frac{\theta_m - x}{\theta_m} = \frac{y}{\theta_1}.$$

θ_1 may be any point on the cooling curve, provided y is taken t seconds later.

From this last equation, θ_m may be calculated, and $\theta_m = \frac{0 \cdot 238}{k} P$; so that, by measuring P, the power, k may be calculated.

Also, from equation (d), $a = \frac{k}{M}$ may be calculated. Thus,

$$a = \frac{k}{M} = \frac{2 \cdot 3}{t} \log_{10} \frac{\theta_1}{y}.$$

Therefore M may be found.

Thus, by the simple experiment of obtaining three excess temperatures x, y, and θ_1; reading t the time; and measuring P the power converted into heat, the values of the radiation coefficient and the water equivalent of the arrangement may be calculated; also, the complete heating and cooling curves may be obtained.

The ordinates of the cooling curve are independent of P, but depend upon the temperature at which cooling commences. Those of the heating curve are proportional to P. A heating and cooling curve drawn for a given arrangement for power P, may therefore be employed for any other power, provided the external medium remains unchanged. Thus, if P is doubled, the ordinates throughout both curves will be doubled; the initial temperature of the cooling curve being doubled in this case.

In the heating of liquids care should be taken to stir the liquid well; a mechanical stirrer worked by a small motor is desirable.

Another important deduction is that of finding the time required to reach a certain proportion of the final excess temperature.

Equation (b) may be put in this form,

$$t = \frac{M}{k} \log \frac{\theta_m}{\theta_m - \theta}.$$

$\frac{M}{k}$ is termed the *heating time-constant* of the arrangement, because this quantity is proportional to the time taken to reach a specified proportion of the final excess temperature.

If

$$f = \frac{\theta}{\theta_m},$$

$$t = 2 \cdot 3 \frac{M}{k} \log_{10} \frac{1}{1 - f}.$$

From this equation the value of t in seconds for *unity* time-constant may be calculated for different values of f. This has been done in the following table:

f	0·1	0·2	0·3	0·4	0·5	0·6	0·8	0·9
t	0·106	0·224	0·36	0·53	0·69	0·92	1·61	2·3

As already described, k and M may be found from experiment, and the time-constant $\frac{M}{k}$ then calculated; θ_m may also be obtained. The time for the arrangement to reach a certain proportion of its final excess temperature may therefore be calculated from the preceding table, or the graph drawn from it. For instance, if $\frac{M}{k}$ is 6000 and θ_m is 90° C.; then the time for the arrangement to reach an excess temperature of 72° C. is

$$6000 \times 1{\cdot}61 \text{ seconds} = 161 \text{ minutes.}$$

The following illustration will show how to determine the radiation coefficient, the water equivalent, the final temperature, and the time-constant of the arrangement used; also, to calculate the time it would take to reach 0·9 or any other specified proportion of its final excess temperature.

A large beaker containing water was heated by a spiral of vestalin carrying 5·65 amperes, and the voltage across the spiral was 8·3. The switch controlling this power was closed when the arrangement was at atmospheric temperature 17·2° C. At 20 minutes after closing the switch, the *excess* temperature was 20° C. When the excess temperature had reached 31·3° C., the switch was opened. After cooling 20 minutes, the excess temperature was 23·5° C. In this case

$$x = 20^\circ \text{ C.,} \quad y = 23{\cdot}5^\circ \text{ C.,} \quad \theta_1 = 31{\cdot}3^\circ \text{ C.,}$$

$$P = 8{\cdot}3 \times 5{\cdot}65 = 47 \text{ watts,}$$

$$\theta_m = \frac{\theta_1 x}{\theta_1 - y} = \frac{31{\cdot}3 \times 20}{31{\cdot}3 - 23{\cdot}5} = 80^\circ \text{ C.}$$

The final temperature attained by the water would then be 97·2° C., that is, it would never have boiled however long the 47 watts were applied to heat it. A larger value of P would be needed to boil the water.

Again, $$\frac{P \times 0{\cdot}238}{k} = \theta_m = 80,$$

so that $k = 0{\cdot}14$, which is the number of heat units radiated out from the surface of the arrangement per second, per degree excess temperature.

Also, $$2{\cdot}3 \log_{10} \frac{\theta_1}{y} = \frac{k}{M} \cdot t,$$

$$2{\cdot}3 \log_{10} \frac{31{\cdot}3}{23{\cdot}5} = \frac{168}{M}.$$

From which $$M = 585.$$

Hence, the time-constant $\frac{M}{k}=4190$, and from the table, when $f=0\cdot9$, the time for the arrangement to become heated to

$$80\times0\cdot9+17\cdot2=89\cdot2^\circ\text{ C.}$$

will be $$4190\times2\cdot3\text{ seconds}=2\cdot68\text{ hours.}$$

The equations of the heating and cooling curves of the arrangement are respectively

$$\theta=80\left\{1-\frac{1}{e^{at}}\right\}$$

and $$\theta=\frac{\theta_1}{e^{at}},$$

a being $\frac{1}{4190}$, and θ_1 any temperature on the cooling curve from which t is measured.

In the preceding case, if θ_m plus θ_0, the outside temperature, was greater than boiling point, the fictitious value of θ_m may be used just as if it were real.

The time of boiling for a given value of applied watts may be calculated from the results of one experiment, such as that already given. Thus M, k, and the time-constant have been determined, and the required time is now for the case when

$$f=\frac{100-\theta_0}{\theta_m}.$$

As an illustration, let the power used be $47\times1\cdot5$ watts instead of 47 as before, then θ_m will be $80\times1\cdot5$, that is, 120° C. M has been found to be 585 and k 0·14. Also, $f=\frac{100-17\cdot2}{120}=0\cdot69$ for boiling point. The time-constant is 4190.

Therefore the time the water would take to boil for 70·5 watts, is

$$4190\times1\cdot18=4950\text{ seconds}=83\text{ minutes.}$$

By selecting values of P as follows, and using the time-constant table or graph, the time of boiling for any given power P′ may be quickly calculated, and the following table constructed. Thus,

$$\theta'_m=\theta_m\frac{P'}{P},$$

and $$f=\frac{100-17\cdot2}{\theta'_m}=\frac{82\cdot8}{\theta_m}\cdot\frac{P}{P'};$$

$$\therefore\ f=\frac{82\cdot8\times47}{80\times P'}=\frac{48\cdot7}{P'}.$$

P′	487	244	162	122	97·4	81·2	60·9	54·1
f	0·1	0·2	0·3	0·4	0·5	0·6	0·8	0·9
T	7·4	15·6	25·0	37·0	48·1	64·2	112	160

T is the time in *minutes* for the water to boil when supplied with P′ watts, and is calculated from the time-constant table, and 4190 the time-constant of the arrangement. For instance, when P′ equals 162 watts,

$$T = 0{\cdot}36 \times \frac{4190}{60} = 25{\cdot}0 \text{ minutes.}$$

The curve relating supply power and time to reach boiling point is given in Fig. 128. The dotted curve gives the *watt hours* required to boil the water at a given power.

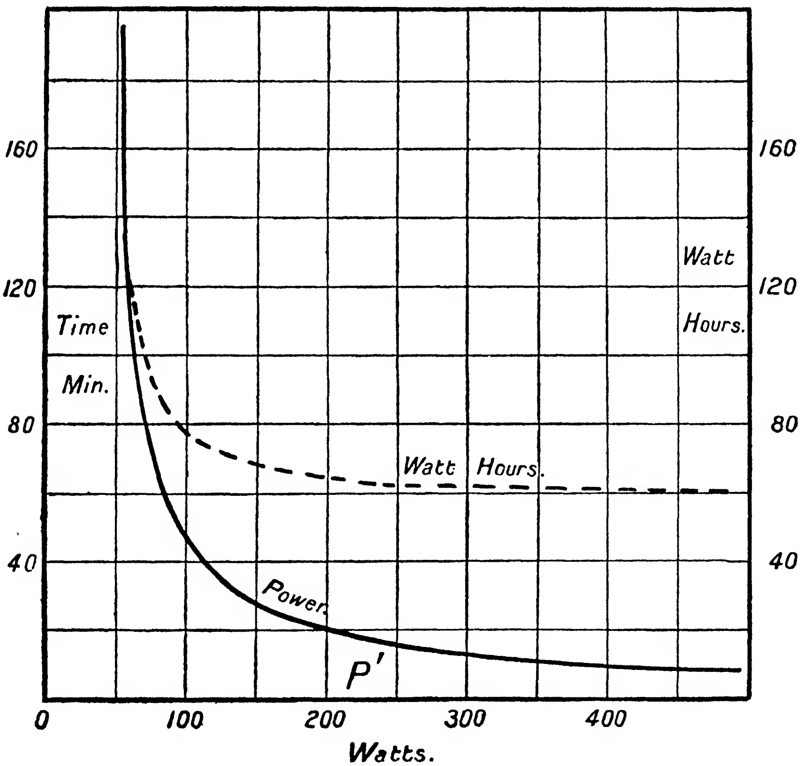

FIG. 128.—The power and watt-hours necessary to boil the water in a given time.

For instance, if a certain quantity of water is boiled in 10 minutes, the energy required is 60 watt-hours, but if boiled in 120 minutes, it requires 120 watt-hours, or *twice* as much energy.

Thus, by one simple experiment, and the derived equations, the behaviour of the arrangement when supplied with any specified power may be predicted, and its heating and cooling coefficients be obtained.

In the heating of electrical machines or apparatus, the same method may be used, the chief practical difficulty in such cases being

the placing of the temperature-measuring instruments in the most representative positions.

The exact temperature rise for the hottest part of a field-magnet bobbin, of the armature winding inside the slots, and at the surface of the commutator, is valuable knowledge from a commercial point of view; for upon such knowledge will largely depend the rating of the machine, whether the manufacturer may catalogue it, for instance, as a 50 or 60 horse-power machine.

Temperatures exceeding 100° C. may be regarded as injurious to the cotton insulation on the conductors, and the usual working temperature at the hottest parts should be much below this value.

Parts like commutators are affected differently by excessive temperature rise. The tendency to spark at the brushes is increased by a heated commutator, chiefly owing to the decrease of contact resistance. Mechanical stresses are also brought into play, due to the different expansions of metal and insulation.

Consider the case in which, under a specified load, the excess temperatures x, θ_1, y and time t of the most representative part of an electrical machine were obtained.

θ_m may be then calculated from

$$\theta_m = \frac{\theta_1 x}{\theta_1 - y},$$

and a calculated from

$$a = \frac{k}{\mathrm{M}} = \frac{2 \cdot 3}{t} \log_{10} \frac{\theta_1}{y}.$$

The time-constant $\frac{\mathrm{M}}{k}$ is therefore known, and, by means of the time-constant table, the number of hours necessary for the machine working on the specified load to reach its final temperature may be calculated.

If this is done for a set of loads, the rating of the machine for loads with respect to time may be determined.

To illustrate this method, consider a field-magnet bobbin, of which the following data is supplied for the hottest layer during a constant over-load run.

The value of x at two hours after switching on the load was 21° C.; θ_1 read just after switching off the load and field current was 40° C.; and y was 33° C. two hours after θ_1 was read. The machine was kept running at rated speed during cooling. Atmospheric temperature was 21° C.

$$\theta_m = \frac{40 \times 21}{40 - 33} = 120° \text{ C.},$$

$$2 \cdot 3 \log_{10} \frac{40}{33} = \frac{k}{\mathrm{M}} \cdot t = \frac{k}{\mathrm{M}} 7200.$$

The time-constant $$\frac{\mathrm{M}}{k} = 3 \cdot 72 \times 10^4.$$

Suppose 93° C., or excess temperature 72° C., is considered as a limiting temperature, then f for this value will be $\frac{72}{120}=0{\cdot}6$, and the load may run continuously for

$$\frac{0{\cdot}92 \times 3{\cdot}72 \times 10^4}{3600}=9{\cdot}5 \text{ hours},$$

without producing injurious effects in the bobbins.

This method may only be expected to give rough results in the case of machines with rotating parts, and its degree of accuracy requires testing for each type of machine.

The heating and cooling curves of the field-magnet bobbins of a direct-current compound generator for a run on a load of 3·5 K.W., at a speed of 1300 R.P.M., are shown in Fig. 191. An approximate value of the time necessary to reach a maximum internal temperature of 90° C. may be obtained as follows.

Assume, first, that the inside temperature at the hottest part is 20° C. higher than the average temperature of the bobbins. Then, since the external temperature was 17° C., the final *average* rise of temperature will be

$$90-20-17=53^\circ \text{ C.}$$

Now, θ_m will probably work out differently for different points on the curve, and several determinations should be made. Let x be taken for 30 minutes, then, from the curve, x is found to be

$$33{\cdot}5-17=16{\cdot}5^\circ \text{ C.}$$

The value of θ_1 will be taken as $39-17=22^\circ$ C., and y, 30 minutes later, is found to be $30{\cdot}5-17=13{\cdot}5^\circ$ C. Therefore, for this set of points,

$$\theta_m=\frac{\theta_1 x}{\theta_1-y}=\frac{22\times 16{\cdot}5}{8{\cdot}5}=42{\cdot}7^\circ \text{ C.}$$

A second set from the curve gave x, for 47 minutes, as

$$39-17=22^\circ \text{ C.};$$

θ_1 as $43-17=26^\circ$ C.; and y, 47 minutes later, as $29-17=12^\circ$ C. Then

$$\theta_m=\frac{26\times 22}{26-12}=41{\cdot}0^\circ \text{ C.}$$

A third set gave x, for 20 minutes,

$$29-17=12^\circ \text{ C.};\quad \theta_1\ 36-17=19^\circ \text{ C.},$$

and y, 20 minutes later, $31-17=14^\circ$ C. From which

$$\theta_m=\frac{19\times 12}{5}=45{\cdot}7^\circ \text{ C.}$$

The hottest temperature inside the bobbin will therefore not exceed about $45{\cdot}7+20+17=82{\cdot}7^\circ$ C., so that the bobbins will never attain a temperature of 90° C.

Suppose it is required to roughly find the time for the hottest part to reach a final temperature of 75° C., that is, an average rise of

$$75 - 20 - 17 = 38° \text{ C.}$$

First the time-constant is found from

$$2{\cdot}3 \log_{10} \frac{\theta_1}{y} = \frac{k}{\text{M}} \cdot t,$$

using values found in the last determination of θ_m, that is, θ_1 19° C., y 14° C., and t 1200 seconds. Then the time-constant $\frac{\text{M}}{k}$ is equal to

$$\frac{1200}{2{\cdot}3 \log_{10}\frac{19}{14}} = \frac{1200}{2{\cdot}3 \times 0{\cdot}132} = 3950.$$

Also,
$$f = \frac{38}{45{\cdot}7} = 0{\cdot}83,$$

and, from the time-constant graph, $t = 1{\cdot}9$. Therefore the time taken by the hottest part to reach a temperature of 75° C. will be

$$\frac{1{\cdot}9 \times 3950}{3600} = 2{\cdot}1 \text{ hours.}$$

The case of intermittent working.—The working of electrical machines is often intermittent and also irregular, and the prediction of temperature rise would be almost impossible, unless a complete set of representative heating and cooling curves for the machine, had been previously obtained.

In such a case it would generally be sufficient to use the results of the constant-load tests, and state for a given irregular load, that the temperature rise after a certain run, would not exceed *that* corresponding to a load of constant value equal to, or a more representative fraction of, the top value of the irregular load, for the same length of run.

The simplest case of intermittent working is that of a machine which alternately works at a constant load for t_1 seconds, and then has no load for t_2 seconds. This is approximately the case of transformers on lighting loads ; they are practically unloaded for a number of hours, and then loaded for the remainder of the twenty-four hours. Another case is that of machines on day work at constant loads, then shut down for the night.

Let the starting *excess* temperature be zero, θ_1 the excess temperature after t_1 seconds, α_1 that after $t_1 + t_2$ seconds, and so on.

Then, referring to Fig. 129,

$$\theta_1 = \theta_m \left\{ 1 - \frac{1}{e^{at_1}} \right\},$$

$$\alpha_1 = \frac{\theta_1}{e^{at_2}},$$

and, by equation (*c*), page 175,

$$\theta_2 = \theta_m \left\{ 1 - \frac{1}{e^{at_1}} \right\} + \frac{\theta_1}{e^{a(t_1+t_2)}},$$

$$\alpha_2 = \frac{\theta_2}{e^{at_2}},$$

$$\theta_3 = \theta_m \left\{ 1 - \frac{1}{e^{at_1}} \right\} + \frac{\theta_2}{e^{a(t_1+t_2)}};$$

$$\therefore\ \theta_3 = \theta_1 + \frac{\theta_1}{e^{a(t_1+t_2)}} + \frac{\theta_1}{e^{2a(t_1+t_2)}},$$

and so on.

Hence
$$\theta_n = \theta_1 \left\{ 1 + \frac{1}{z} + \frac{1}{z^2} + \ldots n \text{ terms} \right\},$$

in which $z = e^{a(t_1+t_2)}$, a quantity always greater than unity.

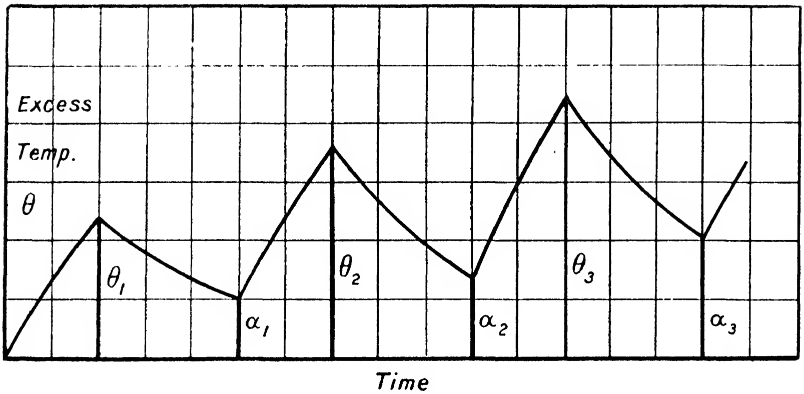

FIG. 129.—The theoretical heating and cooling curves for intermittent working.

The *sum* of the series is
$$\frac{1 - \dfrac{1}{z^n}}{1 - \dfrac{1}{z}}.$$

If n is sufficiently large, and z, as it is, greater than unity, this *sum* becomes
$$\frac{z}{z - 1}.$$

Thus
$$\theta_n = \theta_1 \frac{z}{z-1} = \frac{\theta_m \{e^{at_1} - 1\}}{e^{at_1}} \cdot \frac{z}{z-1};$$

$$\therefore\ \theta_n = \theta_m \left\{ \frac{e^{a(t_1+t_2)} - e^{at_2}}{e^{a(t_1+t_2)} - 1} \right\},$$

$$\theta_n = \theta_m \left\{ 1 - \left(\frac{e^{at_2} - 1}{e^{a(t_1+t_2)} - 1} \right) \right\},$$

which shows that θ_n, the final *maximum* temperature rise for the intermittent working, is, as would be expected, smaller than that for continuous operation.

For lower temperature rises, the true sum of the series must be used, and the exact value of θ_n is given by

$$\theta_n = \theta_m \left\{1 - \frac{1}{e^{at_1}}\right\} \frac{z^n - 1}{z^n - z^{n-1}},$$

z being equal to $e^{a(t_1+t_2)}$.

Consider the case of a transformer so ventilated that its laws of heating and cooling roughly follow those of the ideal case to which the preceding theory applies. Suppose it works at constant load 8 hours on and 16 off. It is required to roughly determine its final temperature rise.

By the experimental method given in this chapter, suppose θ_m was found to be 80° C., and the time-constant 31400. Then

$$a(t_1 + t_2) = \frac{24 \times 3600}{31400} = 2{\cdot}75,$$

$$at_2 = \frac{16 \times 3600}{31400} = 1{\cdot}84,$$

$$\theta_n = 80 \,.\, \frac{e^{2\cdot75} - e^{1\cdot84}}{e^{2\cdot75} - 1} = 51^\circ \text{ C}.$$

Also, $$\theta_1 = 80\left\{1 - \frac{1}{e^{at_1}}\right\} = 48^\circ \text{ C}.$$

Thus, if the load had not been intermittent but continuous, the temperature rise of 51° C. would have been reached in an hour or so beyond 8 hours, whereas with the intermittent load 51° C. rise would never have been exceeded. With the continuous load, the final temperature rise of 80° C. would have been ultimately reached.

A transformer would therefore have a different rating for these two kinds of loading; a lower rating for the continuous load.

The sources of heat in a transformer are those represented by the iron and copper losses. W_0, the iron loss, is practically constant for all loads, and the copper loss in the primary coil is roughly equal to that in the secondary coil, so that P watts, the power transformed into heat, may be roughly expressed by

$$P = W_0 + 2 I_2^2 r_2,$$

r_2 being the resistance of the secondary coil.

To illustrate the case of intermittent loading, the following test was made. The bobbin referred to on page 171 was excited with

3 amperes, and the resistance of the fourth section of the coil was determined at different times.

The excitation was continued for 49 minutes, then stopped for 61 minutes, continued for another 49 minutes, and again stopped. The time T, in *minutes*, and θ_T, the temperature in degrees centigrade, are given in the following table :

Heating	T	0	3	12	20	28	35	43	49
	θ_T	19	24	37	46	55	61	70	—
Cooling	T	49	60	73	89	103	110		
	θ_T	—	61	53	49	47	46		
Heating	T	110	120	130	140	150	159		
	θ_T	46	58	65	74	81	88		
Cooling	T	159	172	184					
	θ_T	88	70	62					

The graph of T and θ_T is shown in Fig. 130.

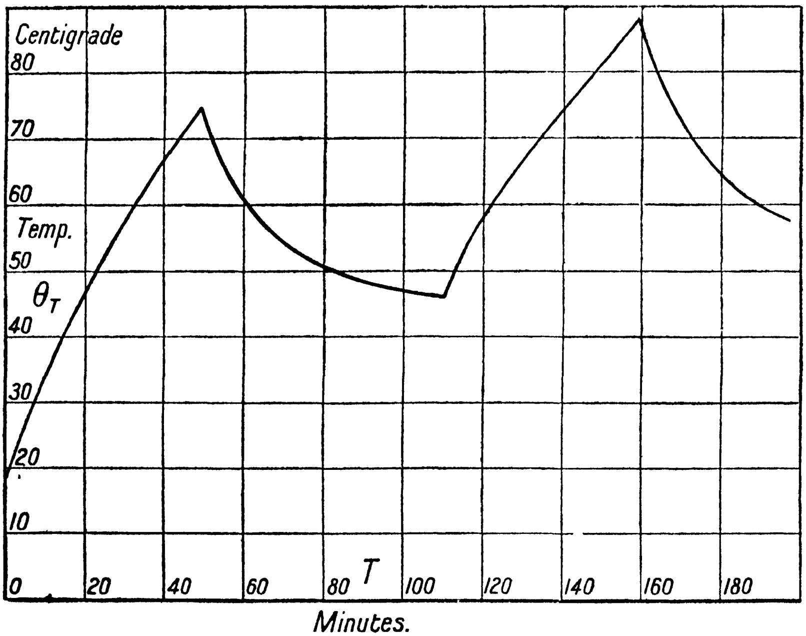

FIG. 130.—Heating and cooling curves of a bobbin for intermittent excitation.

A test was made on the primary coil of a 4 K.W. single-phase transformer. This coil was fed at 346 volts, and the secondary was loaded to give 4·2 K.W. at 80 volts.

The load was kept on for one hour, then taken off for one hour, and so on. During this run, at definite intervals, the resistance of the primary coil was read by an accurate dial box.

At the end of the first half hour, the load was taken off for a minute to obtain the resistance, and thereby a midway point, on the heating curve. The following results were obtained :

	Time.	Resistance.	Temperature.
H	0 min.	0·377 ohm	18·2° C.
	30	0·398	32·2
	60	0·412	41·6
C	75	0·404	36·2
	90	0·399	32·9
	105	0·396	30·9
	120	0·393	28·9
H	180	0·421	47·6
C	195	0·412	41·6
	210	0·406	37·6
	225	0·401	34·2
	240	0·398	32·2
H	300	0·427	51·7
C	330	0·412	41·6

The resistance calculated for 0° C. was 0·350 ohm. Fig. 131 gives the graph of these results.

In this case, $\theta_1 = 41{\cdot}6 - 18{\cdot}2 = 23{\cdot}4°$ C., $x = 32{\cdot}2 - 18{\cdot}2 = 14°$ C., and $y = 32{\cdot}9 - 18{\cdot}2 = 14{\cdot}7°$ C. Then, for *continuous* operation, using the theory of the ideal case,

$$\theta_m = \frac{x\theta_1}{\theta_1 - y} = \frac{14 \times 23{\cdot}4}{8{\cdot}7} = 37{\cdot}7° \text{ C.}$$

Another value of θ_m may be found by taking $x = \theta_1 = 23{\cdot}4°$ C., and y will then be $28{\cdot}9 - 18{\cdot}2 = 10{\cdot}7°$ C.

$$\theta_m = \frac{23{\cdot}4 \times 23{\cdot}4}{12{\cdot}7} = 43° \text{ C.}$$

The discrepancy between these two theoretical values of θ_m, represents to some extent, the difference between the actual case where the temperature is not uniform throughout the coil and the radiation occurs at a surface at lower temperature than any inside the coil, and the ideal case for which the preceding formula for θ_m was obtained.

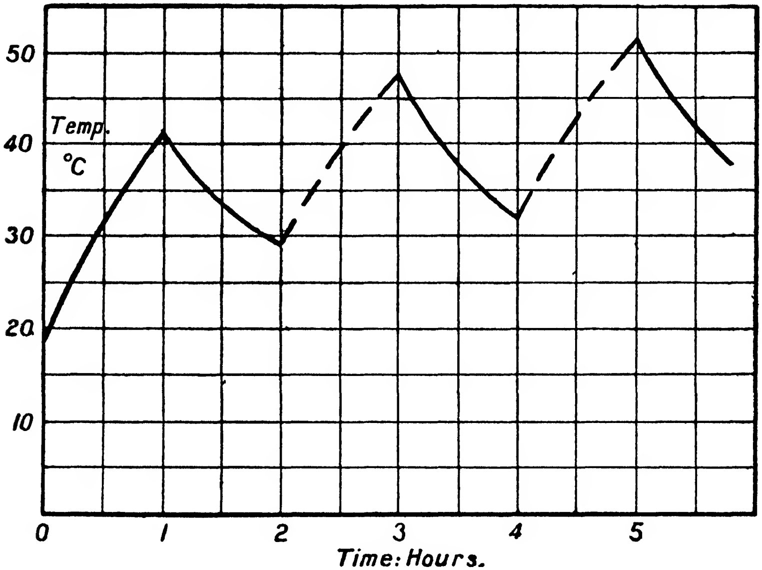

FIG. 131.—The heating and cooling curves of the primary coil of a 4 K.W. transformer on intermittent load.

The curve of the actual results shows that the rise of temperature for intermittent working will gradually reach a final maximum value, to which it will return after each succeeding heating.

By taking a *set* of values, the first heating curve may be drawn accurately, and its equation determined. θ_m, the final rise of average temperature in the primary coil for continuous loading, may then be obtained graphically or from the equation. Also, by taking a sufficient number of intermittent values, the final rise of average temperature for intermittent loading may be obtained.

As previously stated, temperature rise plays a very important part in the *rating* of machines. In some direct-current machines, especially those without interpoles, the commutation limit may be reached before the temperature limit. Also, in some alternators the regulation limit may be reached before the temperature limit. More often, however, the temperature limit defines the capacity or the rating of a machine.

A motor may have a rating of 200 B.H.P. for a continuous run of 6 hours without sparking or excessive temperature rise. If it is to be used for shorter runs, a larger B.H.P., say 250, may be developed by this machine; its rating would be higher. By *enclosing* this motor, which would be necessary in a dusty place or in certain fabric factories, the rating of the machine would be much reduced if it had not forced ventilation; it might only be classed as 100 B.H.P.

The rating of a machine, in which the temperature limit is first reached, depends upon the load, length of run, degree of enclosure, method of ventilation, external temperature, and proximity to hot bodies.

CHAPTER X.

MEASUREMENTS OF SELF AND MUTUAL INDUCTANCE, AND CAPACITY.

INDUCTANCE and capacity are important quantities in alternating-current work, in certain branches of direct-current work, and in wireless telegraphy and telephony. In general these quantities are measured by a bridge method, but other methods are sometimes used. Some of the chief laboratory methods are given in this chapter.

Measurement of self inductance by Maxwell and Rayleigh's method.—In this test a sensitive ballistic galvanometer and an accurate form of dial or post-office box are used. The connections are shown in Fig. 132, diagram (*a*) being for a condition of accurate balance, and (*b*) for the case when the balance is slightly disturbed either by increasing or diminishing R.

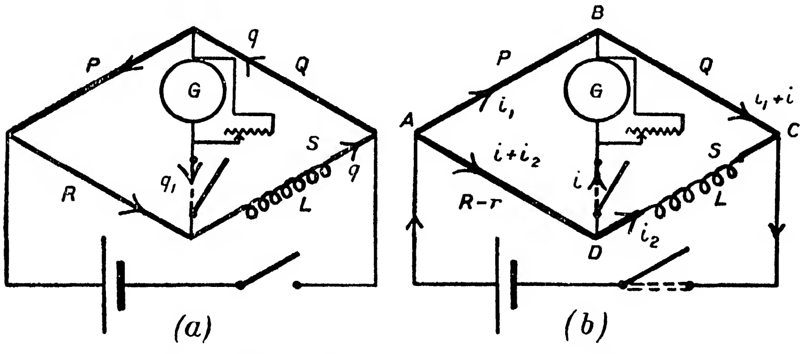

FIG. 132.—An arrangement for determining self inductance; Maxwell and Rayleigh's method.

The theory of the method is as follows. When accurate balance is obtained,

$$\frac{P}{Q} = \frac{R}{S} \quad \ldots\ldots (a)$$

The galvanometer key is now closed, and then the key of the battery; or the latter is released while the former is closed. This produces a *throw* or kick θ of the galvanometer coil.

By this operation a quantity of electricity q is produced in the arm containing L, and this passes through Q and then subdivides, part q_1 going through G and producing the throw, the remainder through P. The proportion going through G is, from page 28, given by

$$q_1 = q \cdot \frac{P+R}{P+R+G}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

The production of q is caused by the magnetic flux collapsing upon the turns of the winding of the coil under test, when the battery key is opened. If L is the self inductance of the coil, and i_2 the steady current in amperes flowing through it just before the battery key is opened,

$$Li_2 \times 10^8$$

is the flux threading the coil multiplied by its number of turns.

Therefore, from equation (4), page 4,

$$q = \frac{Li_2}{X} \text{ coulombs}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

where X is given by $\quad X = Q + S + \dfrac{(P+R)G}{P+R+G},$

and from equation (a), $\quad \dfrac{Q+S}{P+R} = \dfrac{S}{R};$

$$\therefore \; X = \frac{(P+R)\{PS + SR + SG + GR\}}{R(P+R+G)},$$

and $\quad q_1 = Li_2 \cdot \dfrac{R}{S(P+R+G)+GR}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (d)$

The next operation is to change R by either a small increase or decrease in value; suppose it is changed to $R - r$. Both battery and galvanometer keys are then closed, and the steady deflection α read.

To determine the current i passing through the galvanometer and producing this steady deflection, consider diagram (b). From the part ADB of the bridge,

$$Pi_1 = (R - r)(i + i_2) + Gi,$$

and from BDC, $\quad Q(i_1 + i) = Si_2 - Gi.$

Eliminating i_1 from these two equations by multiplying the first by Q and the second by P, subtracting, and using equation (a), will give

$$i(PQ + PG + QG + QR + Qr) = i_2 Qr.$$

Since r is very small, the last term in the brackets will be practically negligible in comparison with the sum of all the other terms, so that

$$i_2 r = \frac{Q(P+R+G)+PG}{Q}$$

and from equation (*a*), $$P=\frac{QR}{S};$$

$$\therefore\ i_2 r=\frac{S(P+R+G)+GR}{S}\,.\,i. \qquad \ldots\ldots(e)$$

From equations (*d*) and (*e*),

$$q_1=\frac{R}{S}\cdot\frac{1}{r}\cdot Li. \qquad \ldots\ldots(f)$$

Now, from equation (*a*), page 90, i is given by

$$i=K_g\frac{\alpha}{\cos\alpha}, \qquad \ldots\ldots(g)$$

and from the equation of the ballistic galvanometer, page 97,

$$q_1=K_g\frac{T}{2\pi}Z\theta. \qquad \ldots\ldots(h)$$

Therefore, from equations (*f*), (*g*), and (*h*),

$$L=\frac{T}{2\pi}\cdot\frac{S}{R}\cdot r\cdot Z\cdot\theta\cdot\frac{\cos\alpha}{\alpha},$$

in which

L is the self inductance in henrys.
T is the period of the galvanometer coil swinging on open circuit.
S is the resistance of the arm containing L.
R is the resistance in the adjustable arm of the bridge.
r is the increment or decrement of R for the disturbed balance.
Z is the correction factor for damping, and must be found for the circuit in which the transient current flowed.

This may be done as follows. When the bridge is arranged for exact balance, a very small resistance is taken out of R, and the battery and galvanometer keys are first closed, then released. The small resistance is now replaced, and the galvanometer key closed and kept in that position. The coil is now moving against the same damping forces which were in operation just after the transient current had passed through it.

Succeeding deflections, d_1 and d_2, are read on opposite sides of zero. This is repeated several times, and the average value of d_1/d_2 obtained. Then Z is calculated from

$$Z=\sqrt{\frac{d_1}{d_2}}.$$

θ is the angle representing the throw of the coil in the first operation.

α is the angle representing the steady deflection of the coil in the second operation.

Generally, $\cos\alpha$ may be taken as unity, and then θ and α may be replaced by their corresponding scale divisions.

G must be a sensitive galvanometer, so that θ may be large, and also, that a very small increment or decrement of R may produce a sufficiently large deflection.

By having r very small, the assumption that i_2 has the same value in both the cases of exact and disturbed balance is justifiable.

The cell or battery should be free from polarisation ; one or more Daniel cells are sufficiently steady.

The throw θ may be doubled by using a reversing key in the battery circuit instead of a tap key as shown.

Exact balance is to be found for the galvanometer unshunted, but a shunt should be used until balance is nearly found.

The value of r required may be less than the lowest value of R, which is usually 1 ohm and sometimes 0·1 ohm. In this case a sliding wire may be used in series with R to give the value of r.

Another method is to shunt R with a resistance box. Thus, if the infinity plug is out of this box, and it is found that the balance requires some value between 19 and 20 ohms, the infinity plug of the shunt arm is replaced, and the latter adjusted for exact balance.

Suppose the shunt resistance is 500 ohms, then the value of the third arm of the bridge is

$$\frac{20 \times 500}{20 + 500} = 19{\cdot}23.$$

To get the steady deflection, suppose this shunt resistance is changed to 450 ohms. The third arm resistance is now

$$\frac{20 \times 450}{20 + 450} = 19{\cdot}15.$$

In this case, $$r = 19{\cdot}23 - 19{\cdot}15 = 0{\cdot}08 \text{ ohm}.$$

With the ordinary sensitive ballistic galvanometers, this method of finding self inductance ceases to be accurate for values of inductance below about 10 millihenrys, as the value of the throw is then very small.

A double commutator called the *Secohmmeter*, devised by *Ayrton* and *Perry*, may be used in conjunction with the preceding method to increase its accuracy and sensitiveness. It consists of two commutators mounted on the same shaft, and driven round by gearing operated by a handle.

The arrangement of connections is shown in Fig. 133. By turning the handle of the secohmmeter, the current through the bridge is periodically reversed ; the connections from the galvanometer are also reversed between each reversal of the current.

An exact balance is obtained with the secohmmeter at rest. The commutators are then revolved at a speed of n revolutions per second. The galvanometer receives a steady succession of quantity q_1 due to

the inductance of the coil under test. As the reversals are four times per revolution, the steady current producing the deflection θ of the galvanometer is $4q_1n$, which is equal to $K_g\theta$, so that

$$q_1 = K_g\frac{\theta}{4n}.$$

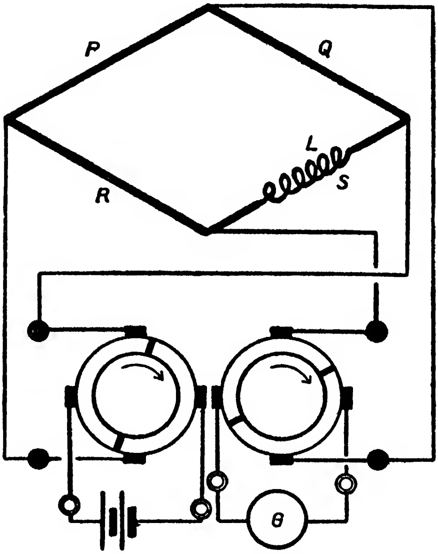

FIG. 133.—The arrangement for determining self inductance by the sechommeter.

On disturbing the balance, by increasing or decreasing R by a small amount r, and with the secohmmeter at rest, let the steady deflection be α.

Then $$i = K_g\alpha.$$

From the preceding method, equation (e),

$$i_2r = \frac{S(P+R+G)+GR}{S} \cdot i = \frac{\lambda}{S} \cdot i,$$

and from equation (d), $$q_1 = Li_2\frac{R}{\lambda};$$

$$\therefore\ Li_2\frac{R}{\lambda} = K_g\frac{\theta}{4n} = i\frac{\theta}{\alpha} \cdot \frac{1}{4n},$$

so that $$Li_2\frac{R}{\lambda} = \frac{Sr}{\lambda}i_2\frac{\theta}{\alpha} \cdot \frac{1}{4n},$$

or $$L = \frac{S}{4R} \cdot \frac{r}{n} \cdot \frac{\theta}{\alpha}.$$

By driving the secohmmeter, if suitably constructed, by a small direct-current shunt motor, the value of n may be accurately obtained. The speed may be varied by altering the resistance of the field circuit of the motor.

Comparison of two self inductances, one of which is adjustable and of known value.—The connections are shown in Fig. 134. An exact balance is first obtained, and then a balance for the throws is found by altering the adjustable self inductance, the resistance of which remains unchanged.

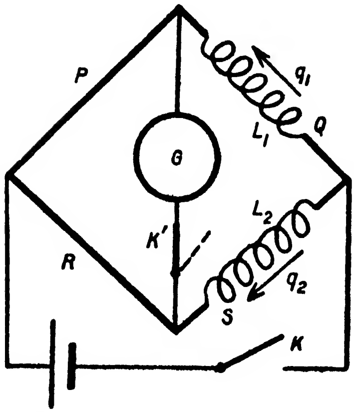

FIG. 134.—The arrangement for comparing self inductances by the bridge method.

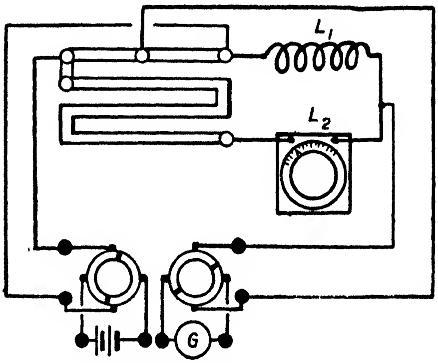

FIG. 135.—The arrangement for comparing self inductances by the secohmmeter.

The theory of the method is as follows. Consider the circuit at the instant K is opened; K′ being closed. The quantity of electricity due to the field collapsing in the turns of self inductance L_1 is q_1, and that for L_2 is q_2.

Now, q_1 flows through the same set of resistances as q_2, but in the opposite directions, and from equation (4), page 4,

$$\frac{q_1}{q_2}=\frac{L_1 i_1}{L_2 i_2}.$$

Therefore, since the same proportions of q_1 and q_2 pass through the galvanometer, the condition for a balance of the throws is

$$\frac{L_1}{L_2}=\frac{i_2}{i_1}=\frac{Q}{S}=\frac{P}{R}.$$

By using the secohmmeter as arranged in Fig. 135, this method is made much more sensitive. The operation is to obtain as before an exact balance, then to rotate the commutators and adjust the value of the known self inductance, until no deflection is observed on the galvanometer scale.

Anderson's method of determining self inductance.—This is one of the best methods for determining self inductance. The arrangement is shown in Fig. 136. L is the self inductance to be measured, C a condenser of capacity C farads, PQR the usual arms of the dial or post-office box, and r a non-inductive resistance box.

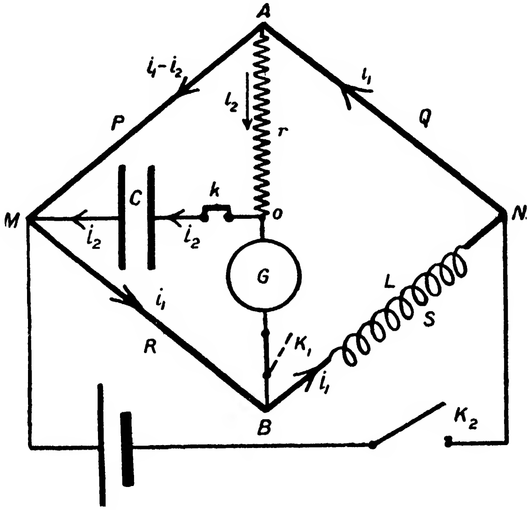

FIG. 136.—The arrangement for determining self inductance by Anderson's method.

With key k opened, an exact balance is first obtained, so that

$$\frac{P}{Q}=\frac{R}{S} \quad \text{or} \quad \frac{Q+S}{P+R}=\frac{S}{R}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Next, k is closed; K_2 and K_1 are also closed; K_1 is kept closed and K_2 released. A deflection will be observed on the galvanometer scale.

An adjustment only of r is then made, and the operation repeated until no throw is observed on the scale. That is, no current passes from O to B during the discharge of the transient currents through the bridge circuits.

One of these transient currents is produced by the discharge of the condenser C, and the other, acting oppositely, by the magnetic field collapsing on the turns of the coil whose self inductance is L henrys.

The currents shown in the diagram are for any given instant just after K_2 is opened.

Consider the triangular circuit MOB. The condenser has at the given instant a quantity of electricity equal to CRi_1, because the potential at O is the same as at B. Thus

$$-Ri_1=\frac{q}{C} \quad \text{and} \quad i_2=\frac{dq}{dt},$$

so that

$$i_2 = -\mathrm{CR}\frac{di_1}{dt}. \quad \text{.................................}(b)$$

Also, from the branches MB and MAO,

$$ri_2 - \mathrm{P}(i_1 - i_2) = \mathrm{R}i_1,$$

or

$$(r+\mathrm{P})i_2 = (\mathrm{P}+\mathrm{R})i_1, \quad \text{.........................}(c)$$

and from the branches BN and OAN,

$$\mathrm{L}\frac{di_1}{dt} + \mathrm{S}i_1 = -\mathrm{Q}i_1 - ri_2,$$

that is,

$$ri_2 = -(\mathrm{Q}+\mathrm{S})i_1 - \mathrm{L}\frac{di_1}{dt}. \quad \text{......................}(d)$$

Therefore, from (a), (b), (c), and (d),

$$ri_2 = -\frac{\mathrm{Q}+\mathrm{S}}{\mathrm{P}+\mathrm{R}}\,.\,(r+\mathrm{P})\,.\,i_2 + \frac{\mathrm{L}i_2}{\mathrm{CR}}.$$

From which equation and (a),

$$\mathrm{L} = \mathrm{C}\{\mathrm{S}(\mathrm{P}+r) + \mathrm{R}r\},$$

L being in henrys and C in farads.

To increase the sensitiveness, P and R should be large, and Q and r small.

Anderson and Fleming's method is a modification of the preceding method, and is more sensitive. The arrangement for making a test is shown in Fig. 137.

An exact balance is found by using the battery B and the galvanometer G; keys k, k_1, and k_2 being open. Then the telephone receiver T.R. is used instead of G, and the buzzer instead of the battery. Keys k_1, k, and k_2 are closed, and k_4, K_2 opened.

The resistance r is adjusted until there is minimum sound or silence in T.R., and the value of L is given by the same formula as obtained before,

$$\mathrm{L} = \mathrm{C}\{\mathrm{S}(\mathrm{P}+r) + \mathrm{R}r\}.$$

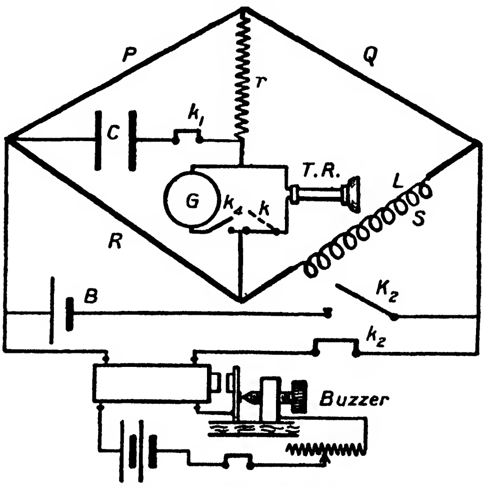

FIG. 137.—The arrangement for determining self inductance by Anderson and Fleming's method.

By using a sensitive telephone receiver, or better a head telephone with a pair of sensitive receivers, very small inductances may be accurately measured. Medium and large values may also be determined with great accuracy. In determining very small inductances, a condenser of small capacity should be used.

Determination of self inductance by means of an alternating current whose wave is sinusoidal.—An alternating current of value I and frequency f is sent through the self inductance of value L and resistance R. Then L is calculated from the formula

$$L=\frac{1}{2\pi f I}\sqrt{E^2-(RI)^2},$$

E being the voltage across the self inductance. This experiment is described in detail on page 66.

Measurement of mutual inductance.—One of the best and simplest methods is that of *G. C. Foster.* The diagram of connections is shown in Fig. 138.

When keys K and K_1 are closed, the condenser has a quantity of electricity q_1 equal to CRi, and the number of magnetic linkages in coil A is $Mi10^8$, if i is the current in amperes flowing through coil B.

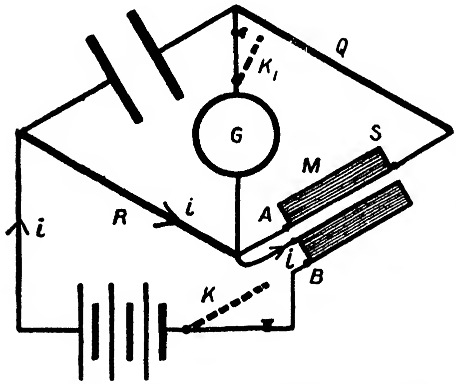

FIG. 138.—The arrangement for determining mutual inductance by Foster's method.

Suppose, now, that K is opened while K_1 remains closed, then the quantity of electricity flowing through the ballistic galvanometer G due to the condenser will be

$$q_2=CRi\cdot\frac{Q+S}{Q+S+G},$$

and that due to the magnetic linkages of A,

$$q_3=\frac{Mi10^8}{10^8(Q+S+G)}.$$

But these quantities flow in opposite * directions through G, so that when they are equal, there will be no deflection of the galvanometer coil.

In this case $M=CR(Q+S)$ henrys.

This method may be used for a large range of values of mutual inductance.

Measurements of capacity.—Exact measurements of capacity can only be made with condensers having a dielectric which has no appreciable absorption effect, such as air or mica. Standard condensers generally have an air or mica dielectric.

* It may be necessary to interchange the connections at the ends of A or B

Condensers with dielectrics of prepared paper and certain other materials exhibit absorption, and have no definite value of capacity. Thus, such a condenser, if charged for one second, will appear to have a certain capacity, while, if charged for one minute or longer, the capacity value will be much larger.

In experimenting with condensers, especially when dealing with high voltages, it is necessary to take every precaution against leakage errors due to defective keys, moisture, dust, and poor insulation.

Fleming and Clinton's commutator method of measuring capacity.— This is one of the best methods of measuring capacity; it is illustrated in Fig. 139. A special commutator of disc form as shown, or

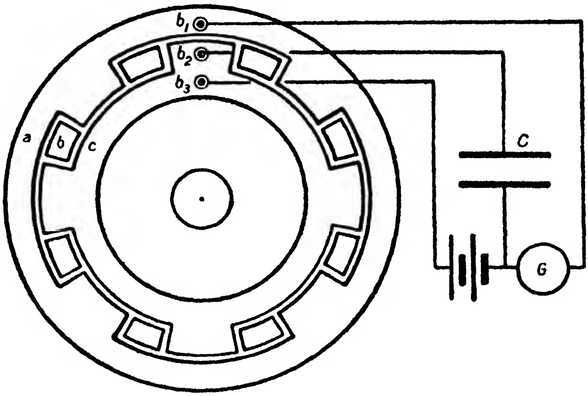

FIG. 139.—The commutator arrangement for determining capacity.

preferably of cylindrical form, is used to charge and discharge the condenser.

This is driven by a direct-current shunt motor, whose speed may be adjusted by altering the resistance in its field circuit. If the commutator is of disc form, the brass pieces *a*, *b*, and *c* may be fixed on a disc of ebonite, which is bored out to tightly fit either the shaft or a pulley on the shaft.

The brushes b_1, b_2, and b_3 are fixed to holders well insulated from each other. The metal parts, *a*, *b*, and *c*, must be as perfectly insulated as possible from each other and the machine.

At one instant the battery charges the condenser, and the next the latter discharges through the galvanometer. This discharge is repeated four times per revolution, or $4n$ times per second; n being the number of revolutions per second.

Thus $4n$CE is the quantity of electricity flowing through G per second, and is, therefore, the current I in the galvanometer circuit. So that

$$4nCE = I = k\theta,$$

in which θ is the deflection in scale divisions and k the galvanometer constant.

The value of k may be found by connecting the galvanometer in series with a high resistance of known value and a shunted cell whose voltage has been obtained by an accurate voltmeter or potentiometer. This arrangement is shown in Fig. 140. Then, if E_0 is the voltage of the cell and θ_0 the scale deflection of G,

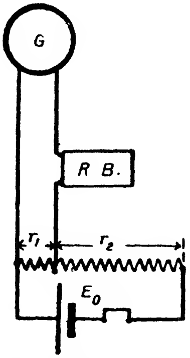

FIG. 140.—An arrangement for determining the constant of a galvanometer.

$$E_0 \cdot \frac{r_1}{r_1+r_2} = k\theta_0(R+G).$$

The range of capacities which may be measured by this method is very large, for, if the capacity is large, a shunted cell may be used to give the voltage E, while, if it is very small, the value of E may be increased to several hundred volts or even higher. Capacities as low as 10^{-4} of a microfarad may be measured in this way.

Anderson and Fleming's method.—If a self inductance of known value is available, the method described on page 196 may be used for determining an unknown capacity.

Leakage method.—If a resistance of very high value is accurately known, the leakage method of page 119 may be used for determining capacities.

Comparison of capacities.—The simplest method is to charge one condenser capacity C_1 from a battery, and discharge it through a ballistic galvanometer, noting the throw d_1. This is repeated for the second one. Then

$$q_1 = C_1E = kd_1,$$

$$q_2 = C_2E = kd_2;$$

$$\therefore \frac{C_1}{C_2} = \frac{d_1}{d_2}.$$

If C_1 is very large compared with C_2, the latter should be charged with a higher voltage than the former. This may be done by shunting a cell or battery with two adjustable resistances r_1 and r_2 in series. The sum of the two resistances should be about 5000, or larger, if the wire they are made of is very fine, such as that in the coils of post-office boxes.

The condenser of larger capacity C_1 is then charged by leads from the two ends of r_1, which should at first be made very small, and then discharged through the ballistic galvanometer, and d_1 the scale reading noted. The other condenser is charged by leads from the two ends of r_1+r_2, and the deflection d_2 read for discharge. The ratio of the capacities is then given by

$$\frac{C_1}{C_2} = \frac{d_1}{d_2} \cdot \frac{r_1+r_2}{r_1}.$$

Comparison of capacities by De Sauty's method.—The two condensers are arranged as in Fig. 141, R_1 and R_2 being adjustable non-inductive resistances. The experiment consists of adjusting R_1

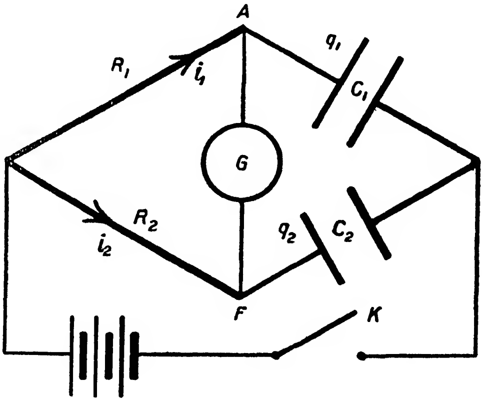

FIG. 141.—The arrangement for comparing capacities by De Sauty's method.

and R_2 until, on pressing K, no deflection of G occurs. The condensers should be as fully discharged as possible before pressing K.

The theory of the method is as follows. Considering the circuits at an instant after pressing K, the currents charging the condensers are as indicated, and the quantities of electricity in the condensers at this instant are respectively q_1 and q_2.

Then, if the potential at A is the same as at F,

$$R_1 i_1 = R_2 i_2, \quad \dots\dots\dots (a)$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}. \quad \dots\dots\dots (b)$$

Differentiating (*b*) with respect to time,

$$\frac{1}{C_1} \cdot \frac{dq_1}{dt} = \frac{1}{C_2} \cdot \frac{dq_2}{dt},$$

that is,

$$\frac{i_1}{C_1} = \frac{i_2}{C_2}. \quad \dots\dots\dots (c)$$

Therefore, from (*a*) and (*c*), $\dfrac{C_1}{C_2} = \dfrac{R_2}{R_1}.$

It is found that the method is more accurate when R_1 and R_2 are made large.

Comparison of capacities by Lord Kelvin's method of mixtures.—The diagram of connections is given in Fig. 142, and a side view of a special key needed in this method is also shown. A and B are the two condensers, and R_1 and R_2 two high-resistance boxes. When Q is pressed, T_2 and T_3 are metallically connected, so also are T_5 and T_6. When P is pressed, T_2 and T_1, also T_5 and T_4, are metallically connected. T_1, T_2, T_3 are as perfectly insulated as possible from T_4, T_5, T_6.

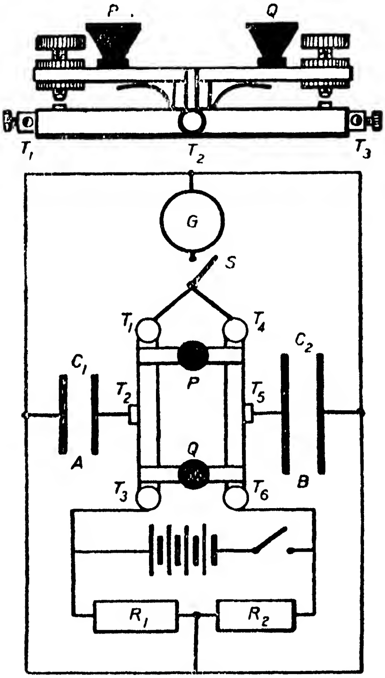

FIG. 142.—An arrangement for the comparison of capacities by Lord Kelvin's method of mixtures.

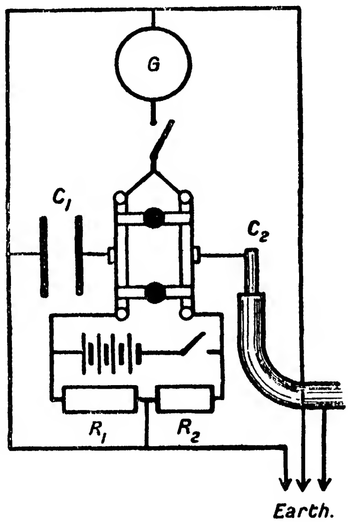

FIG. 143.—The arrangement for determining the capacity of a cable by the method of mixtures.

The battery switch is first closed, Q is then pressed down for a few seconds, and A becomes charged with a quantity R_1IC_1 coulombs; I being the steady current passing through R_1 and R_2. Condenser B likewise receives a charge of R_2IC_2. Also, the *right*-hand plate of A is charged positively, and the *left*-hand plate of B negatively.

Q is released, and P immediately after, is closed down. The charges of A and B then *mix*, and, if the former is larger, the latter will be completely neutralised, and the remaining charge will redistribute itself between the two condensers. In this case the charge on right-hand plate of A is still positive, and that on the left-hand plate of B will also be positive.

Therefore, on closing S, both charges will flow the same way through G, and a deflection will be observed on one side of zero. If the charge on A was less than that on B, by the same reasoning the deflection will be on the opposite side of zero.

When the charges are equal, one will completely neutralise the other on mixing, so that on pressing P no discharge will take place through G. The condition of equality of charge, obtained by adjusting R_1 and R_2, for which the deflection is zero is therefore given by

$$R_1IC_1 = R_2IC_2,$$

or

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}.$$

The key S should be closed quickly after P is pressed down, to avoid as far as possible any leakage from the condensers after the mixing of their charges.

The diagram of connections for determining the capacity of a cable by this method is shown in Fig. 143.

Determination of capacity by the alternating-current method.—A sine voltage E is applied to the terminals of the condenser, and the current I is read by an ammeter. Also, the frequency f is obtained. Then C, the capacity in farads, is given by

$$C = \frac{I}{2\pi f E}.$$

This method may only be regarded as accurate for sinusoidal voltage, and for condensers without appreciable dielectric absorption.

CHAPTER XI.

THE PRINCIPLES OF DIRECT-CURRENT MACHINES.

A KNOWLEDGE of the main principles of electric machines is necessary before they can be tested in a representative manner. It is essential to study armature windings, the effects of armature reaction, commutation, and the equations representing the operation of direct-current machines.

Armature windings.—It will be sufficient to consider the two chief types of armature windings known as the simple-lap or multiplé-circuit winding, and the simple-wave or two-circuit winding.

Simple-lap winding.—A lap winding with 24 conductors and 4 poles is shown in Figs. 144 and 145. The pitches, which must be *odd*, of a lap winding may be found from the formula

$$v=\frac{Z}{p}\pm 1,$$

in which y is the pitch, Z the total number of armature conductors, and p the number of poles.

For the case selected $y=5$ and 7. In the diagram, 5 is taken as the *front* or commutator end pitch and 7 as the *back* pitch.

To *analyse* this winding into its branches or sections, a diagram as shown in the upper part of Fig. 146 may be drawn; the arrows showing the directions of the induced voltages in the conductors.

As conductor 7 is associated with a south pole in exactly the same way as 1 is with a north pole, the induced voltage in each will be of the same numeric value; the poles being assumed to have equal strength. This is true of all similarly placed pairs of conductors, 4 and 10, and so on.

Therefore, if $e_1, e_2, \dots e_6$ are the respective induced voltages in 1, 2, 3, 4, 5, and 6, that is, all the conductors associated with one pole, the voltage distribution of the armature is that given by the lower diagram of Fig. 146.

Thus E, the internal voltage induced in the armature, is the sum of all the voltages induced in the $\frac{Z}{p}$ conductors associated with one pole.

The value of E may be determined from the consideration of the

on. This solenoid produces a magnetic field, which returns by way of the pole pieces, having the dotted lines as its mean path.

The vertical part of this field is at right angles to the main field passing through the armature core, and combining with it, produces a resultant flux, which is inclined to the main field in the direction shown by the dotted straight line.

In this case there is no demagnetising effect produced upon the main field, only pure distortion, which alters the position and uniformity of the magnetic flux through the armature.

The magnetic density at the forward pole tips, A and B, is increased, and at the back ones, C and D, diminished by the distorting effect of this armature reaction.

Generator: positive lead.—This case is illustrated by Fig. 152, and the winding may now be regarded as made up of two solenoids—one horizontal, the other vertical.

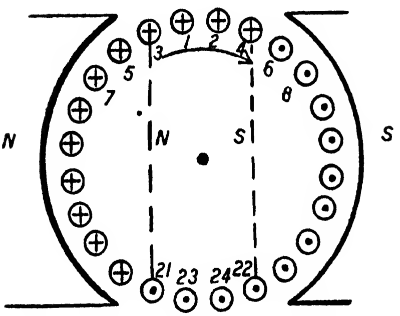

FIG. 152.—Armature reaction; positive lead.

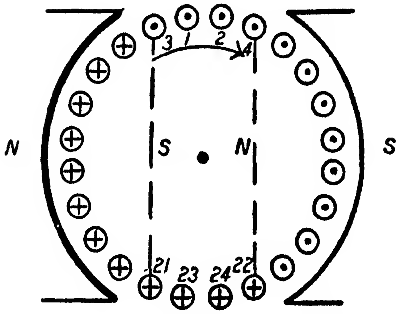

FIG. 153.—Armature reaction; negative lead.

The horizontal one lies between the two vertical lines, and is made up of conductors 3, 1, 2, and 4 at the top, and 21, 23, 24, and 22 at the bottom of the core; 3 and 21 forming the effective limbs of one spiral, 1 and 23 of another, and so on. This solenoid has its north pole adjacent to the north pole of the machine, and hence produces a *demagnetising* effect upon the main field; this opposing field runs parallel in direction to the main field throughout the whole magnetic circuit.

The vertical solenoid is made up of the remaining conductors; 5 and 6, 7 and 8, and so on, constituting the effective limbs of the respective spirals. A *distorting* effect is produced on the main field, and the flux density, as before, is increased in the forward pole tips and decreased in the backward ones.

Generator: negative lead.—The distribution of current in the winding will be that indicated in Fig. 153. In this case the vertical solenoid has the same distorting field as before, but the main field is *strengthened* by the horizontal solenoid. Hence, for the same amount of negative as positive lead, there will not be quite so much distortive

effect on the main field on account of it being strengthened instead of weakened by the horizontal solenoid.

In practice, the brushes of generators have positive lead, which may be very small or even zero when it is of interpole type.

Motor : general case.—In the motor the current enters the positive brushes instead of leaving them as in a generator. Figs. 151, 152, and 153 apply to a motor if crosses are substituted for dots, and *vice versa.* It may therefore be deduced for the motor that *negative* lead produces a *demagnetising* effect upon the field flux, and *positive* lead a *magnetising* or strengthening effect. In the motor the distorting effect is opposite to that of the generator, and the flux density is increased at the back pole tips and decreased at the forward ones.

In practice, motors used for traction, or where they have to revolve in either direction, have no brush lead ; when they are used in only one direction, negative lead is given to the brushes. Since the field is strengthened by plus lead and weakened by negative lead, a displacement of the brushes may be used as a means of varying the speed through a considerable range. The range is much increased by using interpoles to improve commutation.

ARMATURE REACTION IN MULTIPOLAR MACHINES.

Generator : no lead.—This case is illustrated in Fig. 154, in which part of a multipolar machine is shown. There are eight conductors

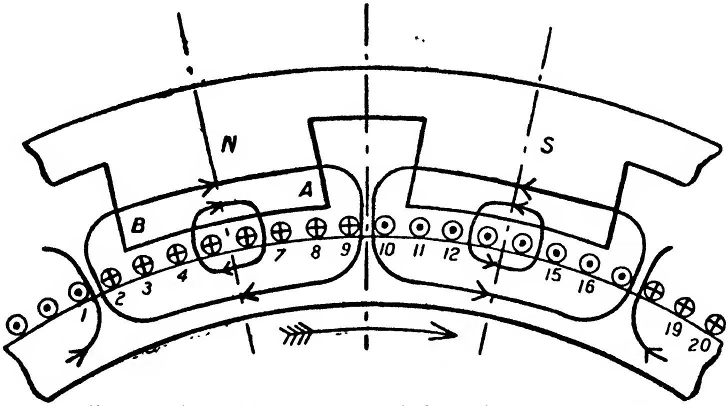

Fig. 154.—Armature reaction in a multipolar machine ; no brush lead.

under the influence of each pole. Conductors 9 and 10 may be regarded in magnetic effect as being a spiral with its bottom face a north pole ; 8 and 11 a second spiral with its lower face a north pole, and so on. Thus conductors from 6 to 13, both inclusive, form *in effect* a nearly flat coil with its lower face a *north* pole, while conductors from 14 to 21, in the same manner, form a similar coil with its lower face a *south* pole.

A series of poles, alternately north and south, are therefore formed around the armature periphery, equal in number to the poles of the machine, but exactly midway between them. The direction of the flux from these is indicated by the closed curves.

Again, there is only distortion, and the magnetic flux density at A is increased, that at B diminished ; in a multipolar motor with no lead, the reverse happens.

Generator : positive lead.—This case is illustrated in Fig. 155, and it will be noted that the alternate poles produced by the armature

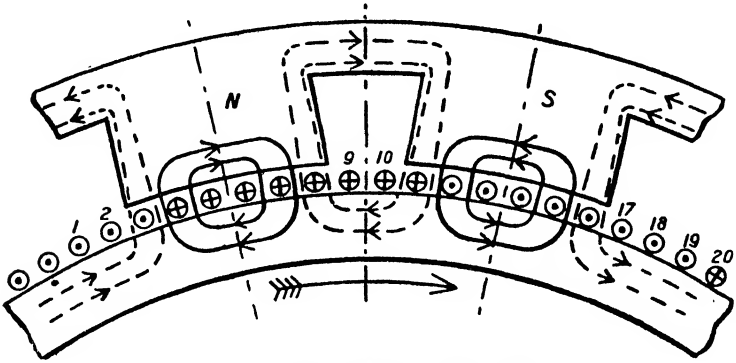

Fig. 155.—Armature reaction ; positive lead.

currents are displaced forward in the direction of rotation. A demagnetising, as well as a distorting effect, is thereby produced on the main field.

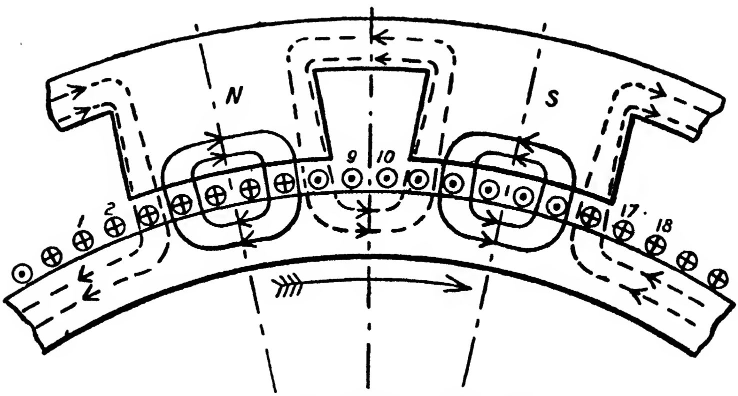

Fig. 156.—Armature reaction ; negative lead.

The spirals producing distortion are those represented by conductors 7 and 12, together with 6 and 13, which form in effect a nearly flat coil having its lower face a north pole ; those represented by 15

and 20, together with 14 and 21, which form a coil having its lower face a south pole; and so on round the armature periphery.

The spirals producing a demagnetising effect on the field flux are those represented by 9 and 2, together with 8 and 3, which form a nearly flat coil having its lower face a south pole; those represented by 10 and 17, together with 11 and 16, which form a coil having its lower face a north pole; and so on round the armature periphery. The faces of these coils adjacent to the main poles are therefore of the *same* polarity, and consequently produce a demagnetising effect on the main field.

Generator: negative lead.—Similarly, a distorting effect is produced in this case, but the main field is strengthened and not weakened, as in the case of positive lead. This is illustrated in Fig. 156.

Motor: general case.—It follows that the main field will be strengthened for positive lead and weakened for negative lead. Also, the flux density at the forward pole tips will be weakened and strengthened at the back ones.

THE AMPERE TURNS PRODUCING DEMAGNETISATION.

The ampere turns on the armature winding producing demagnetisation or magnetisation is proportional to the current in the conductors, to the number of armature conductors, and to the amount of the lead of the brushes.

In a *two-pole* machine, if θ is the angle of lead in degrees, the number of ampere turns producing demagnetisation is clearly equal to

$$2\theta \cdot \frac{Z}{360} \cdot I_c,$$

Z being the total number of ampere conductors in the winding, and I_c the current in the conductors. For a lap-wound armature, $I_c = \frac{I}{p}$, and for a wave-wound armature, $I_c = \frac{I}{2}$, I being the total current coming from or entering the armature.

The same expression is true for a multipolar machine, and may be deduced by the aid of the diagram (Fig. 155) in the same way.

Therefore, if A_r is the number of demagnetising or magnetising ampere turns *per pole* of the armature winding,

$$A_r = \frac{\theta \cdot Z \cdot I_c}{360}.$$

EXAMPLE.—Calculate the number of demagnetising ampere turns per pole due to armature reaction for the case of a six-pole lap-wound armature of 480 conductors when carrying its total full-load armature current of 800 amperes. The angle of lead may be taken as three degrees. *Ans.* 533 ampere turns per pole.

The ampere turns neutralised by armature reaction are generally compensated for by putting more series turns on the main field circuit in the ordinary type; by putting these compensating turns on the inter-

pole for the interpole type; and in high-speed generators, such as turbo-generators, using a special winding placed in slots in the pole shoes.

The action of the interpole in neutralising the effect of armature reaction is shown in Figs. 157 and 158. The former shows the mag-

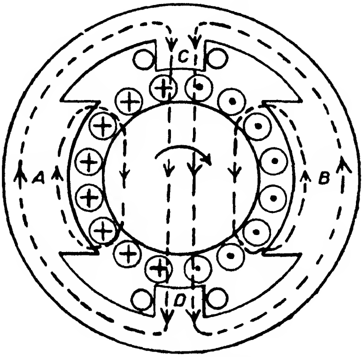

FIG. 157.—The magnetic fields due to the armature winding.

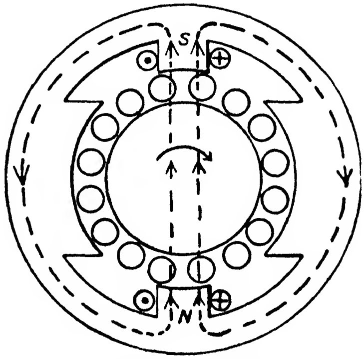

FIG. 158.—The magnetic fields due to the interpole winding.

netic field produced by the armature winding acting alone, while the latter figure shows that due to the interpoles acting alone. These fields are opposite, and tend to neutralise each other. The polarity of the interpole is the same as that of the main pole, which is next to it, but ahead, in the direction of rotation.

In designing interpoles it is generally sufficient to place on each interpole $\frac{ZI_c}{2p}$ ampere turns to neutralise armature reaction, and add about 50 per cent. more for commutation purposes. The interpoles, like the series winding, carry the load current, or a fixed fraction of it.

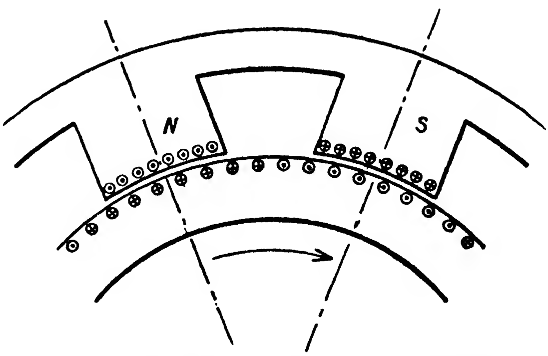

FIG. 159.—A winding for compensating for armature reaction

A special winding to neutralise armature reaction is shown in Fig. 159. This also carries the load current, or a fixed fraction of it.

Such a winding is known as *Deri's* winding, which is named after the inventor.

Commutation in direct-current machines.—The current in an armature coil is changed in direction as the two adjacent commutator segments to which its ends are attached pass from one side of a brush to the other. This refers to a lap winding. In a wave winding, if p is the number of poles, two adjacent segments will be attached to the ends of $\frac{p}{2}$ armature coils in series, the coils ranging round the whole armature periphery. This set of coils will therefore be short circuited as the two adjacent segments pass under a brush. The reversal of the current in the armature coil or coils is termed *commutation.*

The value of the current reversed is $\frac{I}{p}$ for a lap, and $\frac{I}{2}$ for a wave winding; I being the total armature current entering or leaving, and p the number of poles.

The collection of the current by the brushes, the sparking from the brushes to the segments or *vice versa*, and the heating produced at the contact surfaces, are all associated with the act of commutation.

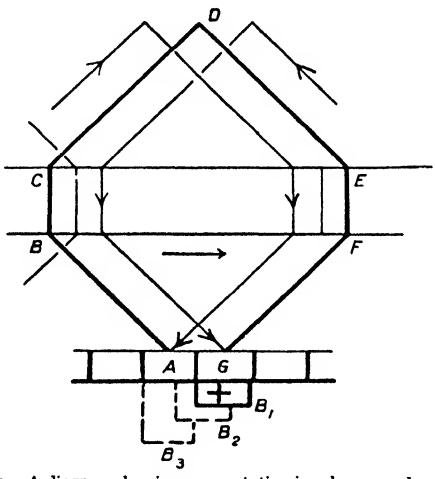

FIG. 160.—A diagram showing commutation in a lap-wound armature.

Sparking at the brushes in a generator generally occurs at the back part of the brush and the edge of the segment just leaving it. When sparking has occurred, this hind edge of the segment is often burnt in appearance, whereas the front edge, which meets the brush first in rotation, is unchanged.

Simple commutation in a lap winding, part of which is shown in Fig. 160, will now be considered.

The positive brush is shown in three different positions, B_1, B_2, and B_3, relative to the commutator surface, the armature being regarded as stationary, and the brush moving backwards.

In position B_1, the coil ABCDEFG, or coil S for brevity, is just beginning to be short circuited by the positive brush, and is then carrying a *clockwise* current of value I_c equal to the current in the armature conductors. For, the current from the left-hand section of the armature on reaching segment A, finds no contact surface to enter the brush by, and it flows clockwise through S to reach the brush. Likewise I_c from the right-hand section reaches segment G, and passes directly into the brush.

In position B_2, let x equal the width of contact surface on segment A, and w the width of the brush. The resistance of the copper part of the coil S will be neglected in comparison with the contact resistance of the brush. Then, when I_c reaches A from the left-hand section, $I_c\frac{x}{w}$ goes directly into the brush, and the remainder $I_c\frac{w-x}{w}$ goes *clockwise* through S. Also I_c, reaching G from the right-hand section, divides into $I_c\frac{w-x}{w}$, which goes directly into the brush, and $I_c\frac{x}{w}$, which goes *anti-clockwise* through S to get to the brush. The small thickness of insulation between the segments is neglected.

Hence, the current in S for position B_2 is clockwise, and of value

$$I_c\frac{w-x}{w} - I_c\frac{x}{w} = I_c\left(1 - \frac{2x}{w}\right).$$

When x is greater than $\frac{w}{2}$, the current in S is anti-clockwise.

In position B_3, the current in S is anti-clockwise and of value I_c. The graph of Fig. 161 shows the variation of the current in the short-circuited coil, assuming there is no self inductance.

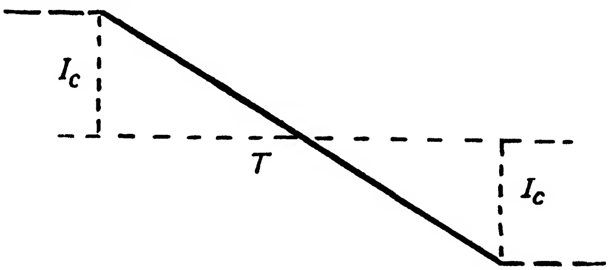

FIG. 161.—Ideal or rectilinear commutation.

The time of a complete reversal, or the time of short circuiting, is given by

$$T = \frac{w}{\pi D n} \text{ seconds,}$$

neglecting the thickness of the mica insulation between the segments of the commutator. D is the diameter of the commutator and n the revolutions per second.

This commutation is termed ideal, straight line, or resistance commutation. It is ideal because with it there can be no sparking; for the current, at the end of short circuiting is of the right value, and in the right direction for the coil to be included in the armature section next to the one it has just left.

Methods for improving commutation, such as lead of brushes or the use of interpoles, aim at realising as far as possible ideal or straight-line commutation.

EXAMPLE.—The diameter of a commutator is 35 inches and the width of one set of brushes is 0·8 inch. Calculate the time of commutation for its speed of 300 revolutions per minute. *Ans.* 0·00145 second.

This reversal of current I_c in the short-circuited coil during the brief interval of short circuiting causes an induced voltage to act in S, and for the whole time of short circuiting this induced voltage will act *clockwise* in the coil. For, the current in S diminishes from I_c to zero in a clockwise direction, and by the conservative principle of page 3, the induced voltage will act in the same direction. The current then rises from zero in an anti-clockwise direction to I_c, and, by the same principle, the induced voltage will act in the opposite direction, that is, still clockwise in S.

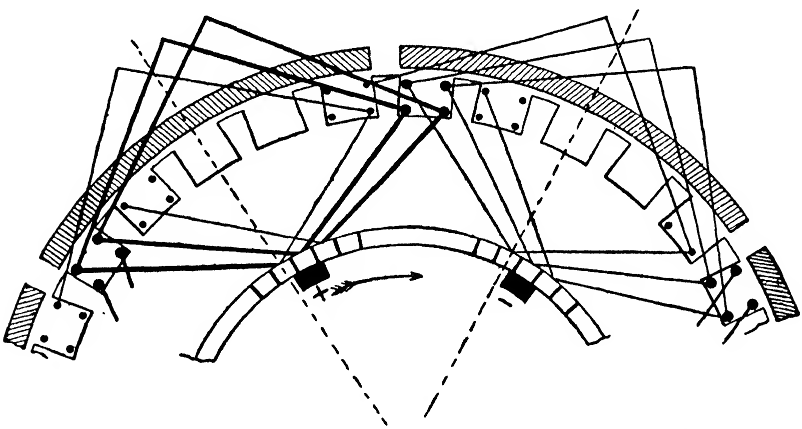

FIG. 162.—The armature coils simultaneously undergoing short circuiting by adjacent brushes.

In practice, this induced voltage, termed the *reactance voltage*, is produced partly by the self inductance of the coil undergoing short circuiting, and partly by the mutual induction of adjacent limbs of other coils, simultaneously undergoing short circuiting by the same or other brushes. Fig. 162 shows the sides, marked black, of the coils of a lap winding which are undergoing short circuiting at the same time.

Reactance voltage may be approximately calculated, and its value is important in the design of direct-current machines.

The function of lead.—A clockwise voltage is induced in the coil S undergoing short circuiting by the positive brush, whether there is lead or not. In Fig. 163 the brush has positive lead, and the coil

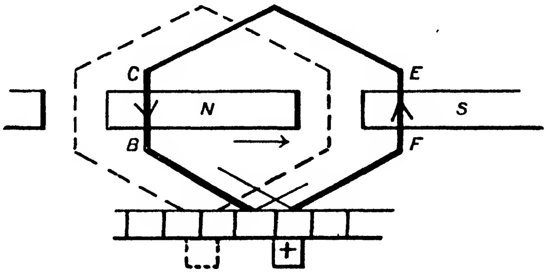

FIG. 163.—Commutation and brush lead.

sides CB and EF of S are cutting the fluxes from the north and south poles respectively; thereby producing in S an anti-clockwise voltage which tends to neutralise the reactance voltage. With no lead, S is in the dotted position, and no neutralising voltage is induced in it.

The function of interpoles.—The coil AB (Fig. 164) is just about to undergo short circuiting. In the position shown, it is threaded by a magnetic flux of $2\phi_i$, that is, by twice the flux threading the armature core from one interpole.

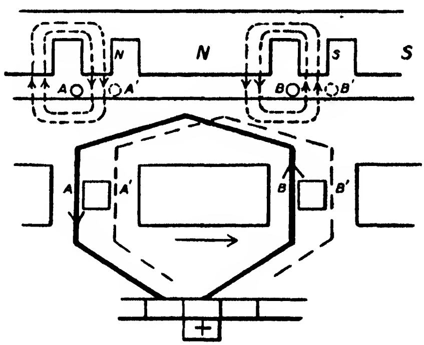

FIG. 164.—A diagram showing the function of interpoles in commutation.

As the coil moves forward, this flux is gradually removed from it, and in position A′B′ there is no *resultant* interpole flux threading it. Such a withdrawal of north-pole magnetism corresponds to a receding movement of a north pole from the back of the coil, which movement would, by an induced voltage, make the back face a south and the front face a north pole, that is, the induced voltage of the interpoles is anti-clockwise; thereby tending to neutralise the reactance voltage.

It is therefore necessary that an interpole should have the same polarity as the next main pole in the direction of rotation.

The function of brushes in commutation.—The contact resistance of a brush diminishes the effect of reactance voltage, as it is part of the circuit in which the latter operates. A machine is designed for a certain class of brush. If a softer brush is used, there may be commutation trouble due to its low contact resistance. If a harder brush is used, the commutator may become over-heated and again

lead to bad commutation. A reversible motor without interpoles would probably need a hard brush. A machine with low reactance voltage would probably work with a graphite brush, when lead or interpoles were used.

Interpoles, lead, and high contact resistance of brush, improve commutation. With interpoles, the tendency will be to use soft carbon and graphite brushes which have a low contact resistance; without interpoles and with lead, a medium carbon brush which has a medium contact resistance; and without lead or interpoles, a hard carbon brush which has a relatively high contact resistance.

The choice of brush will also be limited by mechanical considerations of strength, ability to withstand vibration, and wear and tear at high speed.

The losses in direct-current machines.—These comprise iron, copper, and frictional losses. Of these, the copper loss may be calculated with greater accuracy than the others, which involve a number of uncertain factors.

Iron losses.—There are hysteresis and eddy-current losses in the armature core, in the pole shoes, and in the masses of iron which are threaded by any stray magnetic field with which they have relative motion.

A small cube of the armature core at P (Fig. 165) is threaded by a maximum magnetic flux, which gradually falls to zero when B is reached. At C it has an equal but opposite flux, and so on. The cube therefore passes through a complete magnetic cycle or hysteresis loop in $\frac{2}{pn}$ seconds, that is, the frequency of the magnetic cycles is given by

$$f=\frac{np}{2}.$$

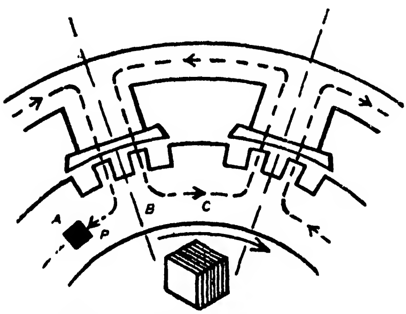

FIG. 165.—The magnetic flux through a small cube of the armature core.

In addition to this hysteresis loss, there is also a loss due to eddy currents being produced, in narrow circuits, at right angles to the lines of magnetic force threading the iron core. These have the same frequency as the hysteresis cycles, and are reduced to a minimum by laminating the core, and insulating the laminations from each other by thin paper, enamel, or varnish.

Another source of iron loss is the eddy and hysteresis action in the pole shoes due to the slots of the armature. For, the parts of the pole face immediately opposite the middle of the slots, are threaded by a minimum number of lines of magnetic force, but when these parts are opposite the middle parts of the teeth, they are threaded

by a maximum number of lines. The material of the pole shoe is therefore subjected to magnetic pulsations of frequency Sn, if S is the number of slots and n the speed of the armature in revolutions per second.

This pulsation produces eddy currents in the material of the pole shoe, and also subjects it to half hysteresis cycles. With large open slots and high speed, this loss may be large, and in some cases it is necessary to laminate both pole shoe, and part or whole of the pole core.

A loss occurs also in the solid parts of the armature, such as the iron clamping rings which hold the core plates together, due to magnetic leakage from the main field.

The iron loss of the machine depends upon the pressure of the armature core plates, the magnetic quality of the iron used, the insulation of the core plates from each other, and whether the edges of the latter are sharp or burred.

It would therefore be very difficult to calculate from theory a representative value for the iron loss of a machine. A more accurate value would be obtained by deduction, from the actual tests on iron loss for different types of machines.

Copper loss.—This comprises the total copper losses in the armature winding, the shunt circuit, the series coils, and the interpole circuit. The ohmic loss due to the brush-contact resistance is sometimes included in this total loss.

The resistance of the armature winding may be calculated from the dimensions of the copper strip for the type of winding used. The length of end connection per conductor may be taken as 1·4 times the pole pitch *plus* 3 inches, measured along the armature periphery.

σ_t, the specific resistance of copper per *inch cube* at temperature t degrees centigrade, may be calculated from

$$\sigma_t = 62{\cdot}5 \times 10^{-8}\{1 + 0{\cdot}00426t\}.$$

The copper loss in the shunt coils plus the ohmic loss in the field rheostat is given by

$$I_s^2 r_s = I_s E_t.$$

The copper loss in the interpole coils, assuming they are in the main circuit, is

$$I^2 r_i,$$

r_i being the resistance of the interpole coils, and which may be calculated from the length and cross section of the copper strip used in the coils.

Likewise, the copper loss of the series coils may be calculated.

The *contact resistance* of the brushes depends upon the current density in them, the pressure on the brushes, the temperature, whether at positive or negative brushes, the condition of the commutator surface, and the bedding of the brush upon that surface. At ordinary

temperatures, σ, the contact resistance per *square inch* of brush contact surface for *full*-load working may be roughly taken as

0·03 for hard carbon brushes,
0·014 for soft carbon brushes,
0·007 for graphite brushes.

If A square inches is the contact area of one set of brushes of a machine, $\frac{\sigma}{A}$ is the contact resistance of this set, and the ohmic loss per set is

$$(2I_c)^2\frac{\sigma}{A},$$

I_c being the current in the armature conductors, and $2I_c$ carried by one set of brushes.

A lap-wound armature of p poles, and armature output current I amperes, would have a brush-contact ohmic loss of

$$4\frac{I^2}{p^2}\frac{\sigma}{A}\cdot p=4\frac{I^2}{p}\cdot\frac{\sigma}{A}.$$

A wave-wound armature of p poles, and two sets of brushes, would have a brush-contact ohmic loss of

$$2I^2\frac{\sigma}{A}.$$

EXAMPLE.—Calculate the sum of the copper losses, including the ohmic contact loss of the brushes, at full load for a compound-wound generator of which the following data is given:

Full-load output 220 kilowatts, full-load terminal voltage 220 armature diameter 27 inches, length of armature conductor not including end connections 14·6 inches, cross section of these conductors 0·12 sq. inch, number of conductors 336, lap winding 4 poles, shunt current 6 amperes, resistance of series coils 0·001 ohm, resistance of interpole coils 0·0028 ohm, graphite brushes, each set having a contact surface of 6 sq. inches. Temperature of armature winding 60° C. *Ans.* 12·88 kilowatts.

Friction and windage losses.—These include the axle- and brush-friction losses and the windage loss. The axle or bearing friction depends upon the size of the bearing surfaces, lubrication, and the speed of the shaft; the windage loss upon the length, diameter, degree of enclosure, and speed of the rotating parts.

Axle friction and windage losses, taken together, increase rapidly with the speed. By doubling the speed in a certain machine, this loss was increased six fold. Above a certain speed, the windage loss increases as the cube of the peripheral speed.

The windage loss is sometimes further increased by the use of fans fixed to the armature spider in order to improve the ventilation, and thus prevent the temperature rise becoming excessive. This extra loss of windage will be accompanied by smaller copper losses in the machine, and the total loss through the use of the fans will

probably be lowered; in addition the machine, in certain cases, will have better operation.

The brush-friction loss depends upon the amount of contact surface, the pressure on the brushes, the coefficient of friction, and the peripheral speed of the commutator. For a rough determination of this loss, the coefficient of friction for carbon brushes may be taken as 0·3, and for graphite brushes 0·2. A pressure of $1\frac{1}{2}$ lbs. per sq. inch may be used for the former, and 2 lbs. per sq. inch for the latter. In certain cases higher pressures than these are used.

The *total* windage and friction losses of a machine may range from roughly 3 to 5 per cent. of the full-load power in small machines, to less than 1 per cent. in large machines of ordinary speed. For turbo-generators this loss may be as large as 4 per cent.

EXAMPLE.—Calculate the brush-friction loss in kilowatts of a turbo-generator having a commutator of diameter 16 inches, 4 brush sets each of contact area 28 sq. inches, and speed 2000 revolutions per minute. The pressure on the brushes may be taken as $1\frac{3}{4}$ lbs. per sq. inch, and the coefficient of friction 0·16. *Ans.* 5·94 kilowatts.

The performance of direct-current machines.—The main characteristics of the operation of a machine may be roughly determined from the equations which more or less represent its actions, and sometimes from first principles.

For instance, the starting torque of a series motor is very large, because at start, both field and armature conductors carry large currents. In the shunt motor it is not so large, because, though the armature conductors would take a large current, the field coils would not.

Series-wound generator.—Except in very rare cases, this machine is not used, but, as a compound-wound generator is a shunt-series machine, its case is important.

Generators are driven at a constant speed n revolutions per second, and it follows that the internal voltage produced is proportional to the effective flux ϕ per pole.

The diagram of connections is shown in Fig. 166, and if E and

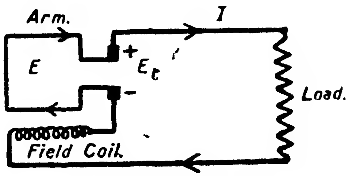

FIG. 166.—A series-wound generator on load.

E_t are the respective *internal* and *terminal* voltages, the equations for the machine are

$$E_t = E - IR, \qquad (a)$$

$$E = k\phi, \qquad (b)$$

R being the sum of the resistances of the series coils and armature, and k a constant.

The relation of ϕ to I is represented by the dotted curve B in Fig. 167, and if armature reaction was absent, the curve A would represent the relation.

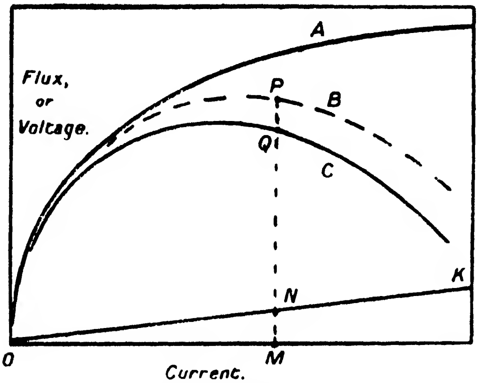

Fig. 167.—The characteristic curves of a series wound generator.

In certain special cases the armature winding is designed to give a very drooping curve beyond a certain output current. This may be done by greatly increasing the ampere turns on the armature with respect to those on the field coils.

The performance curve relating E_t and I, and the total characteristic curve relating E and I, are therefore of the respective shapes C and B. OK is the internal voltage drop curve whose ordinates are values of IR. The line PQ is made equal to NM.

The efficiency is given by

$$\eta = \frac{E_t I}{E_t I + I^2 R + P_1 + P_2} = \frac{E_t}{E + \dfrac{(P_1 + P_2)}{I}},$$

in which P_1 is the *iron loss*, which is very roughly proportional to I for light and medium loads, but nearly constant for full and overloads; and P_2 is the *windage and friction losses*, which are constant for constant speed.

Example.—A series generator has an output of 100 kilowatts at a terminal voltage of 200. Calculate the efficiency of the machine for this output; given that the armature resistance plus that of the field coils is 0·02 ohm, and the iron, friction, and windage losses amount to 3·5 kilowatts. *Ans.* 92·17 per cent.

Shunt generator.—The circuits of this type of machine are shown in Fig. 168, and the equations relating to them are

$$E_t = E - (I + I_s) r_a,$$

$$E_t = I_s r_s,$$

$$E = k\phi.$$

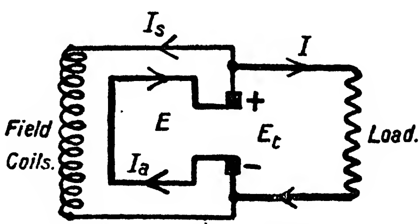

Fig. 168.—Shunt-wound generator on load.

At *no load* I_s is a maximum, because E_t is then a maximum and practically equal to E. I_s diminishes slowly as the load comes on,

and, as a result, E falls in value, because it is proportional to ϕ. E_t will fall in value to a greater extent, as it diminishes with decrease of ϕ and also with increase of armature drop.

Curve B in Fig. 169 indicates the relation between ϕ and I_s. This curve includes the effect of armature reaction, which increases as I_s diminishes, that is, as I_a increases. Curve A represents the relation when armature reaction is neglected.

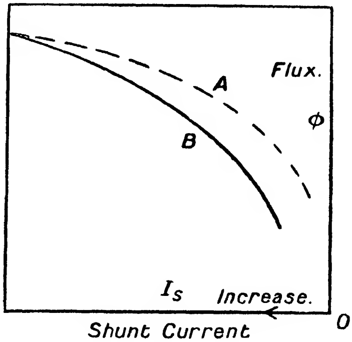

FIG. 169.—A curve relating flux and exciting current in a shunt-wound generator; in curve A armature reaction is neglected, in B its effect is included.

It follows that the E and I curve of the machine will be of the form shown by A in Fig. 170. By subtracting the drop, $(I+I_s)r_a$ equal to MN, from the ordinates of the A curve, the relation between E_t and I will be given by curve B. It will be noted that OK, the drop curve, will not be quite a straight line.

The efficiency is given by

$$\eta = \frac{E_t I}{EI_a + P_1 + P_2}.$$

In this case P_2, the friction and windage loss, is constant, and P_1, the iron loss, is nearly constant.

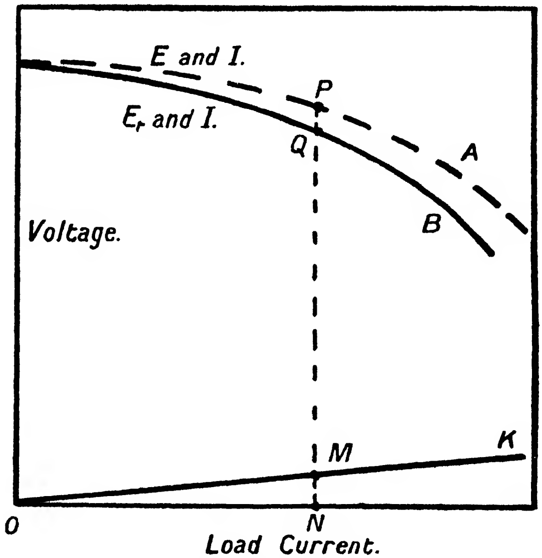

FIG. 170.—Characteristic curves of a shunt-wound generator.

EXAMPLE.—A shunt-wound generator has an output of 200 kilowatts at terminal voltage 250. Its armature resistance is 0·02 ohm, and the shunt current is 5 amperes. Calculate the efficiency of the machine for this output, if the iron, windage, and friction losses amount to 8 kilowatts. *Ans.* 90 per cent.

Compound-wound generator.—The circuits of this machine are shown in Fig. 171, and the equations are

$$E_t = E - R(I + I_s),$$
$$E_t = I_s r_s,$$
$$E = k\phi.$$

R is the sum of the resistances of the series coils and armature.

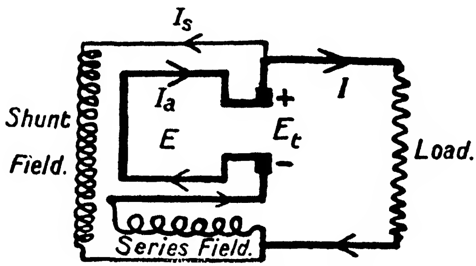

FIG. 171.—Compound-wound generator on load.

The value of ϕ, and consequently that of E, depends upon the sum of the shunt and series ampere-turns. As the shunt ampere-turns diminish through increase of load, those of the series coils increase. Now, if the series ampere-turns increase sufficiently to allow for the decrease in those of the shunt, and also for those neutralised by armature reaction, ϕ would be maintained constant.

Further, if the series ampere-turns were sufficiently large to cause ϕ to increase as the load increases, then E would increase with the load current, and E_t would fall a little in value, or remain practically constant, or also increase with the load, according to the value of the internal drop and the number of excess series ampere-turns.

If E_t increases with I, as in curve A (Fig. 172), the machine is over compounded, and if it decreases with I, as in curve C, the machine is under compounded.

When constant voltage is required at distant points, a certain amount of over-compounding is necessary in the generators supplying the feeders. This amount is often as much as 10 per cent., that is, a generator giving 200 volts at no load will give 220 volts at full load.

The efficiency η is given by

$$\eta = \frac{E_t I}{EI_a + P_1 + P_2}.$$

P_2, the windage and friction loss, remains constant, and P_1 roughly constant.

EXAMPLE.—A compound-wound generator gives a terminal voltage of 550 at *full* load and 500 at *no* load. The full-load output current is 1000 amperes, the armature is lap-wound and has 1800 conductors, the sum of the resistances of armature and series coils is 0·015 ohm, the resistance of the shunt circuit is 55 ohms, and the speed is 90 revolutions per minute Calculate the no-load and full-load flux per pole threading the armature core.

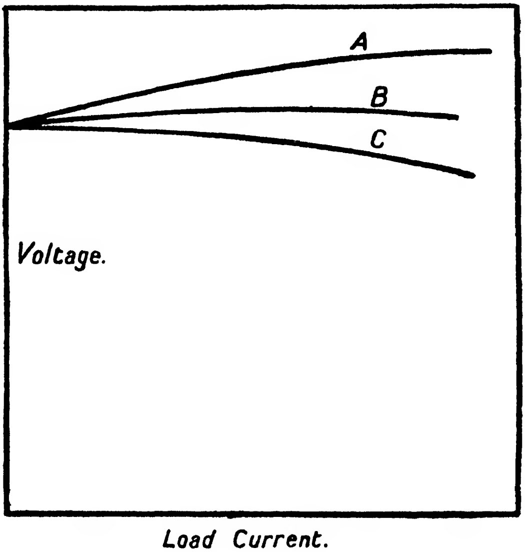

FIG. 172.—Characteristic curves of a compound-wound generator.

Assuming that the curve relating E_t and I is a straight line, and the iron, windage, and friction losses amount to 15 kilowatts, calculate the efficiency of the machine when the output current is 400 amperes, and also when it is 800 amperes. *Ans.* No-load flux per pole threading the armature core 18·5 megalines, and full-load flux 20·9 megalines. 90 per cent. and 93·5 per cent.

Series-wound motor.—The diagram of the series motor is shown in Fig. 173, and the equations relating to its circuits are

$$E - e = I \cdot R, \quad \ldots\ldots\ldots\ldots\ldots (a)$$

$$e = k \cdot \phi \cdot n, \quad \ldots\ldots\ldots\ldots (b)$$

$$T = k_1 \phi I, \quad \ldots\ldots\ldots\ldots\ldots (c)$$

in which k and k_1 are constants, e the counter or back voltage, and T the torque on the armature. Equation (c) follows from equation (10), page 8.

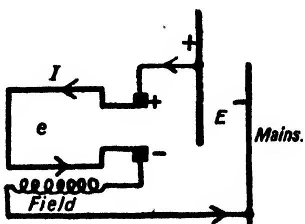

FIG. 173.—A series wound motor connected to supply mains.

Since E, the voltage of the mains, is constant, e and I are related by a straight line law.

For *light and medium* loads it may be roughly taken that

$$\phi \propto \mathrm{I}. \qquad \ldots\ldots\ldots\ldots \qquad (d)$$

Therefore, from (c) and (d), then from (b),

$$\mathrm{T} \propto \phi^2 \propto \frac{e^2}{n^2},$$

and e is nearly equal to E, so that, approximately,

$$\mathrm{T} \propto \frac{\mathrm{I}}{n^2} \qquad \ldots\ldots\ldots\ldots \qquad (e)$$

For *full and over* loads ϕ is roughly constant, and

$$\mathrm{T} \propto \mathrm{I} \propto 1 - \frac{e}{\mathrm{E}} \propto 1 - \mathrm{K}\,.\,n, \qquad \ldots\ldots\ldots\ldots \qquad (f)$$

in which K is a constant.

Equation (f) shows that, when n is zero, the torque is a maximum. From (e) and (f) the relation between T and n may be roughly indicated, as in Fig. 174.

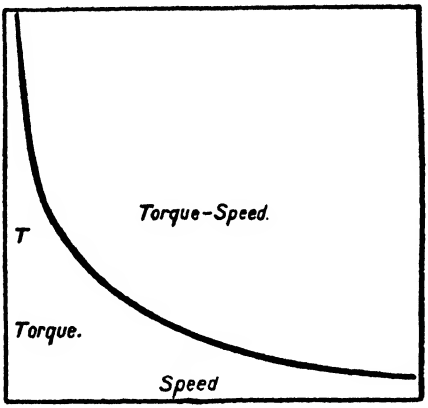

FIG. 174.—The torque-speed curve of a series-wound motor.

Similarly, the approximate relations between T and I, and between I and n, may be deduced.

The efficiency η of the motor is given by

$$\eta = \frac{\mathrm{EI} - \mathrm{I}^2\mathrm{R} - \mathrm{P}_1 - \mathrm{P}_2}{\mathrm{EI}} = \frac{e\mathrm{I} - \mathrm{P}_1 - \mathrm{P}_2}{\mathrm{EI}},$$

$$\eta = \frac{e}{\mathrm{E}} - \frac{\mathrm{P}_1 + \mathrm{P}_2}{\mathrm{EI}}.$$

The power available for turning, that is, for producing torque, is

$$EI - I^2R - P_1 - P_2 = eI - (P_1 + P_2).$$

Therefore
$$\frac{T \cdot 2\pi \cdot 60n}{33000} \times 746 = Ie - (P_1 + P_2),$$

$$T = 0{\cdot}1174 \frac{Ie}{n} - 0{\cdot}1174 \cdot \frac{P_1 + P_2}{n} \text{ lbs.-ft.}$$

By making an allowance for the iron, windage, and friction losses, the torque may be calculated.

EXAMPLE.—1. Deduce the approximate form of the curve of a series motor relating current and speed. In what way will the iron loss change with change of speed? Explain why an unloaded series motor will race up to a dangerous speed.

2. The speed of a series motor, when supplied with 80 amperes at 500 volts, is 600 revolutions per minute. Calculate values for the torque, brake-horse power, and brake efficiency of the motor. The sum of the resistances of the armature and series coils is 0·5 ohm; and the iron, windage, and friction losses amount to 1·5 kilowatts. *Ans.* 414·4 lbs.-ft., 47·36 B.H.P., and efficiency 88 per cent.

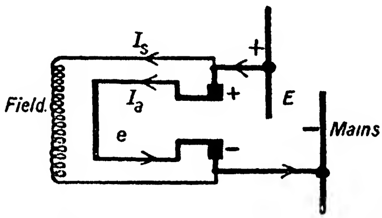

FIG. 175.—A shunt-wound motor connected to supply mains.

Shunt-wound motor.—The diagram of connections for this motor is given in Fig. 175, and the equations in this case are

$$E - e = r_a(I - I_s) = r_a I_a,$$

$$e = k\phi n,$$

$$E = I_s r_s,$$

$$T = k_1 \phi I_a.$$

In this case I_s is *constant*, so that, neglecting the effect of armature reaction, ϕ remains constant, or, allowing for it, ϕ falls slightly in value as I_a increases, being a maximum at no load.

From the second equation,

$$n \propto \frac{e}{\phi},$$

and e is practically equal to E at no load, and slightly falls in value

as I_a increases, while ϕ has a maximum value at no load, and similarly falls in value as I_a increases. It therefore follows that the speed of a shunt motor remains practically constant; this is indicated in Fig. 176.

Also, in such a motor, if the armature reaction is made excessive by using more negative lead of the brushes, or specially designing the armature to have excessive reaction, so that ϕ diminishes more

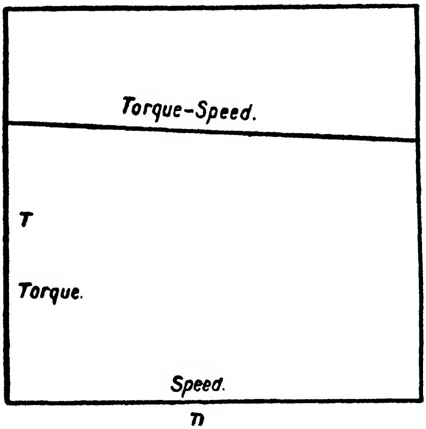

FIG. 176.—The torque-speed curve of a shunt-wound motor.

rapidly than e does, the speed of the motor may be made to *increase* with the load. Designing the armature to have a relatively small resistance will also aid in increasing the speed, as e will diminish less with increase of I_a.

On the other hand, if positive lead is used, then ϕ will increase as I_a increases, so that n will fall more or less rapidly as the load increases.

Hence, in a shunt motor, unless designed as just described to give a variable speed, the speed is nearly independent of the torque and load; and the torque is nearly proportional to I_a.

The efficiency η is given by

$$\eta = \frac{eI_a - P_1 - P_2}{EI}.$$

Both P_1 and P_2 are roughly constant for the shunt motor, and their effect on the value of η will therefore be relatively much more important at light loads.

The torque is deduced, as in the case of the series motor. Thus

$$T = 0{\cdot}1174 \frac{I_a e}{n} - 0{\cdot}1174 \frac{P_1 + P_2}{n} \text{ lbs.-ft.}$$

EXAMPLE.—1. Why is it undesirable to fit a shunt motor with a flywheel?

2. A loaded shunt motor takes a current of 52 amperes from 100-volt mains at a speed of 1200 revolutions per minute. The armature resistance is 0·2 ohm, and the shunt resistance 50 ohms. The iron windage and friction loss amount to 0·4 kilowatt. Calculate the torque, the brake-horse power, and the brake efficiency of the motor. If the machine has four poles and a wave-wound armature of 230 conductors, calculate the flux per pole threading the armature core. *Ans.* 24 lbs.-ft., 5·5 B.H.P., 79 per cent., and 0·98 megaline.

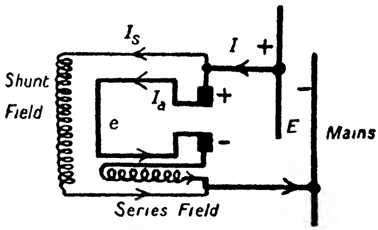

FIG. 177.—Compound-wound motor connected to supply mains.

Compound-wound motor.—The circuits of this type of motor are illustrated in Fig. 177, and the equations relating to them are

$$E - e = I_a R,$$

$$E = I_s r_s,$$

$$e = k\phi n,$$

$$T = k_1 \phi I_a.$$

Since the shunt current is constant, ϕ may be represented by

$$\phi = K + K_1 I_a,$$

K and K_1 being constants.

ϕ therefore increases with the load, that is, with I_a; but this increase is diminished more and more by armature reaction as I_a increases. If the motor is over-compounded, ϕ will increase notwithstanding the effect of armature reaction, and since

$$n \propto \frac{e}{\phi} \propto \frac{E - I_a R}{\phi},$$

the speed diminishes as the load increases, that is, as I_a increases.

The decrease of speed will be greater the greater the over-compounding and the greater the armature drop.

At no-load, the machine will approximately be a shunt motor, and run at the corresponding speed of such a motor.

Also, since $$T \propto \phi I_a,$$

and ϕ increases with increase of I_a, while the latter increase is accompanied by decrease of speed, the torque will therefore increase as the speed diminishes. At no-load the speed is maximum, and the torque is small because I_a is then small. The starting torque is large because of the series field, whose coils will then take a large current.

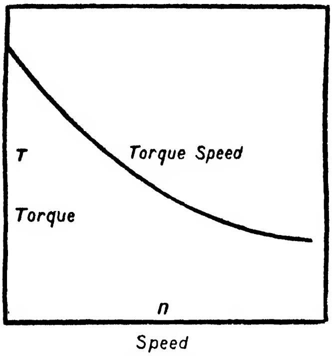

FIG. 178.—Torque-speed curve of a compound-wound motor.

The curve relating T and n will therefore be of the form indicated in Fig. 178.

The *efficiency* and *torque* may be calculated from the same equations as were given for the shunt motor.

Compound-wound motors are used to drive machines subject to sudden heavy loads alternating with light loads, such as rolling mills. A flywheel is used with the motor to store up energy during light load, and give it out when the heavy load comes on the machine. The flywheel supplies what is termed the peak of the load, while the motor is also doing its share.

EXAMPLE.—A compound-wound motor takes 200 amperes at 500 volts, and its speed is 600 revolutions per minute. The respective resistances of its armature, series coils, and shunt coils are 0·03, 0·01, and 100 ohms. Iron, windage, and friction losses amount to 3 kilowatts. Calculate the torque, the brake horse-power, and the brake efficiency of the motor for the given load. *Ans.* 1093 lbs.-ft., 125 B.H.P., and 93 per cent.

CHAPTER XII.

TESTING DIRECT-CURRENT MACHINES.

SOME of the more important tests which may be made with direct-current machines are described in this chapter, and these should preferably be carried out in the sequence given.

Accuracy of results will depend upon the correctness of the instruments used, the care in reading them, and the constancy of the temperature of the parts of the machine during the tests.

In view of the latter, it is desirable to warm up the machine before testing by a full-load run of several hours, when a fairly steady temperature will have been attained.

TESTING A COMPOUND-WOUND GENERATOR.

The drop test.—The machine is connected up as in Fig. 179, and different currents are sent through the armature and series coils,

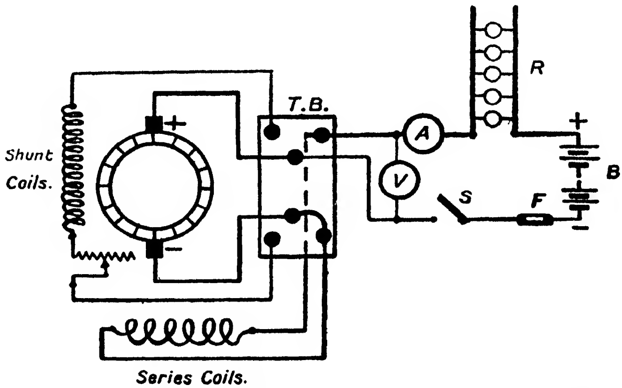

FIG. 179.—The arrangement for the internal voltage-drop test for the generator.

preferably in the *same* direction as in normal operation. These currents should range from a small value to the full-load output current of the machine.

T.B. is the terminal box of the machine, A an ammeter for reading to somewhat beyond full-load current, V a voltmeter of range from zero to ten volts, R a lamp or liquid rheostat, F a fuse, S a switch, and B a battery or direct-current generator. The shunt coils, as shown, are disconnected. The positive sign at the brush indicates its nature for normal operation.

The internal voltage drop in the series and armature windings is read on V, and will be denoted by e and the armature current by I_a. The drop across the series coils *alone* should be taken for two or three values of I_a by connecting V across the terminals of the series coils on T.B. The resistance r of the series coils may thus be calculated, and then r_a, the armature resistance, may also be obtained.

The tabulation of the results for a 5 K.W. compound generator, which for brevity will be termed *machine A*, is as follows :

I_a	5	10	14·6	23·3	31·4	39·2	49·5
e	1·15	2·13	3·0	4·6	5·8	6·65	8·2
r_a+r	0·230	0·213	0·205	0·198	0·186	0·170	0·166
r	0·016	0·016	0·016	0·016	0·016	0·016	0·016
r_a	0·214	0·197	0·189	0·182	0·170	0·154	0·150

The graph of e and I_a is given in Fig. 180.

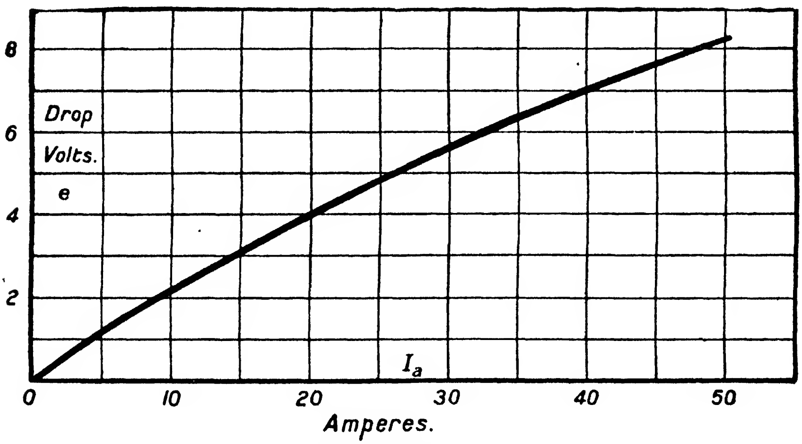

FIG. 180.—The internal voltage drop, and armature current of the generator.

For the higher values of I_a it was necessary to clamp the motor, owing to the motor action of the currents in the series coils and armature.

The copper loss in armature and series coils.—This loss, which includes the electrical loss at the brush contacts, is obtained from the preceding table by multiplying e by I_a. For *machine A*,

I_a	5	10	14·6	23·3	31·4	39·2	49·5
eI_a	5·75	21·3	43·8	107·0	182	261	405

and the graph is shown in Fig. 181.

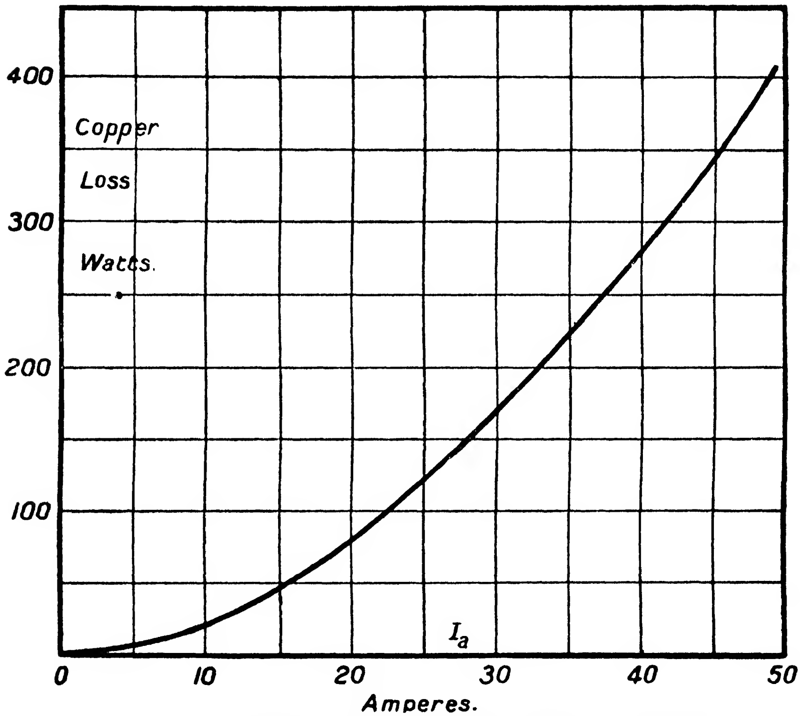

FIG. 181.—The copper loss in the armature and series windings of the generator.

The excitation curve.—The connections for this test are shown in Fig. 182. V indicates volts up to and a little beyond the rated voltage of the machine, and A is a low-reading ammeter. Care should be taken to send the exciting current through the shunt circuit in the *same* direction as it flows for normal operation. The series winding has been omitted.

The machine is run on open circuit at *rated* speed with the shunt field coils separately excited. E_i, the internal voltage for different exciting currents I_s, ranging from small value up to a little beyond full-load value, is read on V. I_s, multiplied by the number of shunt

turns *per bobbin*, which is 720 for *machine A*, gives the ampere turns, A.T., per bobbin, necessary to produce an *internal* voltage E_i.

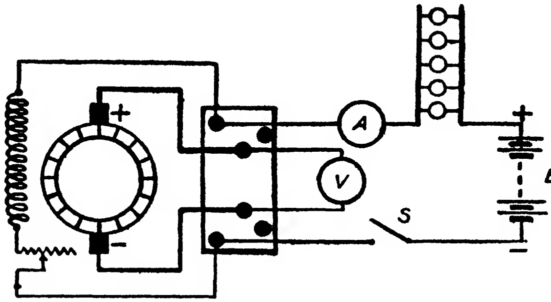

FIG. 182.—The arrangement for the no-load excitation test for the generator.

The values obtained for *machine A* at rated speed 1300 R.P.M. were

E_i	30·7	45·6	57	70	78·9	94	107·5	121	130
I_s	0·5	0·75	0·96	1·17	1·32	1·62	2·0	2·48	2·9
A.T.	360	540	692	842	950	1168	1440	1790	2090

and the graph is given in Fig. 183.

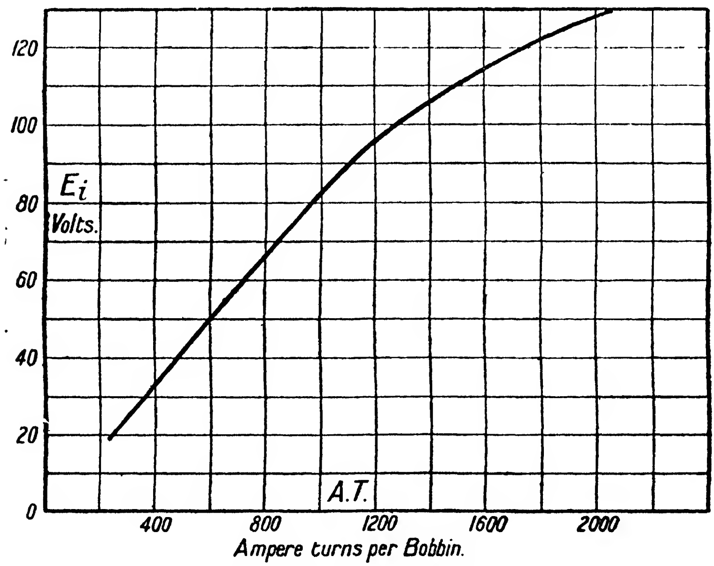

FIG. 183.—The no-load excitation curve of the generator.

This curve also represents the graph of ϕ the flux per pole threading the armature core, and A.T. per bobbin. Thus the ampere turns necessary to produce a certain flux may be determined. For if the machine has a lap-wound armature,

$$E_i = \phi Z n 10^{-8},$$

and if *wave*, $\frac{p}{2}$ times this value. The curve of Fig. 183 may therefore have a second scale for flux.

By dividing ϕ by the pole face area, B_{av} the *average magnetic density* at the pole face may be calculated.

In the case of *machine A*, the armature is wave-wound. The number of armature conductors is 324, $p=4$, $n=\frac{1300}{60}=21{\cdot}7$, and the pole-face area $2\frac{3}{8}$ inches by 6 inches. From these values the average pole-face magnetic density and the flux ϕ in terms of E_i is given by

$$B_{av} = 0{\cdot}5 E_i \text{ kilolines per sq. inch,}$$

$$\phi = 0{\cdot}00712 E_i \text{ megalines.}$$

Thus, when the internal voltage is 100, the average magnetic density at the pole face will be 50,000 lines per sq. inch, and the flux per pole threading the armature core 712,000 lines.

The performance or regulation test.—Connections for this test are shown in Fig. 184. An ammeter *a* is included in the shunt-field

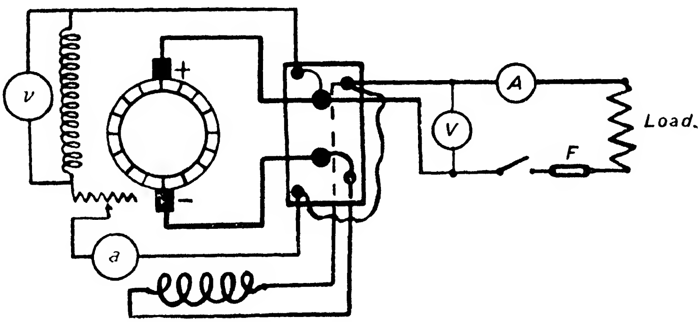

FIG. 184.—The arrangement for the performance or regulation test of the generator.

circuit, and the machine is connected up for normal operation. For the purpose of measuring the temperature rise in the field-magnet bobbins, a voltmeter *v* is placed across the field coils.

The shunt-field rheostat is adjusted so that the machine will give full-load current at rated voltage and speed ; it is then left unchanged during the test. Simultaneous readings of all the instruments for different load currents are taken, and the speed is adjusted to remain

constant throughout the test. The following table gives the results obtained for *machine A* at rated speed 1300 R.P.M.:

I	0	5·3	10·6	15·7	26	36·25	45
I_s	1·85	1·915	1·93	1·92	1·92	1·95	1·9
I_a	1·85	7·2	12·5	17·6	27·9	38·2	46·9
E_t	103	106	107	107	106	106·5	104
e	0·2	1·5	2·6	3·6	5·3	6·8	7·7
E_i	103·2	107·5	109·6	110·6	111·3	113·3	111·7

The values of e, the drop in armature and series coils, have been taken from the drop curve (Fig. 180). In Fig. 185 are shown the

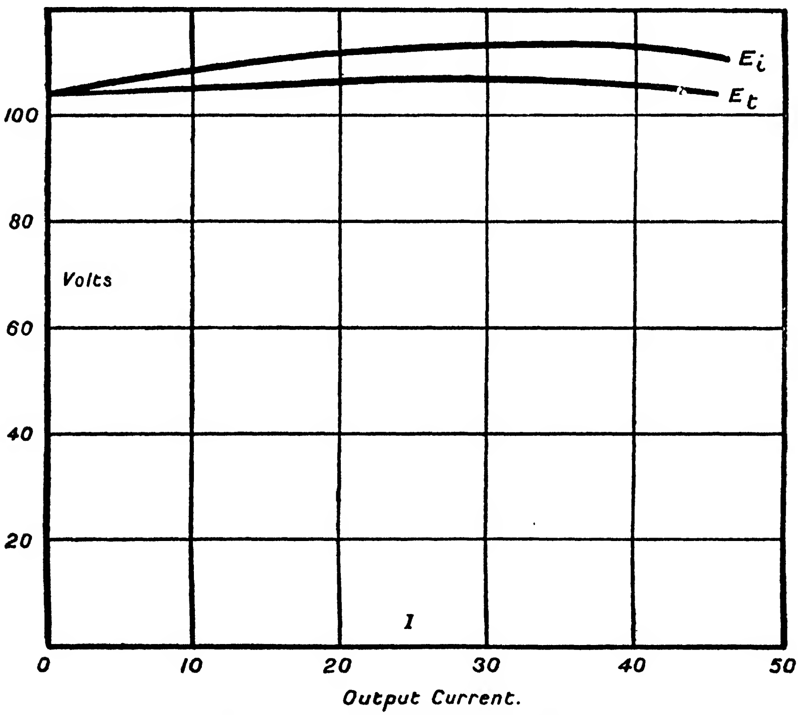

FIG. 185.—The regulation and total characteristic curves of the generator.

regulation curve relating I the load current and E_t the terminal voltage; and also the total characteristic curve relating I and E_i.

The copper loss in the shunt coils and rheostat.—This loss is represented by the product of I_s and E_t obtained in the performance test, and the values for *machine A* are tabulated thus :

I	0	5·3	10·6	15·7	26	36·25	45
I_sE_t	192	203	207	206	204	208	198

The effect of armature reaction.—This is quantitatively represented by the number of ampere turns per bobbin neutralised by armature reaction, due both to its demagnetising and distorting effects on the flux of the field magnets. This number may be determined from certain of the preceding results as follows :

I	0	5·3	10·6	15·7	26	36·25	45
I_s	1·85	1·915	1·93	1·92	1·92	1·95	1·9
I_a	1·85	7·2	12·5	17·6	27·9	38·2	46·9
E_t	103	106	107	107	106	106·5	104
e	0·2	1·5	2·6	3·6	5·3	6·8	7·7
E_i	103·2	107·5	109·6	110·6	111·3	113·3	111·7
S_1I_a	15	58	100	141	223	305	375
S_2I_s	1333	1380	1390	1380	1380	1400	1362
A	1348	1438	1490	1521	1603	1705	1737
B	1336	1428	1472	1496	1515	1562	1520
A − B	12	10	18	25	88	143	217

In this table S_1 is the number of series turns per bobbin, S_2 the number of shunt turns per bobbin, A the total *actual* ampere turns used per bobbin to produce E_i, and B is the number per bobbin necessary to produce E_i found from the excitation curve of Fig. 183. The value A − B will then be the number of ampere turns per bobbin neutralised by armature reaction. In this case S_1 is 8, and S_2 720.

This test requires much care in obtaining a constant and correct speed, and the use of accurate instruments, especially for determining I_s and E_t, in both the performance and excitation tests ; otherwise the results will be very inaccurate. The graph of A − B against I is shown in Fig. 186.

The determination of the iron loss.—The connections for making this test are shown in Fig. 187. A shunt motor M drives G the generator under test at no load, and may therefore, if necessary, have a much smaller output than the generator. It is preferably direct coupled to the generator.

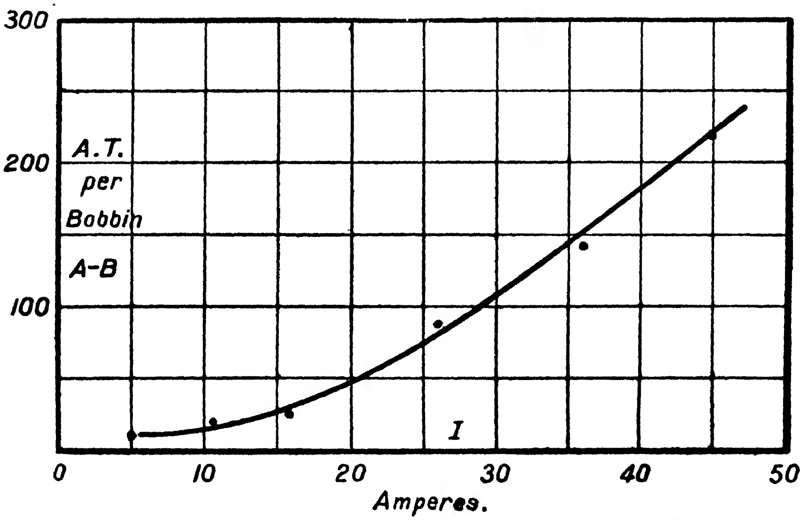

FIG. 186.—The ampere turns per bobbin neutralised by armature reaction, and output current of the generator.

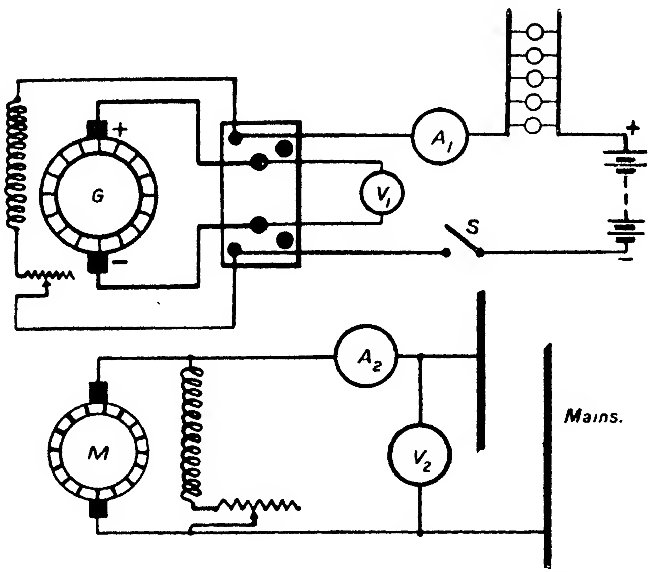

FIG. 187.—The arrangement for determining the iron, windage, and friction losses of the generator.

The shunt circuit of the latter is separately excited, and the voltage of the motor mains is kept constant. By adjusting the shunt-field rheostat of the motor, the rated speed may be obtained.

The input power, $P_0 = I_2E_2$, of the motor is first read when the switch S is *open*. This power is the sum of the iron loss in the motor, the copper loss in its field, a small copper loss in its armature generally negligible, and the friction and windage losses of the two machines.

If the field rheostat of the motor is left untouched after its first adjustment to give rated speed, the sum of all these losses, namely P_0, will remain practically constant; an allowance for the small variable copper loss in the armature of the motor may be made if necessary.

The input P is now read after switch S has been closed. Then $P - P_0$ will give the iron loss in the generator when the latter is producing an internal voltage of value E_i indicated on voltmeter V. A set of such values are taken for different values of I_s, ranging from low values up to a little beyond full-load exciting current, and tabulated.

The following table gives the results obtained for *machine A* at rated speed 1300 R.P.M.:

Motor.			Generator.		
E_2	I_2	P	E_i	I_s	$P - P_0$
99·4	6·70	666	0	0	—
,,	6·90	686	25	0·45	20
,,	7·35	730	54	0·90	64
,,	7·60	755	66	1·11	89
,,	8·10	805	82	1·39	139
,,	8·25	820	91	1·56	154
,,	8·60	855	101	1·80	189
99·4	9·40	935	120	2·39	269

The iron loss $P - P_0$ and E_i are related as shown in the graph (Fig. 188). As the iron loss depends upon the speed, which is constant, and the value of E_i, this graph may be used to determine the iron loss in the load test. Thus, from the graph, the iron loss is read off for the values of E_i found in the load or performance test and entered as follows:

I	0	5·3	10·6	15·7	26	36·3	45
E_i	103·2	107·5	109·6	110·6	111·3	113·3	111·7
Iron loss	194	219	223	230	234	240	235

I and E_i being the performance values of load current and internal voltage.

The windage, axle, and brush-friction, losses.—These may be conveniently found at the time of doing the iron test, and the method will be explained for the case of *machine A*. With S open, the input of the motor was found to be 666 watts. The brushes were lifted from the commutator of *machine A*, and the input was then 613 watts.

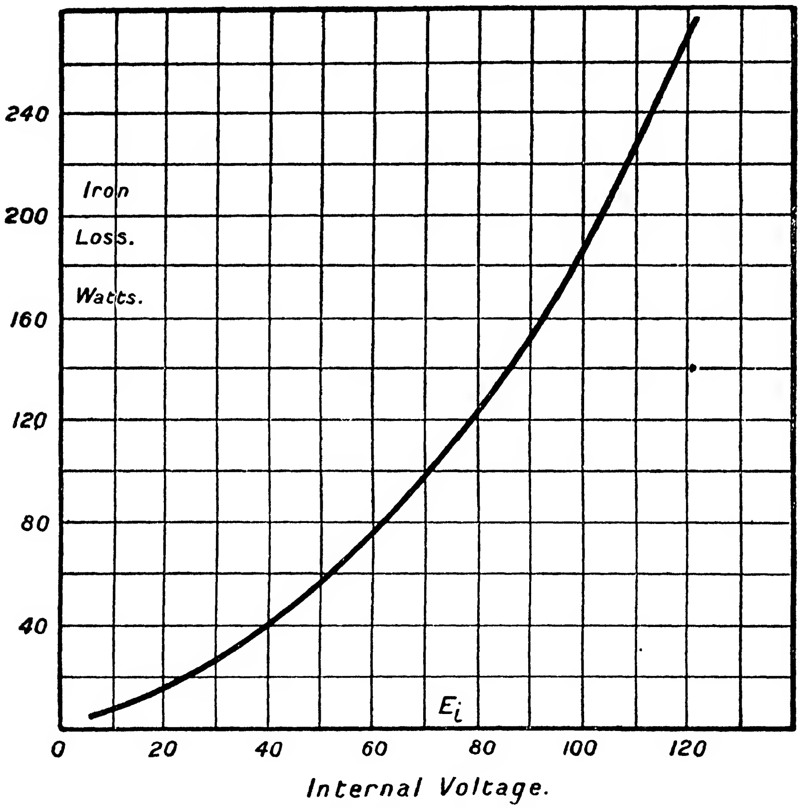

FIG. 188.—The iron loss for different values of the internal voltage of the generator.

The *brush friction* was therefore $666 - 613 = 53$ watts. Next, *machine A* was uncoupled and left stationary, and the motor input found to be 490 watts. Therefore $613 - 490 = 123$ watts was the *windage and axle-friction* loss of the generator.

Determination of the efficiency curve.—The detailed losses may now be tabulated and the efficiency calculated. This is done for *machine A* at rated speed 1300 R.P.M. in the following table, which has been constructed from the results and curves of the preceding tests.

I - - -	0	5·3	10·6	15·7	26	36·25	45
E_t - -	103	106	107	107	106	106·5	104
Output -	0	560	1135	1680	2750	3875	4685
Copper loss -	3·7	10·8	32·5	63	148	260	370
Copper loss -	192	203	207	206	204	208	198
Iron loss -	194	219	223	230	234	240	235
W and A.F. -	123	123	123	123	123	123	123
B.F. - -	53	53	53	53	53	53	53
Total loss -	566	609	639	675	762	884	979
Input - -	566	1169	1774	2355	3512	4759	5664
Efficiency -	0	48	64	71	78·5	81·4	82·5

The first copper loss is that of the series coils and armature winding;* the second is that of the shunt coils, including its rheostat. The latter

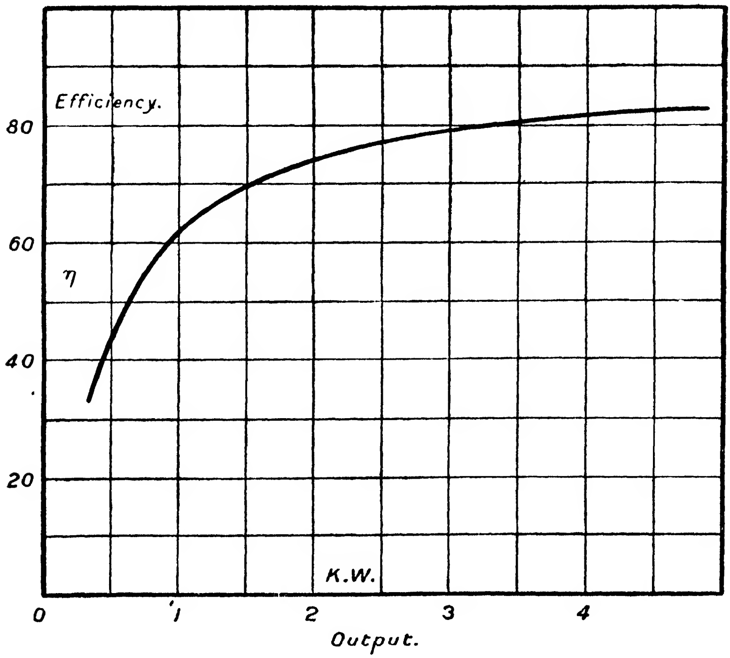

FIG. 189.—The efficiency and output of the generator

* Obtained from curve Fig. 181 for the values of I_a in the table on page 238.

is found by multiplying E_t by I_s in the performance test. W and A.F. is the windage and axle-friction loss, while B.F. is the brush-friction loss.

The input divided by 746 gives the horse-power applied to the generator. When the latter is known for a generator at different loads, the machine may be used as an accurate and useful brake for finding the brake horse-power of any type of electrical machine or engine which may be coupled to it ; provided the latter has the same speed for which the generator was tested. The use of the generator in this respect may be extended by repeating the preceding tests for different speeds.

The graph relating efficiency and output is shown in Fig. 189.

Determination of temperature rise.—The *average* temperature rise in the shunt coils may be determined by placing an ammeter in the shunt circuit, and a voltmeter across the shunt coils *not* including the field rheostat ; both instruments as accurate and sensitive as possible. By dividing the reading of the voltmeter by that of the ammeter, the resistance of the shunt coils is found.

Before starting to run on a specified load for the temperature test, an accurate thermometer is placed inside the machine for a short time, and the temperature noted. The machine is then started, and the readings of voltmeter and ammeter for the shunt coils at once read. The load is put on, and the readings of these two instruments are taken at different times.

When sufficient values have been obtained, the load is removed and the shunt current is reduced as small as possible by the field rheostat, and the shunt circuit broken. While still running at rated speed, the machine is allowed to cool for some minutes, and then its shunt field is made again and the readings of the instruments taken again. The shunt circuit is then broken with the same care as before, and, after suitable intervals of cooling, other readings are similarly obtained. The temperature may then be calculated from the formula

$$t = \frac{r - r_0}{r_0 \times 0{\cdot}00426} \text{ degrees centigrade.}$$

The following values of the resistance of the shunt coils were obtained for *machine A* when loaded with 3·25 K.W. at rated speed 1300 R.P.M. :

Time.	Resist.	Temp.	Time.	Resist.	Temp.
0	25·84	17·0	80	28·65	44·5
10	26·47	23·1	85	28·79	45·7
20	27·174	29·3	Cooling.		
30	27·52	33·3			
40	27·82	37·2	95	28·50	43·0
50	28·11	39·0	105	27·71	35·2
60	28·35	41·5	120	27·31	31·3
70	28·50	43·0	135	27·07	28·9

The time is in *minutes*. The heating was continued for 85 minutes, and the first value for cooling was taken ten minutes after the end of heating.

The value of r_0 was calculated from the preceding formula by using the temperature 17·0° C. and the resistance $r = 25{\cdot}84$ ohms, obtained just before the machine was loaded. It was found to be 24·1 ohms.

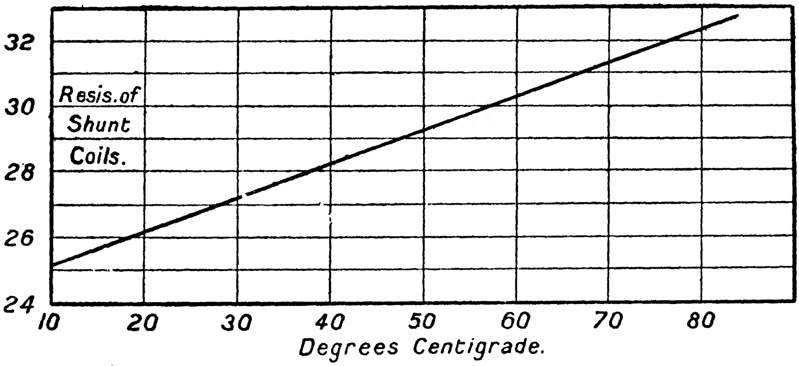

FIG. 190.—The resistance of shunt coils and temperature.

By using this value in the formula, the resistance for a temperature of 50° C. was calculated, and a straight line relating temperature and

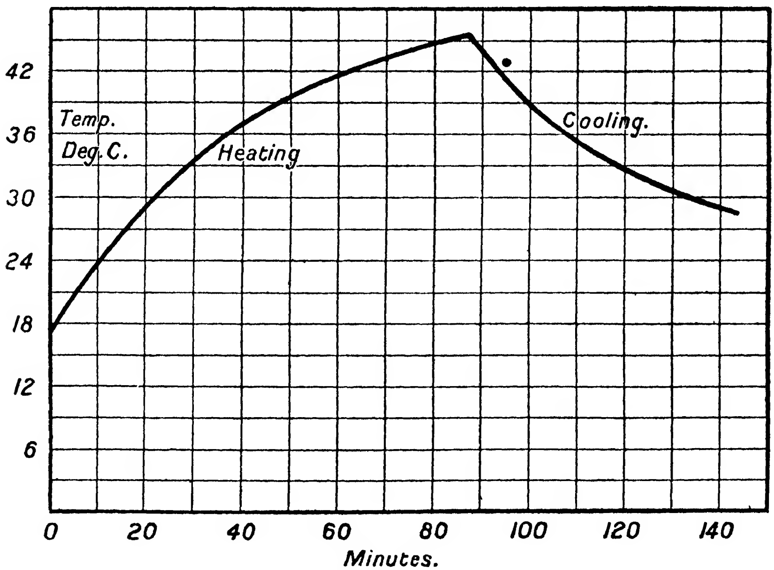

FIG. 191.—The temperature-time curve, heating and cooling of the shunt coils of the generator.

resistance drawn through the two points corresponding to 0° C. and 50° C., as in Fig. 190. From this the temperatures corresponding to the resistances in the table were read off.

The temperature-time curve is shown in Fig. 191.

The temperature rise, average value, was also taken for the armature winding. This was done by lifting the brushes and measuring the resistance between two marked commutator segments, the pitch of which was equal to the span of two adjacent brushes, the winding being wave. Copper contacts were used on the segments, and the resistance measured by a good dial box.

At 17° C. this resistance was 0·088 ohm, and after a full-load run for an hour the machine was shut down, the brushes lifted, and the resistance again measured. This was found to be 0·103 ohm.

As soon as possible, the machine was started, and another hour's run was made, and the resistance once more measured. This was practically 0·103 ohm as before, and showed that the balancing temperature was reached within an hour from the start.

The temperature corresponding to this final resistance, calculated from the temperature resistance formula, was found to be 60° C., giving an average temperature rise for the armature winding of 43° C.

The differential method of determining the efficiency of shunt-wound direct current generators and motors.—This is the most economical way of testing generators or motors for efficiency. In large generators or motors it would be costly to load them up with a wasteful test load in the manner previously described.

With the differential method, power merely equal to the sum of all the losses in the two machines used, is all that is required.

This method is known as Hopkinson's efficiency test, though various modifications of his original method have been used by others. The one about to be described is due to Professor Kapp.

It is essentially a work's test, because two machines built from the same specifications are needed. These machines are direct coupled, one M (Fig. 192) running as a shunt motor and the other G driven by the former, works as a shunt generator supplying current to the motor. Therefore, if extra outside power equal to the losses of the machines is supplied to the motor, the generator may be worked differentially with the motor, for any given output current.

In the diagram given, B is the battery or a generator which may be of small output compared to those of the machines under test.

The theory of the method is as follows: Let r_1 and r_2 be the respective armature resistances of G and M determined by the *drop* test; I_1, I_2, etc., the currents read on A_1, A_2, etc.; E the voltage read on V, which is the terminal voltage of the generator and motor; and 2P the sum of the iron, windage, and friction losses of G and M.

The values of the iron, windage, and friction losses are assumed to be the same in both machines, namely P. The windage and friction losses will be of practically the same value, but there will be some difference between the iron losses.

The iron losses can only be equal when the armature internal voltages are equal, but, in this test, such cannot be the case; that of G must always be larger than that of M, otherwise it could not supply power to the latter.

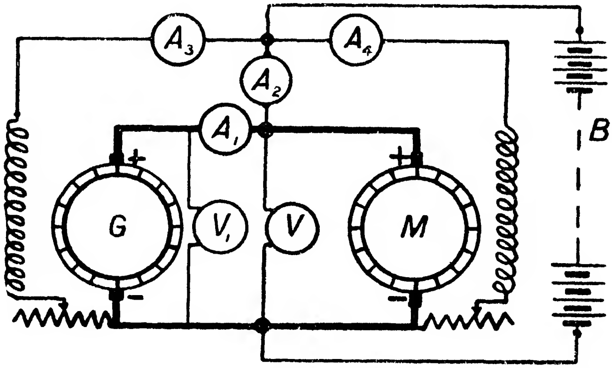

FIG. 192.—The arrangement for the differential method of determining the efficiencies of generators and motors.

Thus the iron loss in G will be somewhat larger than that in M, and, as a consequence, the efficiency of G will be slightly over-estimated, and the efficiency of M similarly under-estimated by making the preceding assumption.

From the diagram it follows that

$$EI_2 = I_1^2 r_1 + (I_1 + I_2)^2 r_2 + 2P,$$

which gives the value of P. Also, the copper loss $I_1^2 r_1$ of the generator, and $(I_1 + I_2)^2 r_2$ that of the motor, may be calculated.

The efficiencies η_g and η_m of generator and motor are therefore given by

$$\eta_g = \frac{I_1 E}{I_1 E + EI_3 + I_1^2 r_1 + P},$$

$$\eta_m = \frac{(I_1 + I_2)E - (I_1 + I_2)^2 r_2 - P}{(I_1 + I_2)E + EI_4}.$$

If necessary, the iron loss may be determined by the previous method of page 239 for different values of E_i for each machine, and also the windage and friction loss. The values of P in the efficiency formulas may then be more accurately represented.

The method of making the test is as follows:

1. The circuits should be well fused, and the *positive* of G joined to that of M, after testing that the machines will run in their right directions.

2. An extra rheostat should be placed in the field circuit of the generator, and a liquid or other rheostat which can be completely

cut out may be placed in the connection containing A_2 (Fig. 193). With the latter there is no need to use a starter for the shunt motor, and this simplifies the arrangement. This resistance should be fully in, on starting the machines, and wholly cut out when measurements are taken.

3. The motor switch is closed, R gradually cut out, and its field rheostat adjusted to give rated speed.

4. Before closing the generator's switch, its open circuit voltage is adjusted to be a little greater than that read on V. After closing this switch, adjustments of the field rheostats of G and M, and the supply voltage, are made to obtain any given output current from G at rated speed and voltage.

Instead of the battery, it is better to use a dynamo for the supply voltage, which may then be much better regulated.

The results obtained for two similar 5 kilowatt shunt machines, one of which was *machine A* previously tested by the detailed loss method, were as follows :

Speed	1240	1240	1240	1240	1240	1240
I_2	7	8·5	9	12	10	17·3
I_1	4·5	13·3	19·2	27	32·5	55·5
$I_1 + I_2$	11·5	21·8	28·2	39	42·5	72·8
I_3	1·8	1·7	2·05	2·0	1·55	1·3
I_4	1·73	1·5	1·70	1·5	2·13	2·11
E	100	100	100	100	100	100
r_1	0·204	0·184	0·172	0·160	0·155	0·138
r_2	0·179	0·156	0·145	0·132	0·129	0·117
EI_2	700	850	900	1200	1000	1730
A $\{ I_1^2 r_1$	4	33	64	117	164	426
A $\{ (I_1 + I_2)^2 r_2$	24	74	116	200	234	620
A	28	107	180	317	398	1046
2P	672	743	720	883	602	684
B $\{$ P	336	372	360	442	301	342
B $\{ EI_1$	450	1330	1920	2700	3250	5550
B $\{ EI_3$	180	170	205	200	155	130
B $\{ I_1^2 r_1$	4	33	64	117	164	426
B	970	1905	2549	3459	3870	6448
η_g	46·5	70	75·3	78	84	86
D $\{ E (I_1 + I_2)$	1150	2180	2820	3900	4250	7280
C $\{ (I_1 + I_2)^2 r_2$	24	74	116	200	234	620
C $\{$ P	336	372	360	442	301	342
C	360	446	476	642	535	962
D – C (output)	790	1734	2344	3258	3715	6318
EI_4	173	150	170	150	213	211
Motor input	1323	2330	2990	4050	4463	7491
η_m	59·5	74·2	78	80·5	83	84·5

r_1 and r_2 were obtained by the drop test for different armature currents.

The efficiency curves are drawn in Fig. 194, that for the generator being higher than the other.

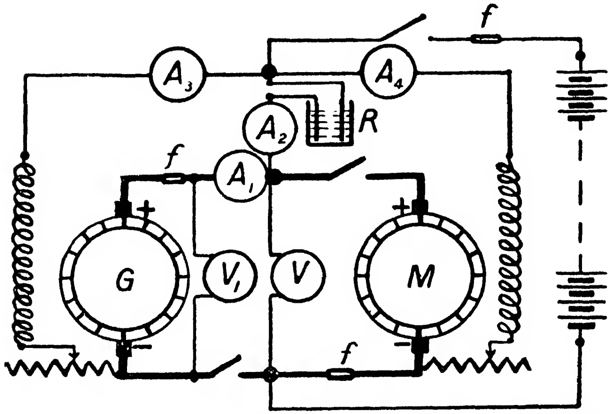

FIG. 193.—The arrangement for Kapp's differential method of determining efficiencies of generators and motors.

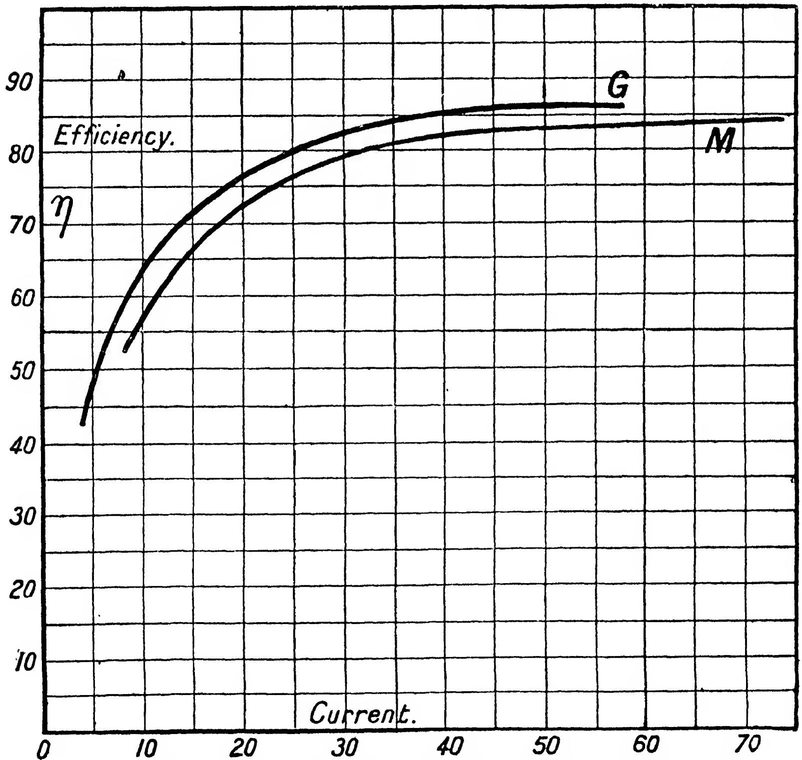

FIG. 194.—The efficiency curves obtained for the shunt-wound motor and shunt-wound generator by Kapp's method.

This equality of speeds for the two arrangements when supplied with the same input current I, may be shown thus.

Let e be the counter voltage of the motor for *ordinary* operation. Then

$$e = k\phi N$$

and

$$e = E - I(r_a + r_m),$$

k being a constant, r_a the armature resistance, r_m the resistance of series coils, N the speed, and ϕ the flux per pole threading the armature core.

$$\therefore \quad N = \frac{E - I(r_a + r_m)}{k\phi}.$$

Again, let e_1 be the counter voltage of the motor connected as in Fig. 196. Then

$$e_1 = k\phi_1 N_1$$

and

$$e_1 = E - Ir_m - I_a r_a ;$$

$$\therefore \quad N_1 = \frac{E - Ir_m - I_a r_a}{k\phi_1}.$$

Now, since I is the same in both cases, ϕ is *nearly* the same as ϕ_1. They would be equal if there was no armature reaction. Actually ϕ is a little smaller than ϕ_1 from this cause. For armature reaction does not affect N_1 nearly so much as it does N, because I_a is much smaller than I.

At light loads the equality of N and N_1 is almost exact. At heavier loads, although the drop $I(r_a + r_m)$ in the first case is larger than $Ir_m + I_a r_a$ in the second, ϕ is smaller than ϕ_1 on account of greater armature reaction. Therefore the equality of N and N_1 is still approximately maintained.

The following results were obtained for *machine B* :

E	100	100	100	100	100	100	100	100
I	15	19·8	24·2	29·7	35·0	39·8	44·8	53·8
I_a	5·9	5·1	4·8	4·45	4·33	4·23	4·2	4·1
i	9·1	14·7	19·4	25·3	30·7	35·6	40·6	49·7
E_1	99·0	98·7	98·4	98·1	97·7	97·4	97·1	96·5
r_a	0·19	0·19	0·19	0·19	0·19	0·19	0·19	0·19
$E_1 I_a$	583	502	472	437	423	412	408	396
$I_a^2 r_a$	6·6	5·0	4·4	3·8	3·6	3·4	3·4	3·2
P	576	497	468	434	419	409	405	393
Speed	2060	1740	1510	1380	1260	1200	1150	1050

r_a, the resistance of the armature, is taken for the values of I_a, and is practically constant for the small range of these values.

P is the iron, windage, and friction loss.

Efficiency, brake horse-power, and torque.—From the preceding table and the curve of the copper loss (Fig. 195), the following table for *machine B* is produced :

E	100	100	100	100	100	100	100	100
I	15	19·8	24·2	29·7	35·0	39·8	44·8	53·8
P	576	497	468	434	419	409	405	393
Q	50	90	140	220	300	375	460	630
P+Q	626	587	608	654	719	784	865	1023
Input	1500	1980	2420	2970	3500	3980	4480	5380
Output	874	1393	1812	2316	2781	3196	3615	4357
η	58	70	75	78	79·5	80	81	81
B.H.P.	1·17	1·87	2·43	3·10	3·73	4·27	4·85	5·85
Speed	2060	1740	1510	1380	1260	1200	1150	1050
Torque	2·98	5·63	8·45	11·80	15·60	18·70	22·20	29·30

Q is the copper loss of the machine. The brake horse-power was obtained by dividing the output by 746. Torque is given in lbs.-ft.

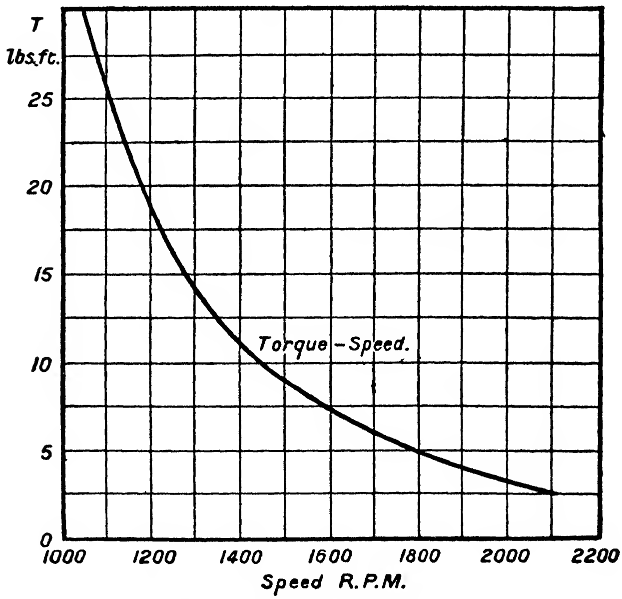

FIG. 197.—The torque-speed curve of the series-wound motor.

The *torque-speed* curve is given in Fig. 197, and the *efficiency-brake*

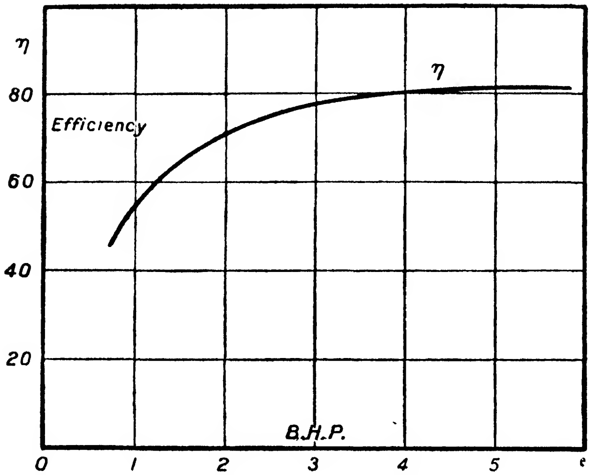

FIG. 198.—The efficiency-B.H.P. curve of the series-wound motor.

horse-power curve in Fig. 198. The speed, efficiency, B.H.P., and torque are also shown plotted against the *input* current I in Fig. 199

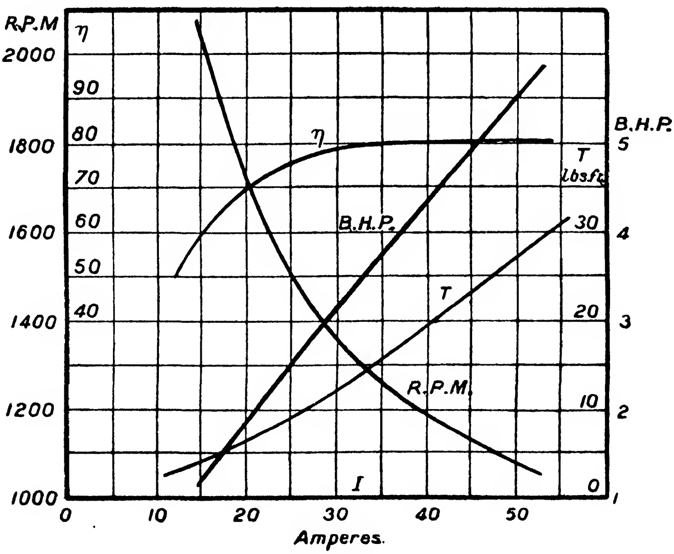

FIG. 199.—The characteristic curves of the series-wound motor.

Speed and temperature.—On page 252 the value of the speed is given by

$$N = \frac{E - I(r_a + r_m)}{k \cdot \phi}.$$

Thus, for the same input current I, ϕ will be constant, but $(r_a + r_m)$ will be greater the higher the temperature. As a consequence, the speed N for a given input current will decrease with increase of temperature.

In the case of the motor under consideration $(r_a + r_m)$ is nearly 0·24 ohm at 25° C. At 90° C. its value is about 0·298. This may be deduced from the value at 25° C., and the formula

$$r_a + r_m = r_0(1 + 0{\cdot}0042t).$$

Taking the test values E 100, I 35 amperes, and N 1260, the speed at 90° C. for the same values of current and voltage will be

$$1260 \times \frac{100 - 35 \times 0{\cdot}298}{100 - 35 \times 0{\cdot}24} = 1230,$$

that is, about three per cent. less.

For an input current of 53·8 the speed was 1050. Therefore the speed at 90° C. will be about

$$1050 \times \frac{100 - 53{\cdot}8 \times 0{\cdot}298}{100 - 53{\cdot}8 \times 0{\cdot}24} = 1010.$$

SHUNT AND COMPOUND MOTORS.

The method of testing a shunt motor for efficiency, and the determination of the total loss of iron, windage, and friction by the Hopkinson's efficiency test has already been explained. The detailed method of testing a compound-wound motor will now be given. A shunt motor may be tested in precisely the same manner.

A brake is required in the load test, and may be a prony or rope brake for motors below about one horse-power, a magnetic brake up to 10 or more B.H.P., or a loaded electrical generator coupled to the motor.

The load may be calculated, as will be shown, quite independently of the brake load. This is an advantage in the case, where an electrical generator is used as the load on a variable-speed machine, such as a compound-wound motor. For, a whole set of efficiency curves at different speeds would have to be obtained for the generator, in order that the latter might be used to directly measure the brake horse-power.

In the present test, the motor was coupled up to a direct-current generator connected to an adjustable load. Its rated full load was 5 B.H.P.

Drop test.—The voltage drop was obtained for the armature alone, and then for the series coils. These values were added together and tabulated thus :

I_a	5	10	20	30	40	50
e	1·0	2·0	3·5	4·9	6·2	7·5

The load test.—An ammeter was put in the motor shunt circuit, and an ammeter and voltmeter were used to measure the input power. The generator was then loaded step by step and values noted. Results obtained in this test were :

E	102	102	101	101	101	101
I	11·7	21·2	28·5	35·6	41·7	47·5
I_s	2·05	2·05	2·01	2·01	2·02	2·02
R.P.M.	1152	1098	1060	1032	1016	1000

Copper losses.—Those in the shunt circuit are found from EI_s for each input current. Those in the armature and series field are obtained from the drop test for the current $I_a = I - I_s$. The latter are thus tabulated :

I_a	9·65	19·15	26·49	33·59	39·68	45·48
e	1·9	3·4	4·4	5·4	6·1	6·9
$I_a e$	18·4	65	117	182	242	315

Iron, windage, and friction loss.—It is required to find this loss for each of the speeds in the load test. To do this the compound-wound motor was coupled to a shunt motor whose speed could be varied, by field adjustment, through the range taken in the preceding load test. This was done in the example under consideration by using the generator, which was loaded in the former test, as a shunt motor.

The shunt coils of the compound-wound motor are connected up for separate excitation, as shown in Fig. 200, and a voltmeter placed across its armature, which is on open circuit.

Now, from the values of the *load* test and the preceding table, the counter voltage E_i and corresponding speed may be tabulated thus :

E	102	102	101	101	101	101
e	1·9	3·4	4·4	5·4	6·1	6·9
E_i	100·1	98·6	96·6	95·6	94·9	94·1
R.P.M.	1152	1098	1060	1032	1016	1000

The field rheostat of the shunt motor is adjusted to give one of these speeds, 1060 suppose, and also the field rheostat of the compound-wound motor running as an unloaded generator is adjusted, until the voltmeter of the latter gives $E_i = 96{\cdot}6$. The input P of the shunt motor is then read. Next, switch S is opened and the input P_1 read.

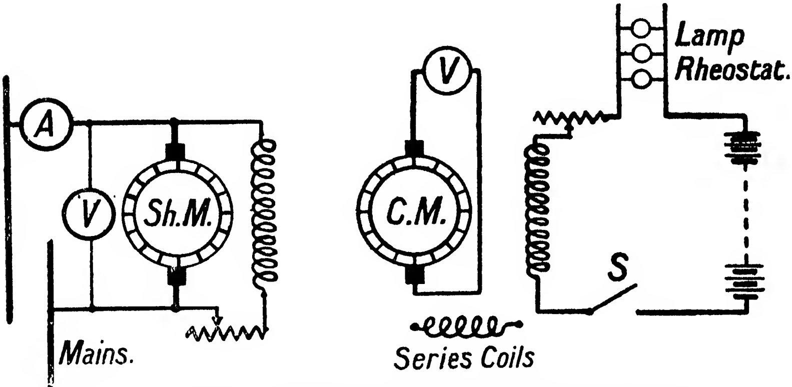

FIG. 200.—Arrangement for the determination of the iron, windage, and friction loss of the compound-wound motor.

The value $P - P_1$ will give the iron loss of the compound-wound motor when its speed is 1060, and its internal or back voltage is 96·6.

A third reading of the input P_2 may be taken after the compound-wound motor is uncoupled from the shunt motor, the latter running at the same speed as before. Then $P_1 - P_2$ will give the windage and friction loss of the compound-wound motor for speed 1060.

The supply voltage of the shunt motor should be kept at constant value in these three operations. A similar procedure is gone through for the other speeds.

The results obtained for the three operations taken in order were :

Speed	1152	1098	1060	1032	1016	1000
E	99	99	99	99	99	99
I	9·0	9·0	9·2	9·2	9·1	9·1
EI	891	891	910	910	901	901
E	99	99	99	99	99	99
I	6·85	7	7·1	7·15	7·0	7·15
EI	680	693	705	710	693	710
E	99	99	99	99	99	99
I	5·25	5·4	5·65	5·8	5·8	5·8
EI	520	535	560	575	575	575
Iron Loss	211	198	205	200	208	191
W & F	160	158	145	135	118	135

A few inconsistencies of not much importance are present in this table, probably due to a slight error, not much larger than one per cent., either in reading the value of I or in the ammeter itself.

Efficiency, B.H.P., and Torque.—The final table for the motor, giving the detailed losses and other quantities, may now be tabulated thus:

E	102	102	101	101	101	101
I	11·7	21·2	28·5	35·6	41·7	47·5
Input	1190	2165	2880	3600	4210	4800
I_s	2·05	2·05	2·01	2·01	2·02	2·02
EI_s	210	210	203	203	206	206
Q	18	65	117	182	242	315
Iron Loss	211	198	205	200	208	191
W & F	160	158	145	135	118	135
Tot. Loss	599	631	670	720	774	847
Output	591	1534	2210	2880	3436	3953
η	49·7	71·0	76·5	80·0	81·5	82·5
B.H.P.	0·8	2·05	2·96	3·86	4·6	5·3
Speed	1152	1098	1060	1032	1016	1000
Torque	3·62	9·7	14·6	19·7	23·6	27·8

Q is the copper loss in armature and series coils.

The graph relating torque and speed is given in Fig. 201; efficiency and B.H.P. in Fig. 202; and the graphs of torque, B.H.P., efficiency, and speed, against the input current in Fig. 203.

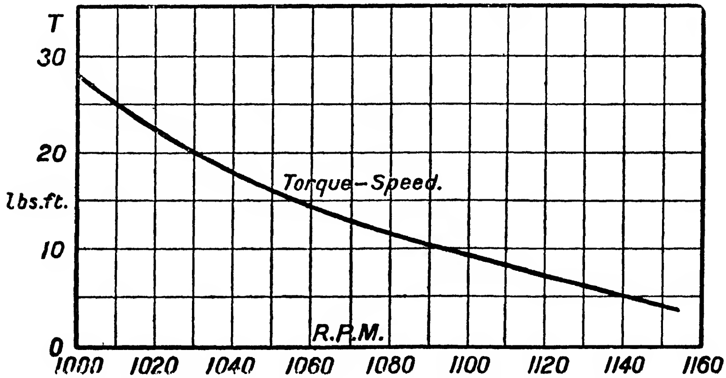

FIG. 201.—The torque-speed curve of the compound-wound motor.

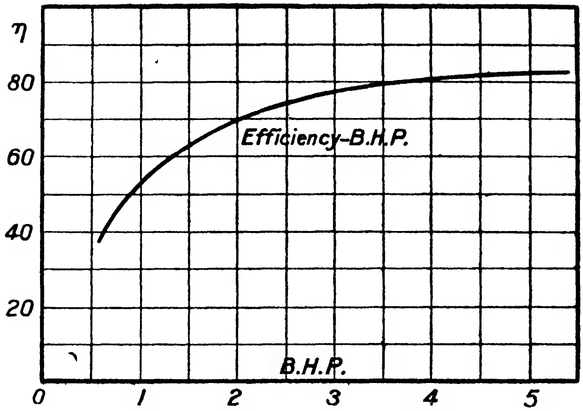

FIG. 202.—The efficiency-B.H.P. curve of the compound wound motor.

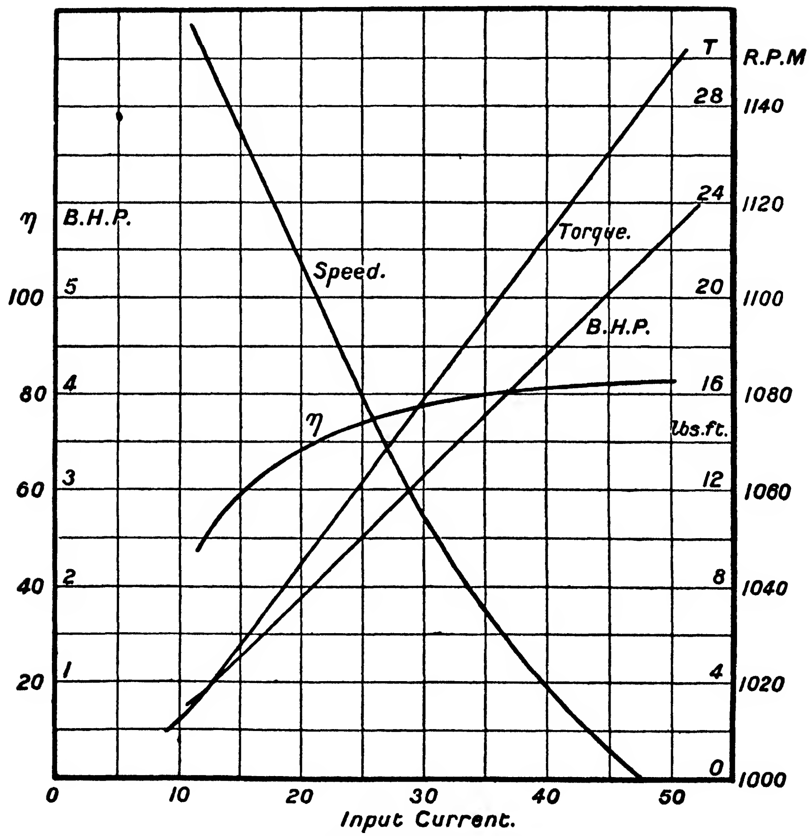

FIG. 203.—The characteristic curves of the compound-wound motor.

CHAPTER XIII.

PRINCIPLES OF ALTERNATING-CURRENT MACHINES.

UNDER the heading of alternating-current machines may be classed those which generate single or multiphase currents, and which are briefly termed alternators; those which are fed with alternating currents of one or more phase and run as motors, such as synchronous motors, induction motors, and single or multiphase commutator motors; and a third class which transforms from alternating current to direct current, or *vice versa*, such as rotary converters, and static transformers which transform alternating currents from a high to a low voltage or *vice versa*.

As stated before, it is difficult to construct satisfactory direct-current machines of higher voltage than about a thousand, and to produce a high *direct* voltage it is necessary to connect a sufficient number of direct-current machines in series, as in *Thury's* system. High voltages, which are so necessary for long distance transmission, are produced by suitable alternators, or the latter used in conjunction with step-up transformers.

Ordinary alternators and synchronous motors have, in general, a revolving separately excited field and a stationary armature or stator. Induction motors have a fixed armature or stator, and a rotating one called the rotor. Single or multiphase-commutator motors have a fixed stator winding and a direct-current armature winding, of which in some cases the brushes are short circuited. Rotary converters have a fixed field system and a rotating armature. The principles of these different types of machines have much in common with one another.

Armature windings.—A few of the different types of windings will be sufficient to illustrate the principles of alternating-current machines.

A direct-current armature can supply alternating current if properly connected to slip rings.—Consider a lap-wound armature winding $Z=24$, $p=4$. The pitches are 5 and 7. If p is the number of poles in a lap-wound winding, then for the production of single-phase currents there will be $\frac{p}{2}$ connections from the armature to one slip ring, and $\frac{p}{2}$ alternate connections to the other slip ring. The

case under consideration is illustrated in Fig. 204; R_1 and R_2 are the rings to which the winding is connected so as to produce alternating currents.

A good method of analysing the action which goes on in the winding is to cut out a strip of paper AB and place on it lines to represent the conductors, having the same spacing apart as the

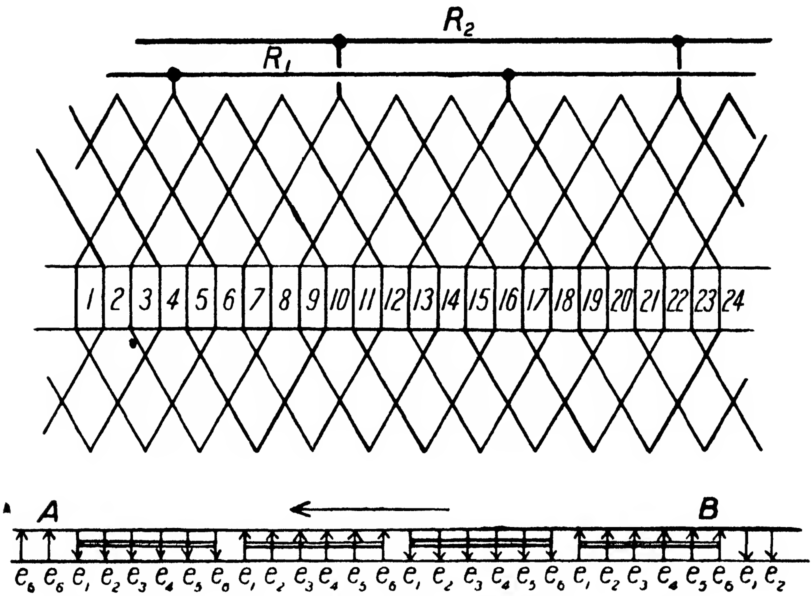

FIG. 204.—A direct-current lap winding, arranged to supply single-phase alternating current.

winding. The induced voltages in the conductors, taken in order, are marked e_1, e_2, up to e_6, and then continued in the opposite direction alternately as shown.

e_1, e_2, up to e_6 are the respective induced voltages in the set of six consecutive conductors associated with each pole at any given instant. By moving the strip of paper in the direction shown across the conductors of the winding, supposed stationary, an exact representation may be obtained of the voltages induced in the conductors as the winding moves from left to right with the poles stationary.

Thus, suppose the line A of the strip of paper coincides with conductor 1 of the winding. At that instant the voltage between rings R_1 and R_2 is

$$e_1 + e_2 + e_3 + e_4 + e_5 + e_6 = E,$$

which is the same as for a direct-current lap-wound armature. Also, there are 4 parallel sections between R_1 and R_2, each of voltage E in the same direction.

When AB has moved through a slot pitch or the spacing between two adjacent conductors, the voltage between R_1 and R_2 is

$$e_2 + e_3 + e_4 + e_5 + e_6 - e_1,$$

that is,

$$e_2 + e_3 + e_4 + e_5.$$

An equal interval later the voltage across the rings is

$$e_3 + e_4 + e_5 + e_6 - e_1 - e_2,$$

which equals

$$e_3 + e_4.$$

Another such interval later, the voltage is

$$e_4 + e_5 + e_6 - e_1 - e_2 - e_3 = 0.$$

After the next interval, the voltage is

$$e_5 + e_6 - e_1 - e_2 - e_3 - e_4 = -(e_3 + e_4),$$

and so on. The voltage curve will therefore be as shown in Fig. 205.

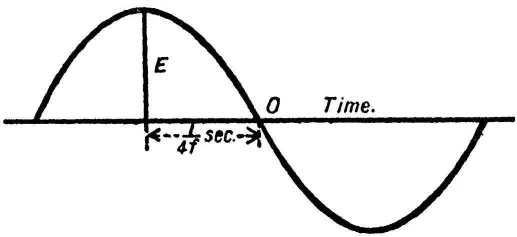

FIG. 205.—A voltage curve of an alternator.

If t is the time for the winding to move through the distance between two conductors, then in time $3t$ seconds, or the time the winding has moved through half the pole pitch, the voltage has changed from maximum value to zero, that is, passed through $\frac{1}{4}$ of a period of the alternation. Therefore

$$\frac{1}{4f} = \frac{1}{2pn} \quad \text{and} \quad f = \frac{np}{2},$$

in which n is the revolutions per second, and p the number of poles.

The R.M.S. value of this alternating voltage is $\frac{1}{\sqrt{2}}$ times the maximum value, assuming it obeys the sine law. Therefore, if the other side of the winding is connected down to a commutator, the ratio of the alternating voltage to the direct-current voltage will be $\frac{1}{\sqrt{2}} = 0{\cdot}707$, and the machine is termed a single-phase *rotary converter.* By running it as a direct-current motor, it will supply alternating current from the slip rings.

This direct-current armature winding may be arranged to give two, three, or more phases, across suitably connected rings. A wave

winding fitted with commutator and slip rings is shown in Fig. 206. It is single phase. The connections down to the rings are diametrically opposite whatever the number of poles. A three-phase wave

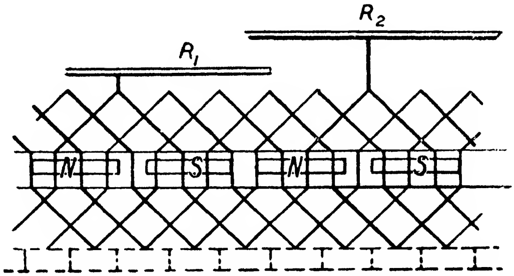

FIG. 206.—A direct-current wave winding, shown with slip rings and commutator.

winding for a rotary converter is shown in Fig. 207. The connections to the three slip rings are three in number and 120 degrees apart. Similarly for six- and twelve-phase windings, which are sometimes used in practice. In the latter there would be twelve rings and, if wave, one connection per ring; the connections being 30 degrees apart.

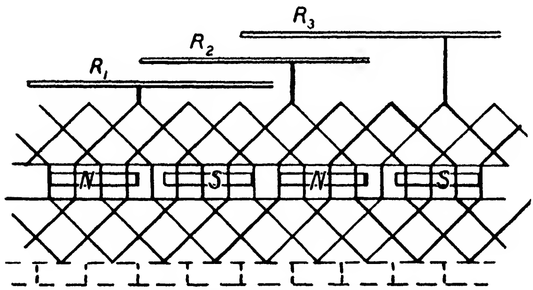

FIG. 207.—A direct-current wave winding, arranged to give either three phase or direct current.

In a lap winding there are the same number of rings for the phases as in the case of the wave winding, but the number of connections to each ring depends upon the value of p, the number of poles. Each ring has $\frac{p}{2}$ connections, which are equally spaced round the winding. Thus, if there are six phases, that is, six rings, there will be a total of $6 \times 3p$ connections equally spaced round the winding. Such a winding of 24 conductors is illustrated in Fig. 208.

By using the *same* strip of paper with the induced voltages marked upon it, as was used in connection with Fig. 204, the production of three-phase alternating voltage may be studied. Starting with the third line, reckoning from the left of the strip, namely e_1, placed on conductor 1 of the winding, the voltage between any pairs of rings

may be determined. The strip is then moved from right to left through steps represented by the spacing distance between two adjacent conductors.

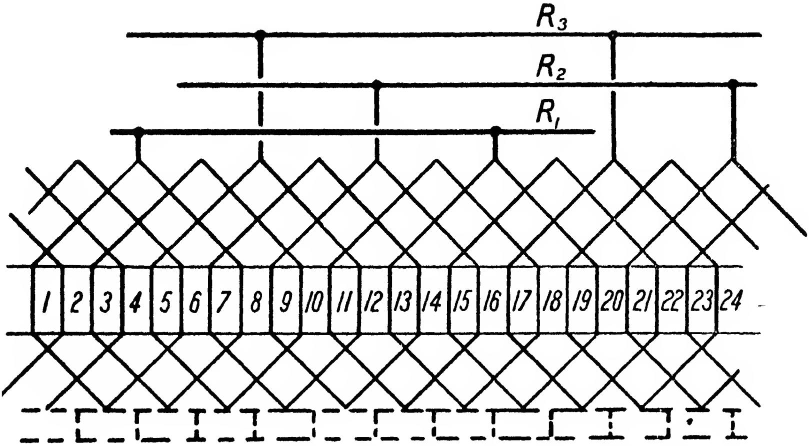

FIG. 208.—A direct current lap winding, arranged to give either three phase or direct current.

Thus the succeeding voltages between rings R_1 and R_3 will be found to be

$$\begin{aligned}
&e_2+e_3+e_4+e_5,\\
&e_3+e_4+e_5+e_6,\\
&e_4+e_5+e_6-e_1=e_4+e_5,\\
&e_5+e_6-e_1-e_2=0,\\
&e_6-e_1-e_2-e_3=-(e_2+e_3),\\
&-e_1-e_2-e_3-e_4,\\
&-e_2-e_3-e_4-e_5,
\end{aligned}$$

and so on.

The frequency of the alternations is therefore the same as in the case of the single-phase rotary converter, namely,

$$f=\frac{np}{2}.$$

During the preceding times the voltages between R_3 and R_2 were

$$\begin{aligned}
&e_6-e_1-e_2-e_3=-(e_2+e_3),\\
&-e_1-e_2-e_3-e_4,\\
&-e_2-e_3-e_4-e_5,\\
&-e_3-e_4-e_5-e_6,\\
&-e_4-e_5-e_6+e_1=-(e_4+e_5),\\
&-e_5-e_6+e_1+e_2=0,
\end{aligned}$$

and so on.

Between R_2 and R_1 the voltages were, for the same times,

$$-e_4-e_5-e_6+e_1=-(e_4+e_5),$$
$$-e_5-e_6+e_1+e_2=0,$$
$$-e_6+e_1+e_2+e_3=e_2+e_3,$$
$$e_1+e_2+e_3+e_4,$$
$$e_2+e_3+e_4+e_5,$$

and so on.

The winding therefore consists of three equal sections, each of which in this case has its two halves in parallel, and may be illustrated as in Fig. 209. This is termed a *mesh* or delta arrangement.

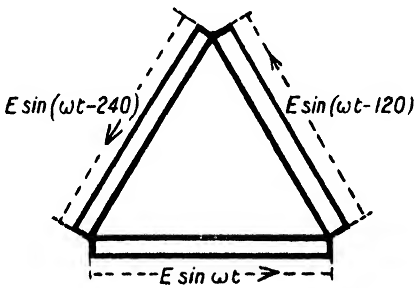

FIG. 209.—The mesh or delta arrangement of the winding of the rotary converter.

In Fig. 210 is given the voltage curves across these sections, that is, across the rings. If the *first* phase is between R_1 and R_3, the *third* phase will be between R_3 and R_2, and the *second* phase between R_2

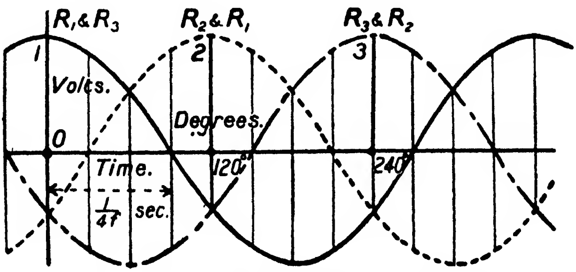

FIG. 210.—The voltage curves for the rotary converter.

and R_1. The second phase is 120 degrees behind the first, and the third phase 120 degrees behind the second.

The maximum voltage across a pair of rings is $e_2+e_3+e_4+e_5$, which is only that of *two-thirds* of the conductors associated with one pole, while the voltage of the direct machine is $e_1+e_2+e_3+e_4+e_5+e_6$.

In the general case, assuming that the voltage variation under a pole is a sine function with respect to its position, then E_d, the direct-current voltage (see Fig. 211) is represented by

$$k\{\sin\frac{a}{2}+\sin\frac{3a}{2}+\sin\frac{5a}{2}+\ldots \text{ to } n \text{ turns}\},$$

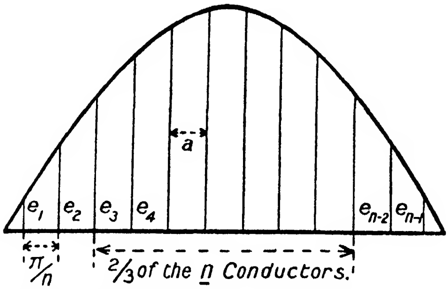

FIG. 211.—Curve for finding the maximum voltage across two slip rings of the rotary converter.

k being a constant and n the number of conductors associated with one pole. Putting in the value of the sum of this series,

$$E_d=\frac{k\sin^2\frac{na}{2}}{\sin\frac{a}{2}},$$

E_a, the *maximum* value of the alternating voltage between a pair of rings, will then be given by

$$E_d-2k\{\sin\frac{a}{2}+\sin\frac{3a}{2}+\ldots\frac{n}{6}\text{ terms}\}.$$

Therefore
$$E_a=E_d-\frac{2k\sin^2\frac{na}{12}}{\sin\frac{a}{2}}.$$

Also,
$$a=\frac{\pi}{n},$$

so that
$$\frac{E_a}{E_d}=\frac{1-2\sin^2 15}{1}=\cos 30=\frac{\sqrt{3}}{2}.$$

The R.M.S. value of the alternating current is thus equal to $\frac{\sqrt{3}}{2\sqrt{2}}$ or 0·612 times the direct voltage of the winding. If Z is the total number of armature conductors, ϕ the flux per pole threading the armature core, and n the number of revolutions per second, the

R.M.S. value of the voltage of a three-phase lap-wound armature across a pair of rings will be

$$0{\cdot}612 Z\phi n 10^{-8} \text{ volts.*}$$

By using the strip of paper for a six-phase winding of 24 conductors, such as that of Fig. 208, but with six rings, it will be seen that the maximum voltage between a pair of rings is $e_3 + e_4$. Thus, for a six-phase lap winding,

$$E_a = E_d - \frac{2k \sin^2 \frac{na}{6}}{\sin \frac{a}{2}},$$

and therefore $$\frac{E_a}{E_d} = \frac{1 - 2\sin^2 30}{1} = \cos 60 = \tfrac{1}{2}.$$

The R.M.S. value of the alternating voltage is thus $\frac{1}{2\sqrt{2}} = 0{\cdot}354$ times the direct voltage.

By drawing a winding with more conductors for 12 phases, the ratio will be similarly found to be 0·183.

The general expression for the ratio of the R.M.S. value of the alternating voltage to the direct voltage for the same winding is given by

$$\frac{1}{\sqrt{2}} \sin \frac{\pi}{m},$$

m being the number of phases or rings. Its frequency will be given by

$$f = \frac{np}{2}$$

in all cases.

A machine with a winding connected to both commutator and slip rings for the purpose of changing alternating current into direct current is termed a rotary converter; if it has to perform the reverse function, it is known as an *inverted* rotary converter. It is used in both these ways in practice.

EXAMPLE.—The flux per pole threading the armature core of a lap-wound 6-phase rotary converter is 2 megalines, the number of armature conductors 364, and the speed 1200 R.P.M. Find the value of the voltage induced in each phase of the winding. *Ans.* R.M.S. voltage 51·5.

Single-phase windings.—One of the simplest types of single-phase armature windings is shown in Fig. 212. It is termed an hemitropic winding, because the number of armature coils is one-half the number of poles. This kind of winding is generally used in each phase of a three-phase winding in order to avoid complicating the end connections, but for a single-phase alternator it is customary to use as many coils as there are poles.

* From equation (a) page 207.

The induced voltage in the winding, when the poles are fixed and the winding moves from right to left, varies as shown in the graph. It will be seen that the frequency of the voltage alternations is given by

$$f=\frac{np}{2}.$$

In the position shown, each coil of the winding is threaded by ϕ, the flux per pole threading the armature core. At a time $\frac{1}{2f}$ second later the coil is threaded by $-\phi$. Therefore, by Faraday's law, and

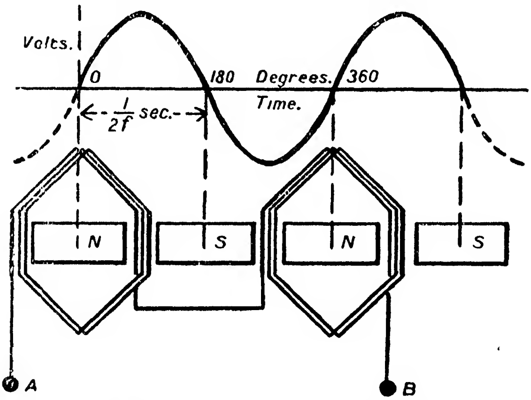

FIG. 212.—A single-phase hemitropic winding, and its voltage wave.

assuming that the ratio of the R.M.S. value to the average value is that of a sine function, namely 1·11, E the R.M.S. value of the induced voltage across the terminals A, B, is given by

$$E=2{\cdot}22\phi Zf10^{-8}\text{ volts,}$$

Z being the total number of armature conductors.

This formula is true for a concentrated winding, that is, for one in which each coil side occupies only one slot. If a coil side is distributed over a number of slots, it is called a distributed winding, and the formula has to be multiplied by k, the breadth or distribution coefficient;

$$\therefore\ E=2{\cdot}22k\phi Zf10^{-8}.$$

Values of k for a single-phase winding are as follows. x is the number of slots per pole per phase.

x	1	2	3	4	6
Case 1	1·0	0·924	0·91	0·906	0·904
Case 2	1·0	0·707	0·667	0·654	0·645

In *Case* 1 only 50 per cent. of the slots round the periphery of the stator or armature are used. They are all used in *Case* 2.

A single-phase winding is generally distributed over $\frac{2}{3}$ of the total number of slots. By using all the slots, the voltage will only be increased by about 10 per cent., and the extra copper needed will be about 50 per cent. greater; an extra expenditure which, in general, is not justified by the small increase of voltage.

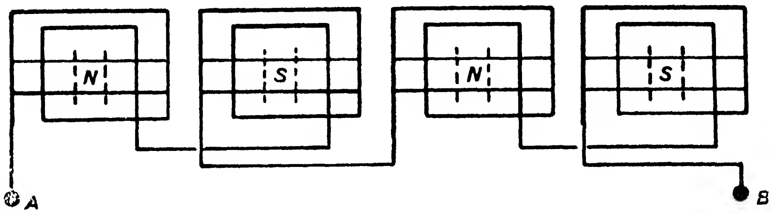

FIG. 213.—A single-phase chain winding.

A *chain* winding is shown in Fig. 213 having the same number of poles as coils, and in which only $\frac{2}{3}$ the total number of slots are used.

Another typical winding, in which the same proportion of slots is used, is shown in Fig. 214.

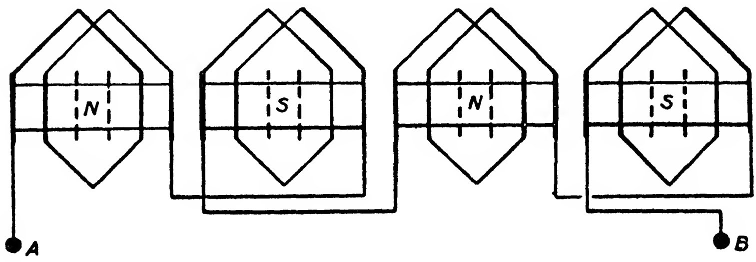

FIG. 214.—A typical single-phase winding.

Typical three-phase windings.—A typical three-phase winding of 24 conductors is shown in Fig. 215. A_1, B_1 are the ends of the first phase, A_2, B_2 those of the second, and A_3, B_3 those of the third.

When the centre line of coil C coincides with that of the north pole, as in the figure, the induced voltage will be zero. Now, supposing the poles fixed, and the winding moving from *right* to *left,* the induced voltage in the first phase will rise in value from zero in an anti-clockwise direction, that is, from B_1 to A_1 through the first-phase section of the winding. This direction will be maintained through a movement equal to the pole pitch, the voltage rising from zero to a maximum, then back to zero. Thus a pole pitch corresponds to

180 electrical degrees or $\frac{1}{2f}$ seconds; f being the frequency of alternation, and of value given by

$$f=\frac{np}{2},$$

in which n is the revolutions per second and p the number of poles.

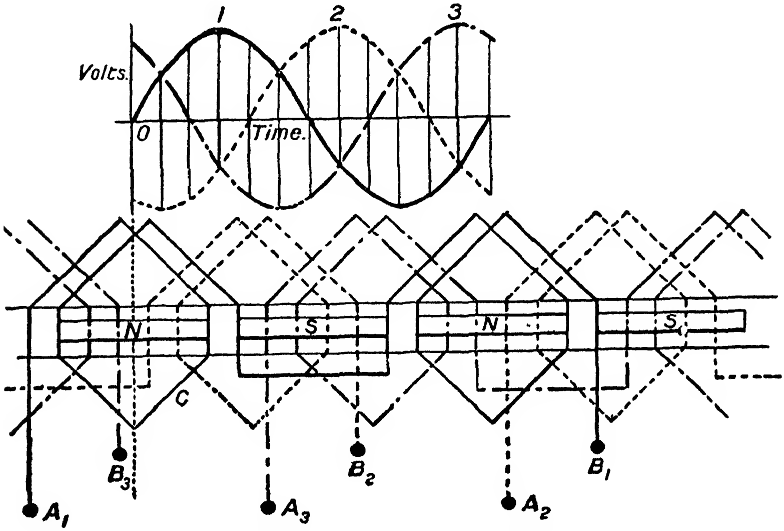

FIG. 215.—A typical three-phase winding.

The second-phase section of the winding has to move through 120 electrical degrees, or $\frac{2}{3}$ the pole pitch, before attaining the same voltage and direction that the first phase section had at the instant represented in the figure, while the third-phase section has to move through 240 electrical degrees, or $1\frac{1}{3}$ the pole pitch, to attain the same result.

Therefore, *reckoning* from B_1 to A_1, B_2 to A_2, B_3 to A_3, the respective voltages in the phases are completely represented by the graphs of Fig. 215, or their equivalent trigonometrical equations

$$e_1=E'_m \sin \omega t,$$
$$e_2=E'_m \sin (\omega t-120),$$
$$e_3=E'_m \sin (\omega t-240),$$

E'_m being the maximum value of the phase voltage.

Star and mesh or delta connection of the three sections of the winding.—A *star* arrangement is made by joining B_1, B_2, and B_3 together to form a common junction, or what is termed the *neutral* point. e_1, e_2, and e_3, the instantaneous voltages in the three phases,

will act from B_1 to A_1, B_2 to A_2, B_3 to A_3, that is, outwardly from the neutral point O of the star, as shown in Fig. 216. These arrows will

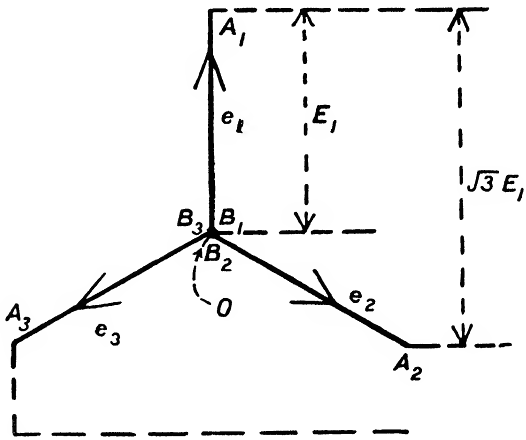

FIG. 216.—The phases of the winding shown star connected.

always point outwardly, and must not be regarded as the direction of the voltage. They are merely directions along which e_1, e_2, and e_3 operate, and these voltages, by their trigonometrical values, include the actual direction of the voltage.

If the winding had moved in the opposite direction to that taken, the arrows would have been all pointing inwardly to O. The first convention will be used.

The *line* voltage for the star arrangement, that is, between A and B, is

$$e = E'_m \sin \omega t - E'_m \sin (\omega t - 120) = \sqrt{3}\ E'_m \cos (\omega t - 60),$$

and its R.M.S. value is $\sqrt{3}$ times that of the R.M.S. value of the phase, or line to neutral, voltage.

A *mesh* arrangement is made by connecting the phases in the form of a triangle (Fig. 217); B_1 is joined to A_2, B_2 to A_3, and B_3 to A_1. The sum of the voltages in the mesh, namely

$$e_1 + e_2 + e_3,$$

is always zero. The *line* voltage is the same as the phase voltage.

Thus an alternator may be arranged to give a terminal or line voltage of one value, or one which is $\frac{1}{\sqrt{3}}$ times that value, according as its phase sections are star or mesh connected.

The R.M.S. value of the phase voltage of a three-phase alternator is determined as in the case of the single-phase machine. Thus

$$E_1 = 2{\cdot}22 k\phi Z_1 f 10^{-8},$$

k being the breadth or distribution coefficient of the winding, ϕ the flux per pole threading the armature or stator core, Z_1 the number of

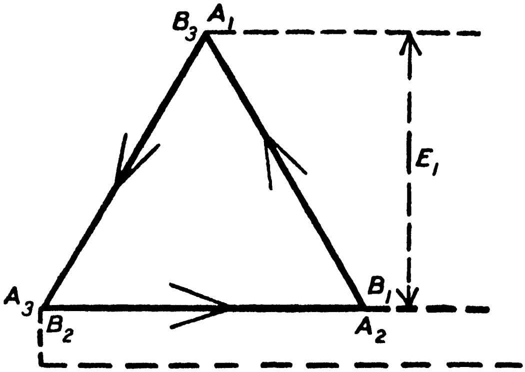

FIG. 217.—The phases of the winding, shown mesh connected.

conductors per phase, and f the frequency. Values of k are tabulated below; x being the number of slots per pole per phase.

Two phase	x	1	2	3	4	6
	k	1·0	0·924	0·911	0·906	0·903
Three phase	x	1	2	3	4	6
	k	1·0	0·966	0·96	0·958	0·956

The *line* voltage is $\sqrt{3}E_1$ for star connection, and E_1 for mesh connection.

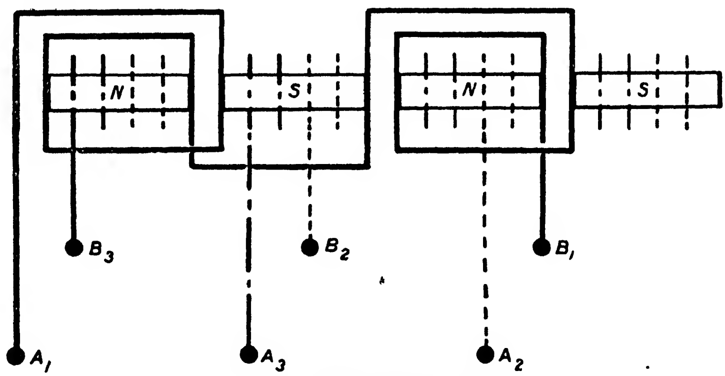

FIG. 218.—A three-phase chain winding.

A three-phase *chain* winding of 24 conductors is shown in Fig. 218. Only one phase A_1B_1 is completely drawn. The second phase, begin-

ning with A_2 and ending with B_2, is drawn exactly as the first, so is the third phase, which begins at A_3 and ends at B_3. The same deductions may be obtained for this type as for the preceding one. The chain winding is not quite so efficient as that shown in Fig. 215, but it is generally easier to wind and more accessible for repairs.

Rotating magnetic field in multiphase machines.—In multiphase synchronous motors, generators, and induction motors, a rotating field is produced in the iron core and across the air gap by the currents in the armature. An approximate form, value, and speed of this field may be obtained as follows.

The distribution of current in alternating and direct-current machines around the periphery of the winding is a set of currents in

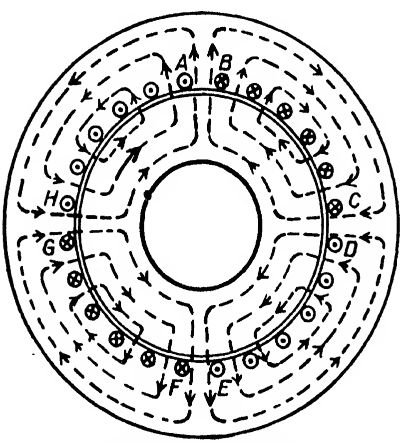

FIG. 219.—The magnetic fluxes produced by the currents in the armature of a four-pole machine.

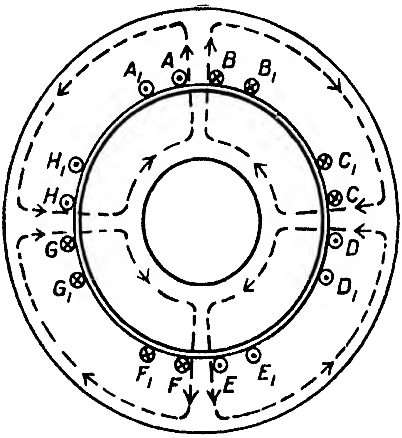

FIG. 220.—The magnetic fluxes due to corresponding pairs of conductors.

one direction, followed alternately by a set in the opposite direction, as shown in Fig. 219; the number of sets depending upon the number of poles. The magnetic fluxes produced by the winding are indicated by the dotted closed curves. Part of the winding is shown in Fig. 220, and it will be noted that four magnetic fluxes are produced across the air gap due to the similarly disposed pairs of conductors, namely A and B, C and D, E and F, G and H. In magnetic effect, each of these pairs of conductors or coil sides is equivalent to that of a flat rectangular coil. As the four fluxes are equal, each pair may be regarded as producing the flux immediately passing through itself.

The magnetic reluctance of the iron part of the magnetic flux will be neglected, as it is small in comparison with that of the air gap. The effect of the broken surfaces due to the slots at the boundaries of the air gap will also be neglected. Afterwards an allowance may be made for these two effects.

Let δ be the radial depth of the air gap, s the pitch of a slot; a the number of conductors per slot, one per slot is only shown in the figure; l the net axial length of the stator core; and I the current in the conductors. All dimensions are in centimetres.

Then, by the M.M.F. law,

$$\phi_{AB} = \frac{4\pi}{10} a\mathrm{I} \cdot \frac{sl}{\delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

that is, the flux in lines threading the air gap between conductors A and B. Similarly for CD, EF, and GH. This flux is distributed uniformly in the air gap across the space it occupies, assuming the boundaries of the air gap as being unbroken.

Likewise, the flux distributed in the air gap due to each of A_1B_1, C_1D_1, E_1F_1, and G_1H_1 is

$$\frac{4\pi}{10} a\mathrm{I} \cdot \frac{3sl}{\delta}.$$

For the next set of pairs of conductors, 3 will be replaced by 5, and so on.

Thus, by knowing the current in the conductors and the disposition of the latter, the flux produced by a certain number of ampere turns may be calculated.

EXAMPLE.—A, B, C, and D are four consecutive slots in the stator core of an induction motor. It has 8 conductors per slot, slot pitch 2 cms., radial depth of air gap 0·1 cm., and an effective axial length of 25 cms. The current in each conductor is 70 amperes, and in one direction for slots A and B, and in the opposite direction for C and D. Neglecting the reluctance of the iron and the effect of the slots, calculate the total magnetic flux in the air gap due to the conductors in the four slots. *Ans.* 1·408 megalines.

The rotating field of a three-phase winding.—It will be assumed that the stator core is fixed in space, as is usually the case, and that the rotor revolves. The material of the rotor core is continuous in the case of induction motors and high-speed turbo-alternators, but not in the case of slow-speed alternating-current generators or synchronous motors, in which projecting magnet poles are used.

In Fig. 221 is shown a three-phase winding carrying currents as indicated. The graphs of these currents are also given. The arrows on the conductors of the winding are the directions of the currents for *instant A* on the graphs.

The distribution of flux through the air gap around its periphery for instants A, B, and C are shown. The conductors of the first phase are indicated by the largest, those of the second by the middle sized, and those of the third by the smallest circles. The winding is a four-pole one, and, as shown by the areas of stepped contour, produces a rotating field of also four poles. Each of these poles span a distance *ab* equal to one-fourth the stator periphery at the air gap.

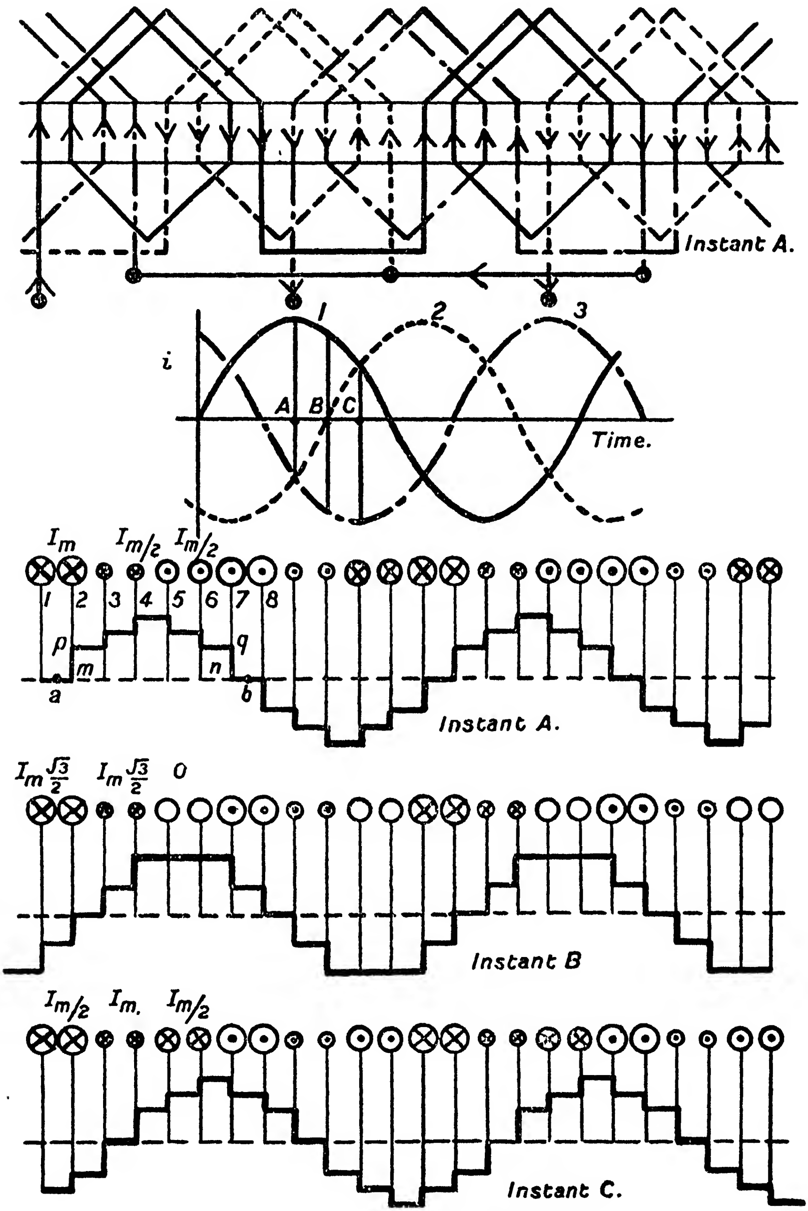

FIG. 221.—The rotating field of a three-phase winding.

The flux threading the air gap due to the conductors in slots 2 and 7 carrying current I_m is, by equation (a), equal to

$$\frac{4\pi a}{10} I_m \frac{l}{\delta} \cdot mn.$$

Therefore, by making pm equal to

$$\frac{4\pi a I_m}{10 . \delta} l = \frac{1{\cdot}26al}{\delta} \cdot I_m,$$

the area of the rectangle $pmnq$ represents the flux threading the air gap due to the current I_m in the conductors of slots 2 and 7.

The next *step* on the curve will similarly be of height

$$\frac{1{\cdot}26al}{\delta} \cdot \frac{I_m}{2},$$

and so on.

ϕ, the total flux per pole through the air gap, is thus represented by the area of the stepped curve. This area should, however, be multiplied by a correction coefficient to allow for the reluctance of the iron and the effect of the slots, in order to give a more exact representation of the value of ϕ. The correction coefficient may range from 0·7 for small machines to 0·9 for large machines, and depends upon whether the slots are enclosed or not.

The shape of the stepped curve changes from *peaked* to *flat*, and then back to peaked, with a frequency of 6 times the frequency of the alternating currents in the winding, that is, the interval between peaked shape and peaked shape again is $1/6f$ second.

During an interval of AC on the graph curves, that is, $1/6f$ second, the field has moved through one-third the pole pitch, and if N is the revolutions per minute of this rotating field,

$$\frac{60}{3pn} \text{ second}$$

is the time taken by the field to move through this distance. Therefore

$$N = \frac{120f}{p}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

which is generally true for multiphase windings.

The value of p, the number of poles of the stator winding, is also found by taking *twice* the number of stator coils per phase.

By interchanging the feeding currents in any two of the three terminals of the winding, the direction of rotation of the field may be shown in the same manner to be reversed.

EXAMPLE.—A three-phase stator winding has 18 coils. Calculate the speed of its rotating field when the former is supplied with currents at frequency 50, and also when the frequency is 25. *Ans.* 500 and 250 R.P.M.

ϕ, the flux per pole.—The flux per pole through the air gap, due to the currents in a three-phase winding, such as that of the stator

of an induction motor, may be calculated from equation (*a*). Fig. 222 shows one of the peaked stepped areas similar to those of Fig. 221, but more representative.

As before, each conductor may be taken to represent a slot containing a conductors. s is the slot pitch, y the number of slots per pole per phase, and l the axial length of the iron of the stator core.

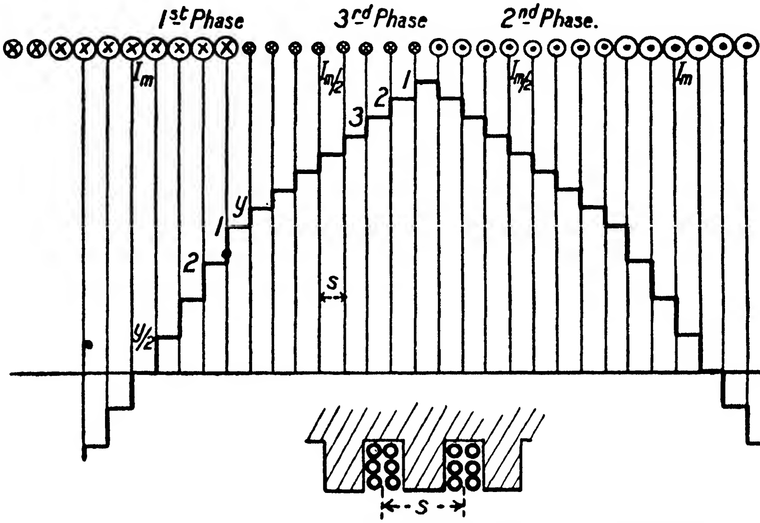

FIG. 222.—The peaked area representing the flux of a pole of a three-phase winding.

The flux represented by the highest rectangle is

$$\frac{1{\cdot}26}{\delta}\, as\, \frac{I_m}{2} l.$$

By the second rectangle from the top, the flux is

$$\frac{1{\cdot}26}{\delta}\, a3sl\, \frac{I_m}{2},$$

and so on. Therefore

$$\phi = \frac{1{\cdot}26salI_m}{\delta}\Big[\tfrac{1}{2}(1+3+5\ldots y \text{ terms}) + \Big\{(2y+1)+(2y+3)+\ldots\frac{y}{2}\text{ terms}\Big\}\Big].$$

The sum of the two series in the brackets is $\frac{y^2}{2}+\frac{5}{4}y^2=\frac{7}{4}y^2$. Also, $I_m=\sqrt{2}I$; the R.M.S. value of the current being I.

Thus, $$\phi = \frac{3{\cdot}11}{\delta}\, saIly^2. \qquad \ldots\ldots (c)$$

If Z is the number of conductors per phase, and D is the mean diameter of the winding periphery,

$$Z = yap \quad \text{and} \quad \pi D = 3spy.$$

Therefore, $$\phi = \frac{3 \cdot 26}{\delta} \frac{DZl}{p^2} \cdot I. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

The area of the flat-topped flux curve is practically of the same value as that of the peaked curve, and the average value does not differ appreciably from that given in the last equation. In the latter, the effect of the broken surfaces of the air-gap boundaries due to the slots, and the reluctance of the iron of the cores, have been neglected.

EXAMPLE.—The stator of a three-phase induction motor has 4 slots per pole per phase, 8 conductors per slot, air-gap depth 0·16 cm., slot pitch 2 cms., and the axial length of iron in its core 20 cms. Its winding carries a current of 60 amperes. Find the total flux per pole crossing the air gap and the average pole face magnetic density. There is no current in the rotor winding. *Ans.* 5·97 megalines and 12·44 kilolines. If a correction coefficient of 0·9 is taken to allow for reluctance of the iron and effect of the slots, the values given will be multiplied by 0·9.

In the case of a multiphase alternator, the speed of its stator rotating field is the same as that of its field-magnet system, and the relative displacement of these two fields depends upon the self inductance and resistance of the machine itself, and the circuits it is feeding.

The case of a single-phase winding is different, its coils producing an alternating magnetic flux across the air gap. However, this alternating flux is equivalent to two equal rotating fields, each having *half* the amplitude of this flux. These fields move in opposite directions around the periphery of the air gap with speeds of the same value as that of the field magnets of the machine. Thus, one is moving in the same direction and with the same speed as the poles of the machine, while the other field moves in the opposite direction with a relative speed of twice that of the poles.

One rotating field is stationary with respect to the poles. The other cuts across these poles with twice the speed of the poles, and thereby induces in the coils of the field magnets, currents, which by their magnetic action, react on this rotating field producing them, and practically damps it out.

Armature reaction in multiphase generators.—The armature reaction in the case of a three-phase generator is illustrated by the diagrams of Fig. 223. A shows the graphs of the alternating currents in the machine, supposing they are in phase with the induced voltages of the winding. The directions of these voltages are indicated by dots and crosses in diagram (*l*).

In the position of the poles of the field magnet shown in (*l*), the voltage induced in the first phase, *large circles*, is at maximum value, as its coil sides are then cutting the maximum number of lines of

magnetic flux per second. This position is represented by instant *a* on the graph A.

The rotating field of the winding is built up as described before, and its reaction on the field magnets is one of distortion only ; being

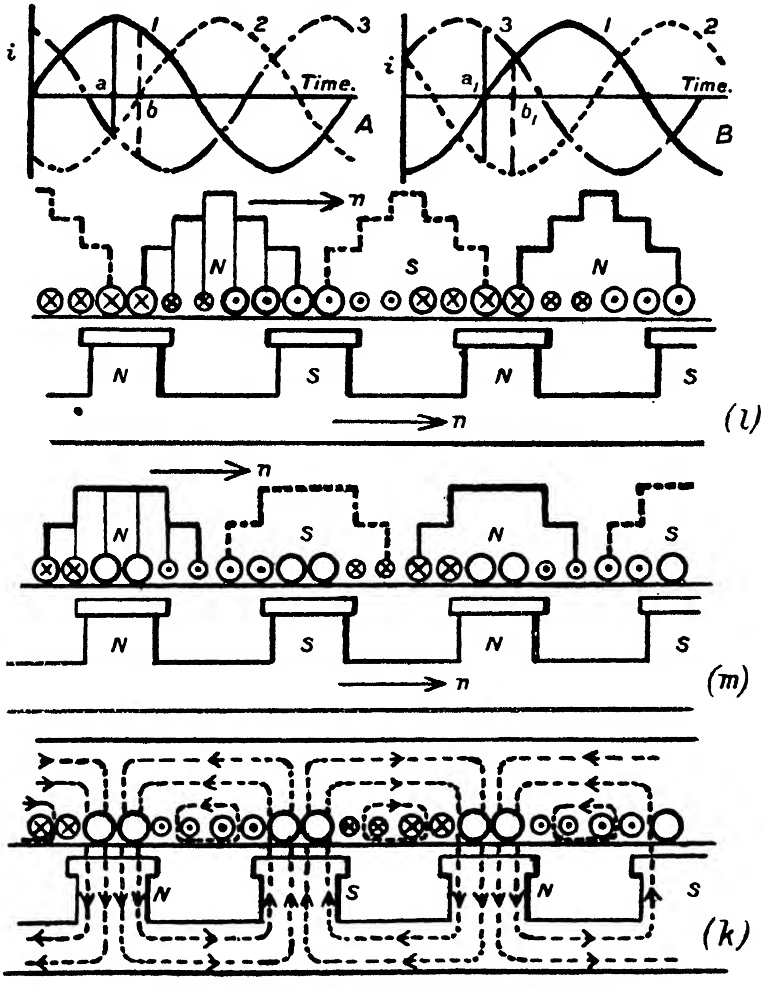

FIG. 223.—The armature reaction in a three-phase generator.

located in the pole shoes and stator core. It moves with the same speed as the field magnets.

B shows the graph of the currents in the winding for the *same* position of the field-magnet poles, supposing the currents lagged

90 degrees behind the induced voltages. This position is represented by instant a_1 on the graph of B.

The current distribution in the winding is shown in diagram (m), and the rotating field is also given; being flat-topped.

If the instants b and b_1 had been chosen instead of a and a_1, the curve of (l) would have been flat-topped and that of (m) peaked. In this case the poles of the field magnets would have been a slot pitch ahead, that is, from left to right, of their position shown.

Diagram (k) gives the direction of the armature magnetic flux for diagram (m), and shows that when the *lag* of current is 90 degrees, the reaction is purely one of demagnetisation. Similarly with a *lead* of 90 degrees, a purely *magnetising* effect is produced by the winding.

In the diagram it will be noted that not all the armature ampere turns have a demagnetising effect; some of them confining their action to the stator core. This number of demagnetising ampere turns will therefore depend upon the ratio of the pole arc of the field-magnet poles to the pole pitch; and the greater this ratio, the smaller will be the number of demagnetising ampere turns.

The number of demagnetising ampere turns per pole of a three-phase alternator winding.—Let θ be the ratio of the pole arc to the pole pitch, usually from 0·6 to 0·7; Z the number of conductors per phase; and I the R.M.S. value of the current in the conductors. The poles of the rotating field of the winding are fixed with respect to those of the field-magnet system, and they merely change their form from flat to peaked topped every $\frac{1}{12f}$ of a second, f being the frequency of the alternating currents. Both flat- and peaked-topped curves have practically the same area in typical machines.

In Fig. 224 is shown one of the armature flux curves for a three-phase winding having 4 slots per pole per phase. The part of the flux represented by this curve which produces demagnetisation in the field-magnet system is the shaded area. The shaded part is taken for $\theta=0{\cdot}7$, and its area, that is, the demagnetising flux per pole of the armature winding, is

$$\frac{1{\cdot}26}{\delta}\,\mathrm{I}_m\,sal\{16{\cdot}8+8\}=44{\cdot}2\,\frac{\mathrm{I}sal}{\delta},$$

I being the R.M.S. value of the current.

The *average* value $\mathrm{A.T.}_{av}$ of the ampere turns per pole necessary to produce this flux in the air gap is, by the M.M.F. law, equal to

$$\frac{10}{4\pi}\times\frac{\delta}{8{\cdot}4sl}\times\frac{44{\cdot}2}{\delta}\,sa\mathrm{I}l=4{\cdot}2a\mathrm{I}. \quad \ldots\ldots\ldots\ldots\ (e)$$

If x is the number of conductors per pole, $\frac{x}{12}$ is the value of a, the number of conductors per slot. Therefore the number of demagnetising ampere turns of the armature winding is given by

$$\mathrm{AT}_{av}=0{\cdot}35\,\mathrm{I}x. \quad \ldots\ldots\ldots\ldots\ (f)$$

A similar relation is true for all multiphase machines in which the currents lag 90 degrees behind the induced voltages.

EXAMPLE.—A three-phase alternator has two slots per pole per phase, θ equal to 0·6, 8 conductors per slot, and a phase current of 120 amperes R.M.S. value. Calculate by the preceding method the average value of the demagnetising ampere turns per pole of its winding when its currents lag 90 degrees behind the induced voltages. *Ans.* 2116 ampere turns.

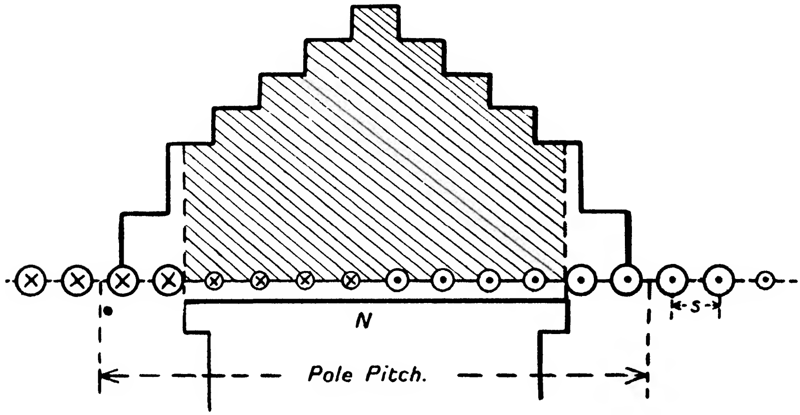

FIG. 224.—The armature reaction in a three-phase alternator for zero power-factor; current lagging.

The demagnetising ampere turns for different angles of lag may be determined in the same manner. Distortion will then be a component of the armature reaction. Equation (*f*) gives the maximum value for the armature, and is attained when the power factor is zero, which is roughly the case when the armature is short circuited.

ARMATURE REACTION IN SINGLE-PHASE ALTERNATORS.

As described on page 278, armature reaction in single-phase alternators is similar to that in two or three-phase alternators. The amplitude of the rotating flux, moving with the same speed as the field poles and in the same direction, is one-half that of the alternating magnetic flux produced by the currents in the winding.

Part of the rotating flux curve of the winding of a single-phase alternator, having 12 slots per pole, of which only two-thirds of them are used, is shown in Fig. 225.

This flux curve is stationary with respect to the poles of the field magnets, and is shown for the case of the currents lagging 90 degrees behind the induced voltages, that is, for zero power factor; a case roughly realised when the armature is short circuited. The flux curve in this case remains throughout of constant form.

It will be noted that, if all the slots were used, the armature demagnetising effect would be considerably increased.

The shaded part represents the demagnetising flux per pole of the winding; its area is

$$\frac{1\cdot26}{\delta}\frac{I_m}{2}sal\,\{16\cdot8+12\}=25\cdot7\frac{Isal}{\delta},$$

the ratio θ of the pole arc to the pole pitch being taken as 0·7. I_m is divided by 2, because the amplitude of the effective rotating flux of the winding, is one-half that due to the alternating currents.

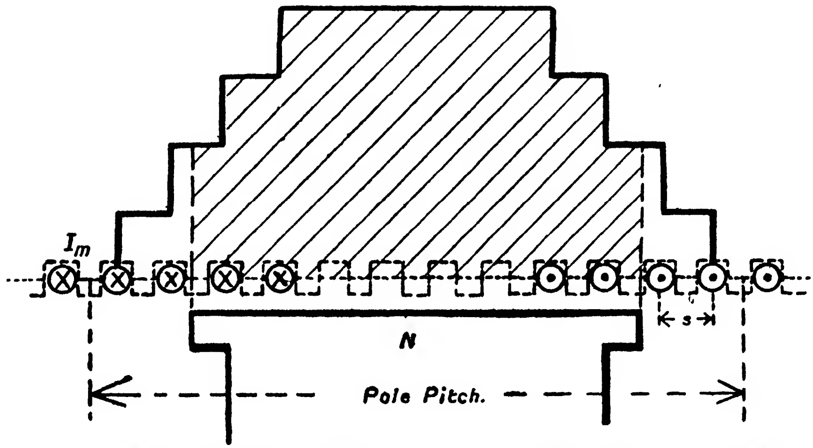

FIG. 225.—Armature reaction in a single-phase alternator; zero power-factor, current lagging.

The average value of the ampere turns $A.T._a$ necessary to produce this flux in the air gap is, by the M.M.F. law, equal to

$$\frac{10}{4\pi}\cdot\frac{\delta}{8\cdot4sl}\times\frac{25\cdot7Isal}{\delta}=2\cdot45aI.$$

If x is the number of conductors per pole, which are distributed in 8 out of 12 slots, then $a=\frac{x}{8}$.

Therefore the average value of the demagnetising ampere turns per pole of the winding for zero power factor and $\theta=0\cdot7$ is given by

$$A.T._{av}=0\cdot306\,Ix.$$

This value becomes $0\cdot325Ix$ when $\theta=0\cdot6$. For other power factors the reaction will be smaller in demagnetising effect, but a distorting effect upon the main poles will be an added component.

EXAMPLE.—Calculate the average value of the demagnetising ampere turns per pole of the armature winding of a single-phase alternator when its current lags 90 degrees behind its induced voltage. The machine has 8 poles, 48 slots of which one-third are not used, 12 conductors per slot, an armature current of 120 amperes, and a ratio of pole arc to pole pitch of value 0·6. *Ans.* 1870 ampere turns.

CHAPTER XIV.

PRINCIPLES OF ALTERNATING-CURRENT MACHINES—Continued.

In certain types of alternating-current machines there are two distinct armature windings: a stator and a rotor winding. The two chief types are alternating-commutator machines and induction motors. An important representative of the former type is the single-phase commutator machine known as the repulsion motor, and the other type is well represented by a three-phase induction motor.

The repulsion motor.—The principle of a repulsion motor may be expressed as follows: "A *closed* coil placed in an alternating magnetic field always tends to move so as to be threaded with a *minimum* number of the lines of the alternating flux; and will rotate, if arranged to do so, in that direction which will bring it *soonest* into the position just stated."

An alternating flux is acting in a direction parallel to the dotted line of diagram (*a*) (Fig. 226). Coil A will therefore rotate in the direction indicated by the arrow until its plane is coincident with the dotted line. Coil B will rotate in the opposite direction.

The coil is in unstable equilibrium when its plane is at right angles to the direction of the field, but is in a *locked* position when parallel to this direction, and requires force to displace it from that position.

By using a commutator with the rotor winding of a repulsion motor and short circuiting the brushes, a set of short-circuited coils are formed in the rotor, having their axes inclined to the direction of the alternating fluxes, and fixed in space. Thus, a constant turning effort is produced on the rotor. Changing the position of the brushes on the commutator will alter the value of this effort, and also, if necessary, the direction of rotation.

Diagram (*b*) gives the graph of the alternating flux, which for a given position of coil A, induces in it a voltage 90 degrees behind the flux as shown by curve e. A current i due to this voltage is produced in A, lagging θ degrees in phase behind it on account of self inductance.

Now i, as shown in diagram (*c*), has two components, i_2 in *quadrature* with the flux and i_1 in exact *opposition* to it. The current i_2 has no average turning effect on the coil, but the mutual action of i_1 and the flux causes the coil to rotate through a certain angle.

The flux produced by i_1 is in phase with this current, and, as indicated in diagram (*c*), is therefore in opposition to the alternating flux. Thus, at the instant when the direction of the latter is from left to right, face F_1 of A or B is a north pole and F_2 a south pole,

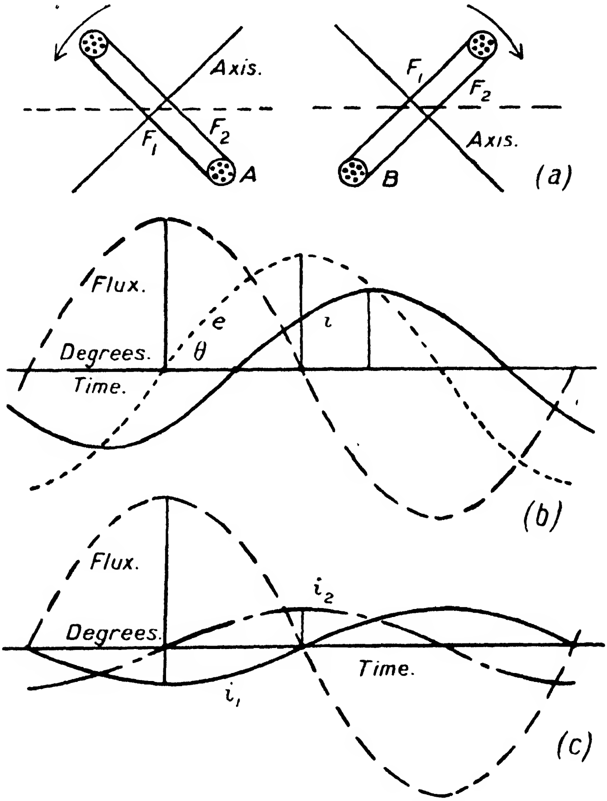

FIG. 226.—Curves for illustrating the fundamental principle of a repulsion motor.

and the coils regarded as magnets will move as indicated, that is, a repulsive turning force is produced. Similarly when the flux is from right to left, F_1 is a south and F_2 a north pole, and the turning action is the same.

A repulsion motor has a fixed stator, which carries a single-phase winding embedded in slots, and a rotor consisting of a direct-current winding connected to a commutator with its brushes short circuited. An arrangement for shifting the brushes round the commutator is provided. Fig. 227 shows the direct-current winding for a two-pole machine. It is a wave winding of pitch 11, and there are 24 conductors.

The *black* conductors form one closed coil having an axis along AB, and the *white* ones a second closed coil with axis also along AB. The first closed coil, spirals round the armature core from the left-hand brush to 2, 13, com. seg., 24, 11, com. seg., 22, 9, com. seg., 20, 7,

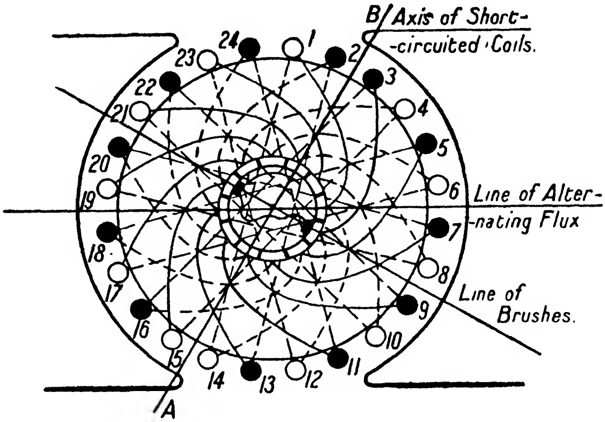

FIG. 227.—The direct-current armature of a repulsion motor.

com. seg., 18, 5, com. seg., 16, 3, com. seg., and left-hand brush. The remaining conductors and connections spiral round the core to form the second closed coil.

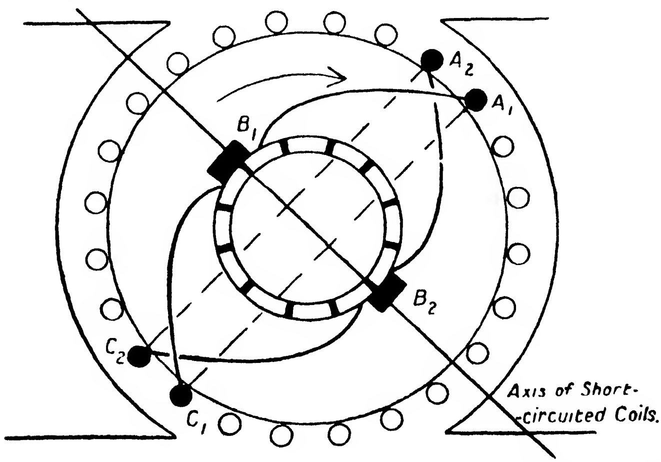

FIG. 228.—A diagram showing the opposing turning action of the coils, short circuited by the brushes, in a repulsion motor.

Each of these coils is quite independent of the other, their circuits being only joined at what may be termed one point. Since the axis

of these two coils is AB, the rotor will move *anti-clockwise* as indicated. Though the conductors of the armature change in position, the closed coils have their axis in a fixed position, as a result of using the commutator.

Those coils short circuited by the brushes, namely $A_1B_1C_1$ and $A_2B_2C_2$ (Fig. 228), have their axes at right angles to the axis of the two main closed coils, and thus tend to move *clockwise*. Hence an opposing torque is produced by the coils short circuited by the brushes.

The induction motor.—The stator of an induction motor is similar to that of an alternator. Fig. 221 may be regarded as a typical stator winding with its rotating field; in some cases the chain type of winding is used.

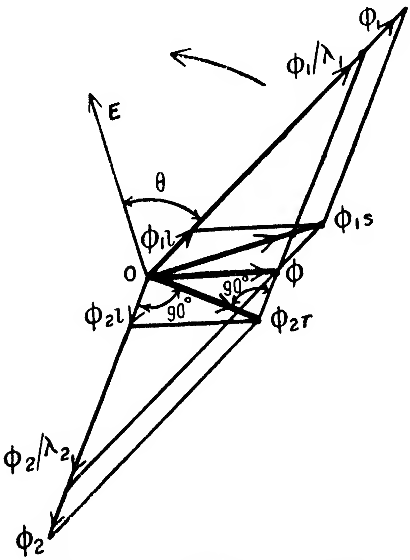

FIG. 229.—The flux diagram of an induction motor.

Its rotor winding is generally a modified direct-current wave winding, or a winding similar to that of the stator, and is supplied with starting resistances in its phases. In small machines a simple winding of short-circuited coils, or bars in parallel, known as a squirrel-cage winding, is generally used.

The rotor is wound for the same number of poles as the stator. A squirrel-cage rotor accommodates itself to the stator, and its currents produce the same number of poles as in the stator.

Both windings have rotating fields, of the same number of poles, and the same direction and speed in space. These two fields are therefore stationary with respect to each other; their relative displacement depending upon the load on the machine. The reaction of the two sets of poles upon each other causes the rotation of the rotor.

In many respects the induction motor is analogous to a static transformer with large leakage coefficients for its windings due to the air gap. The performance of an induction motor may be approximately predicted from its magnetic flux, voltage, and current vector diagrams.

To obtain these diagrams for normal operation, the effects of each of the rotor and stator currents, assumed as acting alone, are added together vectorially, and their resultant or actual effect is obtained.

In Fig. 229 are shown the various magnetic fluxes produced in the different parts of the machine. The current I_1 in the stator

acting alone will produce a magnetic flux ϕ_1 threading the stator winding of value

$$\phi_1 = k_0 Z_1 I_1,$$

according to equation (*d*), page 278 ; k_0 being a constant. This flux is in phase with I_1.

The whole of ϕ_1 will not thread the rotor winding, because part of it does not cross the air gap, and only threads the stator winding by crossing at the tops of the teeth and through the slots of the stator core.

ϕ_1 divided by the part of it which threads the rotor winding will be called the *leakage coefficient* of the stator and be represented by λ_1. ϕ_1/λ_1 is the flux which threads the rotor winding due to the current I_1 in the stator winding.

Similarly for the current I_2 in the rotor winding, only the part of its flux equal to ϕ_2/λ_2 threads the stator winding, and

$$\phi_2 = k_0 Z_2 I_2.$$

$\phi_{1l} = \phi_1 \left\{ 1 - \frac{1}{\lambda_1} \right\}$ is the leakage flux of the stator which does not thread the rotor and $\phi_{2l} = \phi_2 \left\{ 1 - \frac{1}{\lambda_2} \right\}$ is the leakage flux of the rotor.

ϕ_{1s} is therefore the *actual* flux threading the stator winding when the stator is carrying a current I_1 and the rotor a current of I_2. ϕ is the flux in the air gap, and is common to both windings.

ϕ_{1s} is also the resultant of ϕ_{1l} and ϕ, and ϕ_{2r} the resultant of ϕ_{2l} and ϕ. The angle between ϕ_{2r} and ϕ_2 is a *right* angle, and ϕ_{1s} is practically constant for all loads. This may be shown by means of the voltage diagram (Fig. 230).

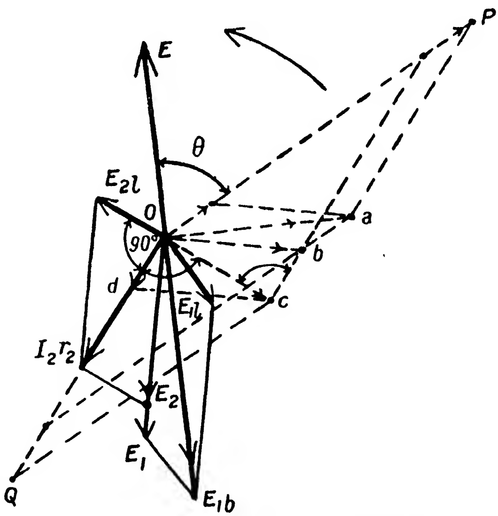

FIG. 230.—The voltage diagram of an induction motor.

In this the fluxes are drawn dotted, and the voltages produced by them are in thick lines. These voltages are always 90 degrees behind the fluxes which produce them. O*a* is ϕ_{1s}, the actual stator flux threading its winding, and which produces E_{1b}, the back voltage in the stator winding.

Assuming that the stator voltage drop is negligible compared with the applied voltage E per phase, then E_{1b} is equal and opposite to E

as shown. Now, E_{1b} is proportional to ϕ_{1s} which produces it, and $E_{1b}=E$, which is of constant value. Therefore Oa, namely ϕ_{1s}, is constant.

In the rotor winding there are two induced voltages whose resultant is equal to the rotor drop I_2r_2. These voltages are E_{2l} and E_2. The first is produced by O$d=\phi_{2l}$, and lags 90° behind this flux. E_{2l} is at right angles to Od and proportional to it. E_2 is similarly at right angles to Ob and equally proportional to it. Therefore, since Oc is the resultant of Ob and Od, I_2r_2 the resultant of E_{2l} and E_2, must be at right angles to OC$=\phi_{2r}$ and proportional to it. Thus the angle COQ is a right angle. From this and the fact that Oa is constant, the point P will be found to move on a circle.

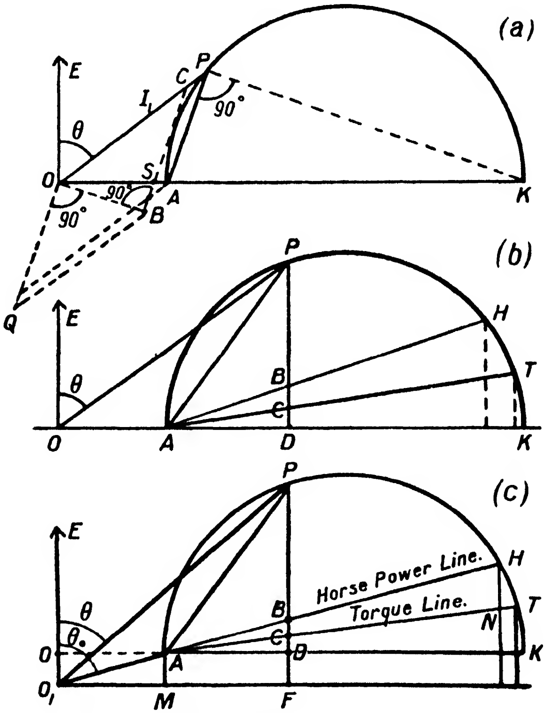

FIG. 231.—The circle diagram of an induction motor.

A current diagram, as in (*a*) of Fig. 231, may be drawn from the flux diagram by dividing the flux lines by a constant. For

$$\phi_1=k_0Z_1I_1,$$
$$\text{and}\quad \phi_2=k_0Z_2I_2.$$

If the number of slots per pole per phase is different in the two windings, k_0 will not be quite the same in value, as it includes the

distribution or breadth coefficient of the winding. This difference, however, may be practically neglected, especially as the leakage coefficients cannot be determined with great accuracy, and also on account of the assumptions of negligible drop in the stator and constancy of permeability of the magnetic circuits of the cores.

Dividing the flux values by $k_0 Z_1$ will make OP equal to I_1, and AP will be equal to $\frac{Z_2 I_2}{Z_1 \lambda_2}$, that is,

$$I_2 = \frac{Z_1 \lambda_2}{Z_2} . AP. \qquad (a)$$

The *circle diagram.*—From P draw PK at right angles to AP. The triangles OSB and AKP are similar. Therefore

$$\frac{OS}{AK} = \frac{SB}{AP} = \frac{BC - SC}{AP} = \frac{BC}{AP} - \frac{SC}{AP} = \lambda_2 - \frac{1}{\lambda_1}.$$

Also, $$\frac{OS}{OA} = \frac{1}{\lambda_1}; \text{ or } OA = OS . \lambda_1,$$

and $$\therefore \frac{OA}{AK} = \lambda_1 \lambda_2 - 1 = \sigma.$$

But OA is constant; therefore AK is constant, and represents the diameter of the semi-circle drawn.

σ is sometimes termed the *dispersive coefficient* of the motor. Its value ranges from 0·08 in small machines to 0·03 in large ones.

In *diagram* (*b*) OP is the stator current, θ the phase difference between the phase current and applied phase voltage. The input power of a three-phase motor is $\sqrt{3}\, I_1 E_l \cos\theta$, E_l being the line voltage and I_1 the line current. Now, $I_1 \cos\theta = PD$ on the circle diagram. Therefore the input power is

$$\sqrt{3}\, E_l \times PD,$$

or PD represents the input power of the machine.

The *copper loss* lines AH and AT are drawn by making

$$CD = \frac{3 . OP^2 r_1}{\sqrt{3}\, E_l} = \frac{\sqrt{3}\, OP^2 r_1}{E_l}$$

and $$CB = \frac{3 I_2^2 r_2}{\sqrt{3}\, E_l} = \frac{\sqrt{3}\, I_2^2 r_2}{E_l},$$

in which $$I_2 = \frac{Z_1 \lambda_2}{Z_2} . AP.$$

BD is therefore the sum of the total copper losses in the motor divided by $\sqrt{3} . E_l$.

Since CB $\propto$ AP², and from diagram (*b*)

$$AP^2 = PD^2 + AD^2 = AD \, . \, DK + AD^2 = AD \, . \, AK,$$

$$\therefore \; CB \propto AD \, . \, AK \propto AD.$$

So that, if AT is a straight line, AH will also be a straight line.

For practical purposes, AT is usually taken as a straight line, which it is approximately; for $OP^2 = OA^2 + AP^2$ nearly, and so

$$\frac{3r_1 OP^2}{\sqrt{3}\,E_l} = \frac{3r_1 OA^2}{\sqrt{3}\,E_l} + \frac{3r_1 AP^2}{\sqrt{3}\,E_l};$$

$$\therefore \; CD = K + \frac{3r_1 AP^2}{\sqrt{3}\,E_l}.$$

Thus, $$CD - K \propto AP^2 \propto CB \propto AD.$$

If, then, CD is taken equal to the stator copper loss, *minus* the no-load stator copper loss, both terms being divided by $\sqrt{3}\,E_l$, AT will be very nearly a straight line.

Diagram (*c*) gives a more complete circle diagram, as diagram (*b*) is drawn on the assumption that there are no iron, windage, and friction losses. Let P be the sum of these three losses, and also the small no-load copper loss. P is practically constant for all loads, as the motor is nearly a constant speed machine, and the actual flux in the stator and rotor cores is nearly constant.

Now, P being an actual loss, must be represented by a component of the stator current *in phase* with the applied voltage E, and this component from the power formula for three-phase balanced loads is

$$\frac{P}{\sqrt{3}\,E_l}.$$

OO_1 in the diagram is therefore made equal to this component.

Also, since OP is the stator current neglecting P, then O_1P, which is the resultant of OO_1 and OP, is the true stator current.

For a *given stator current* O_1P, the detailed losses in the machine are $BC \, . \, \sqrt{3}\,E_l$ the rotor copper loss; $CD \, . \, \sqrt{3}\,E_l$ the stator copper loss; $DF \, . \, \sqrt{3}\,E_l = P$ the iron windage, friction, and no-load copper loss. Also, $PF \, . \, \sqrt{3}\,E_l$ is the input of the motor. $PB\sqrt{3}\,E_l$ is therefore the power absorbed by the load on the machine, so that the

$$\text{B.H.P.} = \frac{PB\sqrt{3}\,E_l}{746}.$$

The *efficiency* is also given by $\frac{PB}{PF}$ and the *power factor* by $\frac{PF}{O_1P}$.

If N_1 is the synchronous speed of the induction motor, and N_2 that of the rotor, *s* the slip, is given by

$$s = \frac{N_1 - N_2}{N_1},$$

N_1 in revolutions per minute being found from the relation

$$N_1 = \frac{120f}{p}.$$

Relation between slip, torque, and rotor copper loss.—In *diagram* (*c*) (Fig. 231) the line PB represents the B.H.P. when the input current is O_1P. By inserting a *certain* extra resistance of r ohms in each of the rotor phases, and keeping the torque on the motor constant, the rotor may be just brought to rest, still exerting the same torque. By this insertion practically no change has been produced in either the stator or rotor currents, and the input is the same as before. For though the iron loss has been increased because the rotor iron is stationary, the windage and friction loss has become zero. Roughly this increase and decrease balance, and any difference will be comparatively small and practically negligible.

With r in the rotor phases, the slip is unity and the line AH coincides with AP. If T was the torque in lbs.-ft. and s the corresponding normal slip,

$$T \times 2\pi N_2 = T \times 2\pi(1-s)N_1 = 3I_2^2 r \times \tfrac{33000}{746},$$

$$TN_1(1-s) = 7 \times 3I_2^2 r. \quad \text{......................................} (b)$$

For the electrical load $3I_2^2r$ has replaced the brake horse-power, which was developed before r was inserted.

Again, since I_1, I_2, and their phase differences, with respect to each other and the voltages of the windings, are unchanged by thus bringing the rotor to standstill, the flux diagram of Fig. 229 is also unchanged. In the voltage diagram (Fig. 230), E_2 and E_{2l} are changed because the frequency of the fluxes producing them, instead of being f_2 as in normal operation, is f_1; the rotor being at rest.

The voltages E_{2l} and E_2 are thus f_1/f_2 times as large as before, but their directions, and also the direction of I_2, are the same as for normal operation.

The resultant of E_{2l} and E_2 for the case of standstill is $I_2(r_2+r)$, and this also must be f_1/f_2 times the value for normal operation, namely I_2r_2. Therefore

$$r_2\frac{f_1}{f_2} = r_2 + r.$$

Now, $$N_1 - N_2 = \frac{120f_2}{p}$$

as $N_1 - N_2$ is the relative speed of the rotor winding with respect to the rotating flux, and $N_1 = \frac{120f_1}{p}$.

Therefore, $$s = \frac{N_1 - N_2}{N_1} = \frac{f_2}{f_1}.$$

Also, $$r_2 \frac{1}{s} = r_2 + r,$$

so that $$r = r_2 \cdot \frac{1-s}{s}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

From equations (*b*) and (*c*),

$$N_1 Ts = 7 \times 3I_2^2 r_2, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

T being the torque in lbs.-ft., and s the corresponding slip for normal operation.

In *diagram* (*c*) (Fig. 231)

$$PB\sqrt{3}\,E_l = 3I_2^2 r \quad \text{and} \quad BC\sqrt{3}\,E_l = 3I_2^2 r_2;$$

$$\therefore \frac{PB}{BC} = \frac{r}{r_2} = \frac{1-s}{s}.$$

From which, $\frac{BC}{PC} = s$ and $\frac{PB}{PC} = 1 - s = \frac{N_2}{N_1}$.

Thus, $\frac{PB}{PC}$ is the ratio of the *actual*, to the synchronous speed of the motor.

Also, $$T \times 2\pi \frac{PB}{PC} N_1 = \frac{33000}{746} \sqrt{3}\,E_l\,PB;$$

$$\therefore T = 7\sqrt{3} \cdot \frac{E_l}{N_1} \cdot PC.$$

Thus PC represents the torque, and AT is therefore termed the *torque* line. The maximum torque is called the *pull-out* torque. HN represents the starting torque, if there is no extra or starting resistances in the rotor winding.

EXAMPLES.—1. The slip of an induction motor is 2 per cent. when its rotor current is of value 80 amperes. The rotor resistance is 0·02 ohm per phase. If the machine has 6 poles, and is supplied with voltage at frequency 50, calculate the torque for this slip. *Ans.* 134·4 lbs.-ft.

2. An induction motor is supplied from voltage mains at frequency 50. It has 8 poles, and the rotor resistance is 0·04 ohm per phase. The slip is 3 per cent. when the load is 40 B.H.P. Find the rotor current and speed for this load. *Ans.* 87·8 amperes and 727·5 R.P.M.

An example to illustrate the use of the circle diagram.—A four-pole three-phase induction motor, when supplied from mains of 346 volts and frequency 50, has a no-load current of 8 amperes at power factor 0·2; a full-load current of 27 amperes; and an *ideal* maximum current, O_1K on the circle diagram, of 175 amperes. The resistance per phase of the stator winding is 0·25 ohm, and that of the rotor 0·07 ohm. The ratio of the numbers of stator and rotor conductors is 2. Obtain data of its full-load performance; the value of the starting torque and the maximum or pull-out torque; and the maximum brake horse-power.

The iron, friction, windage, and no-load copper loss has a total value of

$$\sqrt{3} \times 346 \times 8 \times 0{\cdot}2 = 960 \text{ watts.}$$

AM, shown in Fig. 232, is $OA \times 0{\cdot}2 = 8 \times 0{\cdot}2 = 1{\cdot}6$ amperes. OM is nearly equal to 8 amperes, so that S the centre of the circle is given by $\frac{175-8}{2} = 83{\cdot}5$ amperes, measured from A along AZ.

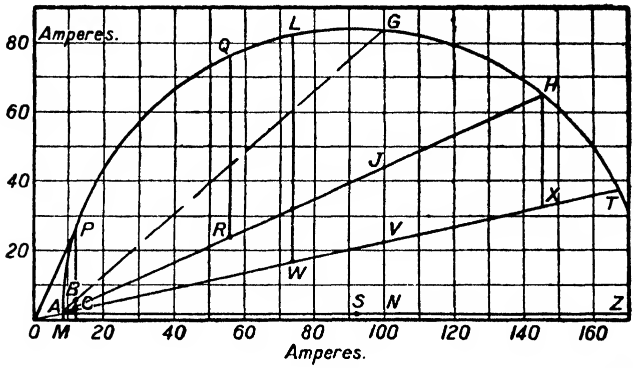

Fig. 232.—The circle diagram of an induction motor, to illustrate its use.

AH, the horse-power line, and AT, the torque line, are obtained by taking any suitable point G, measuring OG, and erecting VN equal to

$$\frac{3OG^2 \times 0{\cdot}25}{\sqrt{3} \times 346} = \frac{OG^2 \times 0{\cdot}25}{200}.$$

JV is also made equal to

$$\frac{3I_2^2 \times 0{\cdot}07}{\sqrt{3} \times 346} = \frac{I_2^2 \times 0{\cdot}07}{200},$$

I_2 being equal to $AG\lambda_2 \frac{Z_1}{Z_2}$, and λ_2 taken as about $1{\cdot}03$.

In Fig. 233 is shown a small part of the circle diagram on a larger scale, from which the performance at full and smaller loads may be more accurately obtained. From this diagram, the following values for *full* load were found:

$$\textit{Efficiency} = 100 \, . \, \frac{PB}{PF} = 85{\cdot}5 \text{ per cent.}$$

$$\textit{Power-factor} = \frac{PF}{OP} = 0{\cdot}895.$$

$$\text{B.H.P.} = \frac{PB\sqrt{3} \times 346}{746} = 16{\cdot}6.$$

$$\textit{Synchronous speed } N_1 = \frac{120f}{p} = 1500 \text{ R.P.M.}$$

$$\textit{Actual speed } N_2 = N_1 \frac{PB}{PC} = 1430 \text{ R.P.M.}$$

$$\textit{Slip} = \frac{N_1 - N_2}{N_1} . 100 = 4{\cdot}7 \text{ per cent.}$$

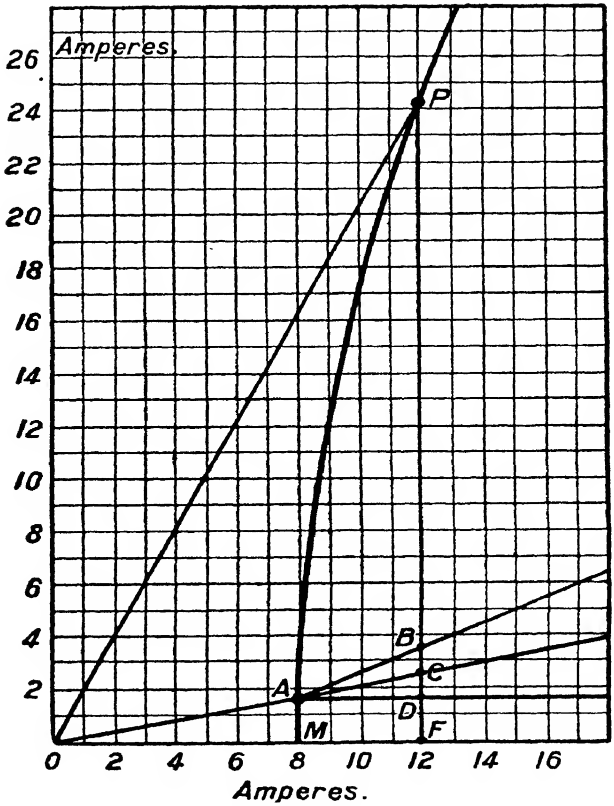

FIG. 233.—The part of the circle diagram representing normal operation.

Torque T *lbs.-ft.* is given by

$$\text{B.H.P.} = \frac{T \times 2\pi N_2}{33000};$$

$$\therefore \; T = 61 \text{ lbs.-ft.}$$

$$\textit{Input power} = PF \times \sqrt{3} \times 346 = 14{\cdot}52 \text{ K.W.}$$

Then, from Fig. 232, the *maximum output* is represented by QR, and the *over-load capacity* is

$$\frac{QR}{PB} = 2{\cdot}5 \text{ times the full-load B.H.P., and of value } 41{\cdot}5 \text{ B.H.P.}$$

The *maximum* or *pull-out torque* is represented by LW, and is

$$\frac{LW}{PC} = 3 \text{ times the full-load torque, and of value } 183 \text{ lbs.-ft.}$$

The *starting torque* is HX, and is equal to the full-load torque multiplied by $\frac{HX}{PC} = 1{\cdot}45$, that is, equal to 88 lbs.-ft.

In the same way the data for other loads may be found, and graphs may be drawn illustrating the performance of the machine.

EXAMPLE.—A 12-pole three-phase induction motor, when supplied from mains of voltage 500 and frequency 50, has a no-load current of 60 amperes at power factor 0·25, a full-load current 180 amperes, and an *ideal* maximum current of 1200 amperes. The resistance per phase of the stator winding is 0·05 ohm, and that of the rotor 0·012 ohm. The ratio of the numbers of stator and rotor conductors is 2. Obtain by the preceding method the complete graphs of speed, power factor, efficiency, and torque *against* brake horse-power. Also indicate the full-load point on each curve, and determine the pull-out torque, and over-load capacity of the motor.

ROTARY CONVERTER.

The principle of a rotary converter has been described in Chapter XIII. page 260. It has, however, one feature somewhat different to other machines due to the superimposition of the alternating and direct currents in the winding. This in general produces a *diminution* in the copper loss, heating, and armature reaction.

This action will now be considered in the case of the winding illustrated in Fig. 234.

To do this, a good method is to cut out a piece of cardboard, as shown in Fig. 235, to represent the poles, brushes, and induced voltages due to the clockwise rotation of the winding. This is moved from right to left, and the winding is kept stationary. The dotted lines represent the conductors over which the cardboard is to be moved.

The arrows on the cardboard indicate the direction of the induced voltages e_1, e_2, etc., in the conductors situated with respect to the poles as shown. When fed by direct current, the latter enters at the *positive* brushes and leave at the *negative* ones.

The cardboard should only have arrows and the winding none. Those shown in Fig. 234 should be regarded as on the cardboard.

One of the parallel sections, namely from A to B between rings R_1 and R_3, will be considered, and the starting position of the

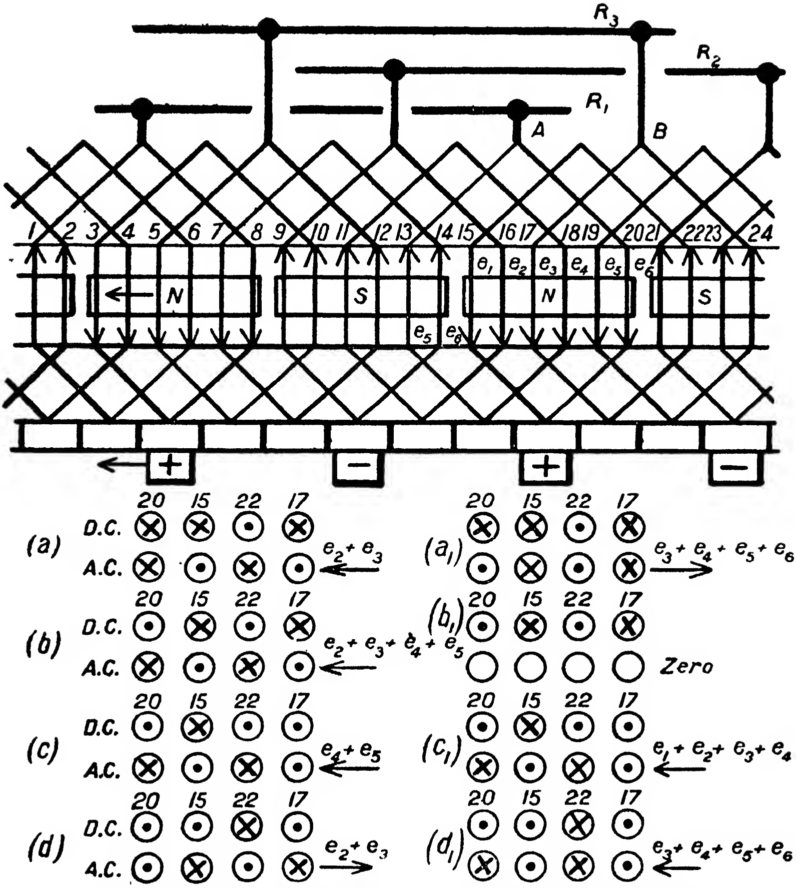

FIG. 234.—The directions of the direct and alternating currents in the armature of a rotary converter, for different power factors.

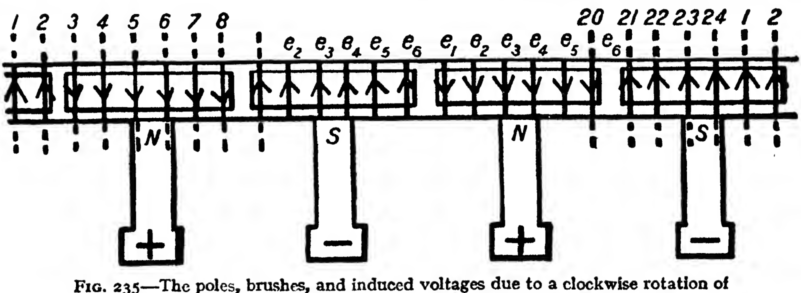

FIG. 235—The poles, brushes, and induced voltages due to a clockwise rotation of the armature, for a rotary converter.

movement of the cardboard will be from that occupied by the poles and brushes shown in Fig. 234.

Direct current is fed into the armature, and its direction through the conductors 20, 15, 22, and 17 in series between A and B is shown by dots and crosses in the first line of each of the diagrams (*a*), (*b*), (*c*), and (*d*); the second line gives the direction of the alternating current at the same instant.

Power factor unity.—In this case the alternating currents are in phase with the induced voltages. Diagrams (*a*), (*b*), (*c*) and (*d*) are for intervals represented by twice the slot pitch, or $\frac{1}{3}$ the pole pitch, or $\frac{1}{6f}$ second; f being the frequency of the alternating currents.

It will be noted that at certain instants the conductors nearest the slip rings have both alternating and direct currents in them in the *same* direction, while those in the other positions, that is, between the rings, have them opposed. This may be more clearly shown by taking the interval as represented by *one* slot pitch instead of two slot pitches.

Thus the conductors nearest the rings will become hotter than those midway between the rings. As the number of phases is increased, this difference of heating will be found to be less pronounced. In a single-phase rotary converter it is considerable, and constitutes one of its chief disadvantages. The heating is much more uniform in a six-phase rotary converter.

Power factor zero.—Here the alternating current lags 90 degrees behind the induced voltages. The cardboard will then have to be moved back from left to right through three slot pitches to give the alternating currents corresponding to the same direct currents of diagrams (*a*), (*b*), (*c*), and (*d*).

Diagrams (a_1), (b_1), (c_1), and (d_1) give the relative distribution for zero power factor. In this case the alternating and direct currents are nearly always in the same direction. This more clearly appears by taking the interval of the displacement of the cardboard as represented by one instead of two slots. The heating, copper loss, and armature reaction will therefore be excessive; much greater than if the converter is used either as a pure alternating or direct machine of the same output.

In the same manner, by moving the strip of cardboard backward through a smaller displacement, the cases of intermediate power factors may be studied.

The heating will be found to diminish rapidly as unity power factor is approached, becoming much smaller than that of the converter when used as a direct-current machine of the same output. In a three-phase converter the copper loss for unity power factor will be about 0·6 of that, when it is used as a direct-current machine. For low-power factors the copper loss will be excessive.

The rotating field of the winding of the rotary converter.—In rotary converters of two or more phases a rotating field similar to that of an induction motor is produced by the rotating armature. This

field rotates with speed $\frac{120f}{p}$ in the opposite direction to the rotation of the winding which revolves with N revolutions per minute ; f being the frequency of the alternating currents in the winding.

Now $f=\frac{N}{60}\cdot\frac{p}{2}=\frac{Np}{120}$. Therefore the rotating field moves across the gap with the same speed that the rotor does, but in the opposite direction, that is, this field due to the alternating currents in the winding is fixed in space like the poles of the machine.

Regarding the direct and alternating currents as acting alone. Each will produce a magnetic field of the same number of poles as the machine and fixed in space. The combined effect of these fields upon the main field, will represent the armature reaction of the machine.

THE SYNCHRONOUS MOTOR.

A synchronous motor is of the same construction as an alternating-current generator. In general its field poles revolve, and the armature or stator is fixed. This stator has a winding similar to that of an alternator or an induction motor.

In synchronous motors of two or three phases the alternating currents fed into the stator winding produce a rotating field, similar to that of an induction motor, which moves in the same direction and with the same speed as the revolving poles.

This type of motor must be first run up to a speed which produces alternating voltages in its winding of practically the same frequency, phase, and value as those of the supply mains, before it is switched on to them ; this is termed *synchronising* the motor.

By altering the exciting current of the field poles when the machine is on a given load, the power factor of the input power may be changed. The power factor will increase as the exciting current is raised from a low value until a certain value will give unity power factor. Further increase of the exciting current will be accompanied by a decreasing power factor.

It is found that the input current lags behind the applied voltage for weak excitation, and leads it for strong excitation. Also, since the applied voltage is constant, the exciting current which gives unity power factor will in general give maximum efficiency, as the input current is then a minimum for the given load. This likewise applies to the case of a rotary converter fed on its three-phase side and supplying direct current.

Armature reaction has been already considered in the case of a three-phase alternator, and it was shown that a current which lagged behind the induced voltage in the winding produced a demagnetising effect of maximum value when the lag was 90 degrees. In a similar way it may be shown that if the current leads the induced voltage, a magnetising effect would be produced.

Suppose this generator is synchronised and switched on to the

supply mains. It is now a synchronous motor, and continues to run in the same direction as before with the same speed, so that the induced voltage is practically unchanged in value and phase. The phase currents in the winding now lie between the applied voltage and the induced voltage, for the resultant of these two voltages is the phase drop.

It therefore follows that when the currents of the synchronous motor lag behind the applied voltage, its armature reaction will have a magnetising component, and when they lead the applied voltage, a demagnetising component.

Diagram (*a*) in Fig. 236, represents the machine running as a generator before being synchronised. The dots and crosses represent

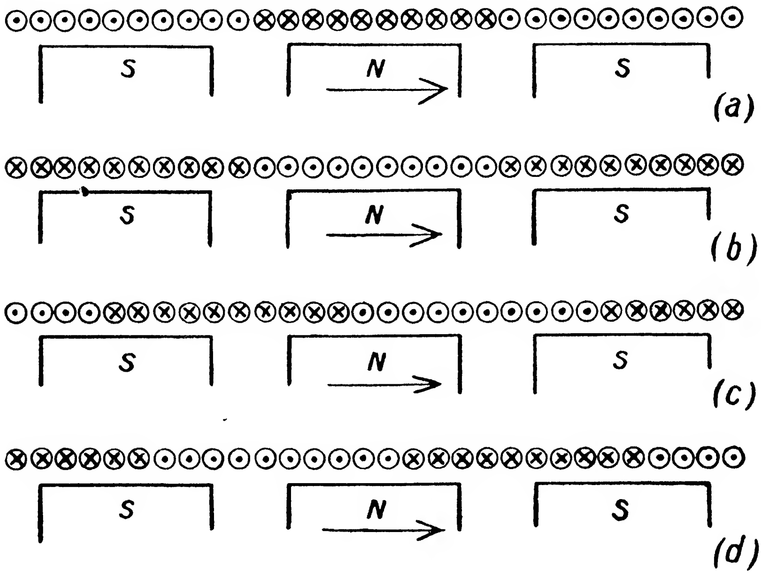

FIG. 236.—The directions of the armature currents in a synchronous motor, and their magnetic action upon the field poles, for lagging and leading currents.

the directions of the induced voltages. Next, the machine is supposed to run as a synchronous motor.

Diagram (*b*) represents the case in which the feeding currents from the mains are in phase with the applied voltage. The effect of armature reaction is pure distortion.

Diagram (*c*) shows the winding with input currents leading the applied voltage. In this case the armature reaction has a demagnetising as well as a distorting effect.

Diagram (*d*) shows the magnetising effect of input currents lagging behind the applied voltage.

Consider a synchronous motor on constant load. An alteration

in its exciting current will alter the strength of the field poles, and tend to change the back or induced voltage in the winding. This voltage, however, cannot change to any appreciable extent, because it practically balances the constant applied voltage, as the armature drop is almost negligible compared with either of these voltages.

It follows, then, that the use of a large exciting current must be accompanied by a large demagnetising effect of the winding, which can only occur by the input currents becoming leading ones, that is, leading the applied voltage.

Similarly for a small exciting current, in order that the induced voltages may be maintained at balancing value, a magnetising effect is needed from the winding. This is brought about by the currents lagging behind the applied voltage.

Since the load is constant, a change of exciting current will produce a change only in the *wattless* component of the input current; the effective component representing the load being constant. This is illustrated in Fig. 237 for the cases of lagging and leading input

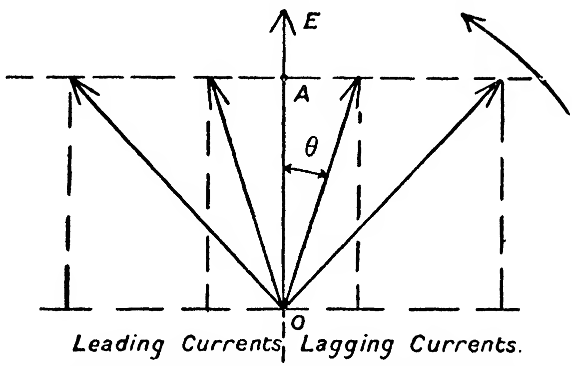

FIG. 237.—The variation of input current with power factor, for a synchronous motor on a constant load.

currents; OA being the effective component of each current shown by the full lines.

Synchronous motors are sometimes employed to improve the power factor of supply systems by this method of over-excitation. In the case of a plant consisting largely of synchronous motors, a power factor of unity may be easily obtained by field current adjustment.

Synchronising a synchronous motor or rotary converter.—There are several methods of synchronising a rotary converter, synchronous motor, or an alternator to work in parallel with others. For laboratory testing, one of the best and simplest method is that of *Siemens* and *Halske*, in which three lamps arranged to form the corners of a triangle are used. A voltmeter across the terminals of one lamp enables one to more exactly determine the nearest approach to synchronism.

The arrangement is shown in Fig. 238. A, B, and C represent the three phases feeding into the supply mains; and A_1, B_1, and C_1 the

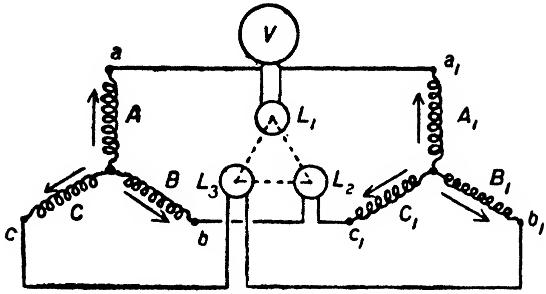

FIG. 238.—The lamp arrangement for synchronising.

three phases of the machine to be switched on to the mains after it is synchronised. If a is joined to a_1 as shown, b to b_1, and c to c_1, then, when the machine has the same voltage, phase, and frequency, the lamps will all be dark and the voltmeter at zero; the machine would

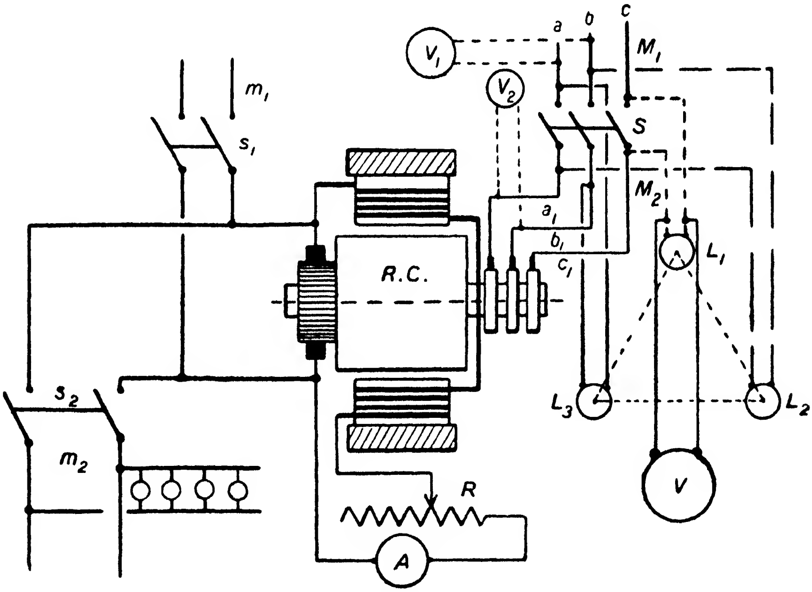

FIG. 239.—The synchronising arrangement for a rotary converter.

then be in perfect synchronism with the mains. When the frequencies differ, the lamps will all darken simultaneously and gradually brighten,

then darken, and so on. If this is very slow, the switch may be closed at maximum darkness.

Instead of connecting b to b_1 and c to c_1, if they are connected as in the diagram, there will be a rotation of darkness and brightness. For instance, L_1 will be bright first, then L_2, next L_3, or the reverse rotation will take place; the direction depending upon the speed of the machine to be synchronised. When the rotation is *very slow*, the switch is closed when lamp L_1 is of maximum darkness, or the deflection of V at, or near zero.

These lamps, L_1, L_2, and L_3, are placed across the ends of the triple pole switch S shown in Fig. 239, which illustrates the method of synchronising a rotary converter. This machine is run up to synchronous speed from direct-current mains m_1 by closing s_1, operating the starter not shown, and adjusting R. When V_2 and V_1 are indicating about the same value, S is closed at the instant L_1 is quite dark or when the reading of V is nearly zero. At the same time s_1 is opened, and then, if necessary, the machine may be put on load by closing s_2.

The rotation of brightness and darkness in the three lamps, forming the synchroniser, may readily be shown graphically, as in Fig. 240.

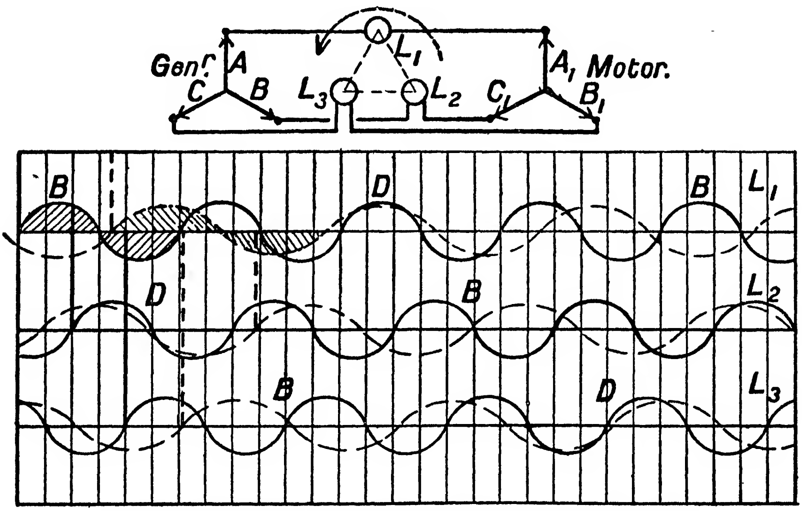

FIG. 240.—The rotation of brightness and darkness in a lamp synchroniser.

The frequency of the generator is considered to be 60, and that of the motor 45, that is, the speed of the machine to be synchronised is too small.

Now, lamp L_1 is fed by current proportional to the resultant voltage in the phases A and A_1. It is therefore bright at B, because the two-phase voltages are then in series; it is dark at D.

After a time represented by 24 spaces, L_1 is at maximum brightness again. The interval between brightness and brightness is therefore

$$\tfrac{24}{360} = \tfrac{1}{15},$$

that is, L_1 is bright 15 times a second.

Generally, if f_1 is the supply frequency and f_2 that of the machine, $f_1 - f_2$ is the frequency of the state of maximum brightness for the lamps.

A similar consideration applies to lamps L_2 and L_3. The rotation of maximum brightness or darkness will in this case be *anti-clockwise*, or in the order of L_1, L_3, L_2. Similarly, if the speed of the incoming machine is too high, the rotation will be clockwise. The same deduction may be made when the armature of the machine to be synchronised is mesh connected, as in the case of a rotary converter.

In the case of high-voltage mains, the use of small step-down transformers becomes necessary to feed the synchronising lamps.

EXAMPLE.—A rotary converter is nearly synchronised with supply mains of frequency 50. The interval between maximum brightness and maximum brightness of the synchronising lamps is 2 seconds, and the synchroniser indicates that the speed of the converter is too low. Calculate this speed if the machine has 6 poles. *Ans.* 990 R.P.M.

CHAPTER XV.

THE TESTING OF ALTERNATING-CURRENT MACHINES.

THE testing of alternating-current machines is generally somewhat more difficult than those of direct current, chiefly on account of the effects of self inductance, frequency, number of phases, and sometimes capacity. There is, however, much in common in the methods of testing these two classes of machines, and in some tests they are identical. The testing of some of the chief types of alternating-current machines will be considered in this and the next chapter.

TESTING A SINGLE-PHASE ALTERNATOR.

The machine is run at rated speed and separately excited, either by a small direct-current generator on the shaft of the alternator or from outside direct-current mains. The exciting current, which at rated speed produces rated voltage and current for a non-inductive load, is known or should be determined. In a 3 K.W. single-phase alternator, which will be called *machine C*, this exciting current was 1·8 amperes.

Resistances of armature and field coils: copper losses.—The resistances r_a and r_f may be found from the drop test or by a post-office resistance set. In *machine C*, the drop test was used and r_a taken for different currents. The values agreed very closely. r_f was also obtained by the same method. The values found were 0·234 ohm for the armature and 40·5 ohms for the field coils.

To allow for the effect of eddy currents in the conductors, the armature resistance was increased by 5 per cent. For conductors of large diameter a further allowance for skin effect would be needed. In this case the conductor was three strands of wire 0·045 inch in diameter. r_a will be taken as 0·246 ohm.

The *copper losses* for the armature and field coils will therefore be

$$0{\cdot}246 I^2 + 1{\cdot}8^2 \times 40{\cdot}5 = (0{\cdot}246 I^2 + 131) \text{ watts},$$

I being the output current.

Self inductance of the armature winding.—An alternating current is supplied to the winding in series with a rheostat and ammeter;

a frequency meter is also used. The field coils are excited with rated current. A voltmeter indicates e, the drop across the winding, for different currents. The value of e and also I is found to change by moving the armature through a small angle, and the average value of the maximum and minimum values of e and I should be taken each time.

The following table is for *machine C.* In this case f, the frequency of the supply current, was 51·5, and L_a, the self inductance of the winding, was calculated from

$$L_a = \frac{1}{2\pi f I}\sqrt{e^2 - (Ir_a)^2} \text{ henrys.}$$

During the test the field coils were excited with rated current 1·8 amperes.

I	10·0	14·3	18·8	26·1
$2\pi f I$	3230	4630	6090	8440
e	7·8	11·5	15·1	21·9
e^2	61	132	227	480
$(Ir_a)^2$	6	12	21	41
Diff.	55	120	206	439
Sq. root	7·4	10·9	14·4	20·9
L_a (henrys)	0·00229	0·00235	0·00237	0·00247

Performance or regulation test on a non-inductive load.—In this test, rated exciting current 1·8 amperes and frequency 50 were used. The values obtained for *machine C* were:

I	0	6	12	17	21·6	26	30
E_t	118·5	116	113	110	106·3	103·5	100

The frequency was read on a frequency meter and kept constant. It was also calculated from the speed 1500 R.P.M. and the number of poles, which was 4. Thus

$$f = \frac{np}{2} = \frac{1500}{60} \times \frac{4}{2} = 50.$$

The *regulation* for constant excitation of value 1·8 amperes is therefore

$$\frac{118·5 - 100}{100} \times 100 = 18·5 \text{ per cent.}$$

The *internal voltage* E_i is given by

$$E_i = I\sqrt{(R+r_a)^2 + (L_a \cdot 2\pi f)^2},$$

the load being non-inductive and of resistance R. For each current used in the test, $R = \frac{E_t}{I}$. The table for *machine C* is :

E_t - -	118·5	116	113	110	106·3	103·5	100
I - -	0	6	12	17	21·6	26	30
R - -		19·3	9·42	6·47	4·93	3·99	3·33
$(R+r_a)^2$ -		381	93·4	45·1	26·8	18·0	12·8
$(L_a 2\pi f)^2$ -		0 53	0·55	0·55	0·56	0·6	0·6
Sum -		381·5	94·0	45·7	27·4	18·6	13·4
Sq. root -		19·6	9·7	6·75	5·25	4·32	3·66
E_i - -	118·5	117·6	116·4	114·5	113·5	112·1	110·0

The regulation curve relating E_t and I, and the total characteristic curve relating E_i and I are shown in Fig. 241.

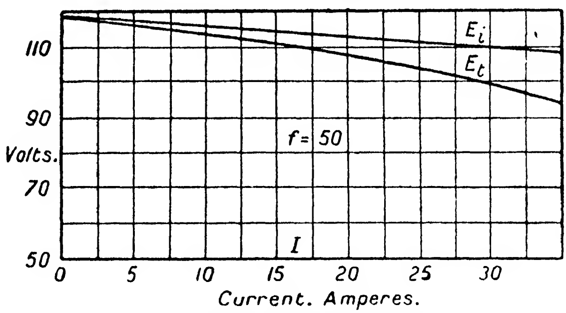

FIG. 241.—The regulation and total characteristic curves of the single phase alternator, of rated output 3 K.W.

No-load excitation test.—In this test the machine is run at *rated speed*, and the voltage E_i across its open terminals is read for different values of I_f, the exciting current. A.T. are the ampere turns *per pole*. The results for *machine C* at frequency 50 were :

E_i -	53	62	75	91	109	118	128
I_f -	0·6	0·71	0·9	1·13	1·5	1·78	2·21
A.T. -	300	355	450	565	750	890	1105

and the turns per pole were 500. Fig. 242 shows the graph relating E_i and A.T.

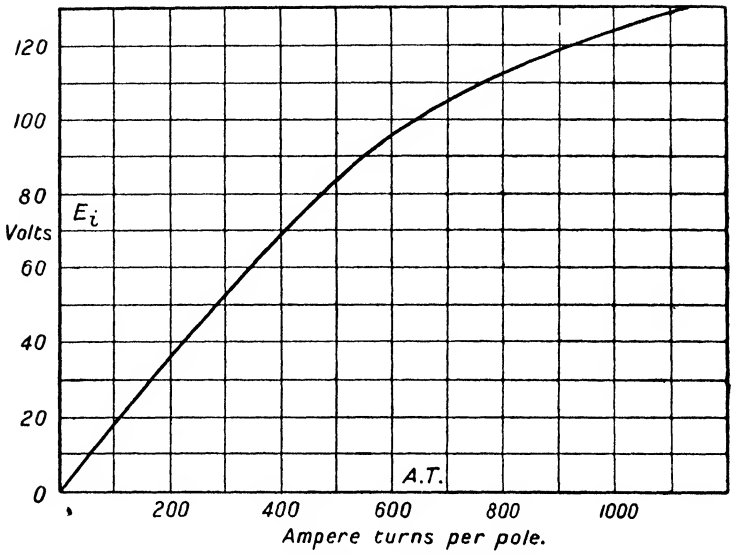

FIG. 242.—The no-load excitation curve of the single-phase alternator.

Armature reaction: non-inductive load.—From the results of the load and excitation tests the following table, which is for *machine C*, may be constructed. I_f in this test was 1·8 amperes.

I - - -	0	6	12	17	21·6	26	30
E_i - - -	118·5	117·6	116·4	114·5	113·5	112·1	110·0
Actual A.T. -	900	900	900	900	900	900	900
Effective A.T. -	900	880	860	830	810	790	760
Q - - -	0	20	40	70	90	110	140

The effective ampere turns are obtained from Fig. 242, and Q is the difference between the actual ampere turns used in the load test and those effective to produce E_i. Q is therefore the number of ampere turns *per pole* neutralised by armature reaction. The graph of Q and I is given in Fig. 243.

Regulation and armature reaction on loads of different power factors.—Let R be the load resistance and L its self inductance; E_t the terminal voltage and E_i the internal voltage of the machine, for this load at rated speed. Then the power factor $\cos\theta$ of the load circuit is given by

$$\cos\theta = \frac{IR}{E_t}.$$

This follows from the relation

$$I^2R = E_t I \cos\theta,$$

in which each term represents the external load. Thus, by measuring R, I, and E_t, the power factor may be calculated.

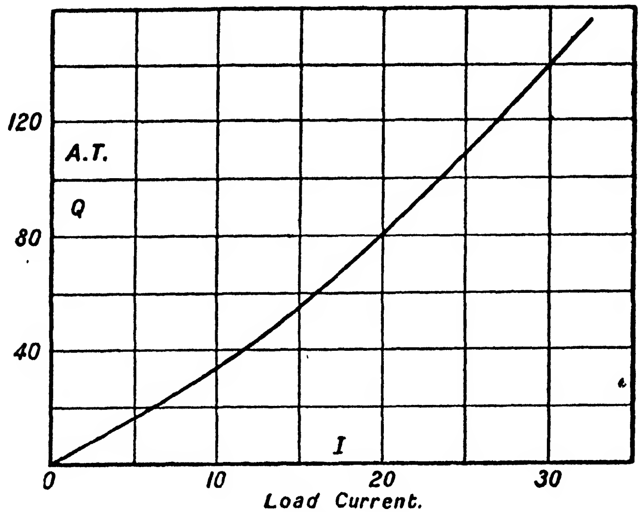

FIG. 243.—The ampere turns per pole neutralised by armature reaction in the single-phase alternator; the power factor being unity.

A quicker method is to use a wattmeter, ammeter, and voltmeter on the load circuit, and find *cos θ* from

$$\cos\theta = \frac{P}{E_t I},$$

P being the reading of the wattmeter and the value of the load.

For *regulation*, E_0, the no-load voltage at rated speed and excitation should be read; I_f should also be noted. An inductive load of value P is then put on the machine. If P is of low power factor, it must not exceed a certain proportion of the rated non-inductive load of the machine. It may be a half, three-quarters, or full load according to the smallness of the power factor.

Thus a load of power factor 0·5 takes about *twice* the output current that the same load does, if of unity power factor. So that in testing the regulation for low power factors, only a proportion of the machine's rated output should be used.

In the case of *machine C*, a set of inductances of negligible resistances were used, and also a set of non-inductive resistances, whose values were determined by the fall of potential method, using alternating current at frequency 50, that is, the frequency used in the regulation test.

The alternator was then loaded to give an output of *one* kilowatt for different power factors, and a set of values of I, E_t, and $\cos\theta$ were taken at rated speed, that is, at frequency 50. In this test the exciting current was kept constant at 1·8, the same value as in the non-inductive load test. The values obtained were:

E_T - -	115	111	109	108	105	98
Cos θ -	1·0	0·9	0·81	0·78	0·68	0·54
Regulation	3·0	5·8	8·7	9·7	12·8	21·0

The no-load voltage was 118·5, and the regulation is calculated from

$$\frac{E_0 - E_t}{E_t} \times 100 \text{ per cent.}$$

The graph relating regulation and power factor for the *given* load is shown in Fig. 244.

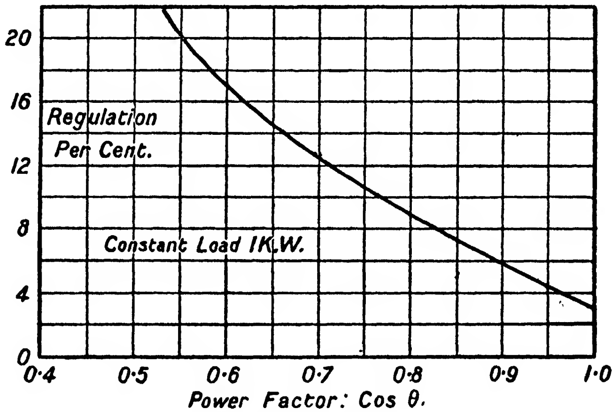

FIG. 244.—The regulation and power-factor curve of the single phase alternator, on a constant load of one kilowatt.

For *armature reaction* and power factor, E_i the internal voltage must be obtained. Its value may be found from the following equations:

$$E_i = I\sqrt{(R + r_a)^2 + \{(L_a + L)2\pi f\}^2},$$

$$I^2R = P,$$

$$\cos\theta = \frac{P}{E_t I} \quad \text{and} \quad \tan\theta = \frac{L2\pi f}{R}.$$

R, the resistance of the inductive load, is found from the second equation, P being the reading of the wattmeter. *Tan* θ may also be determined, and L, the self inductance of the load, calculated.

From the excitation test the effective ampere turns to produce E_i may be found. This number is subtracted from the actual ampere

turns used, and the result is Q, the number neutralised by armature reaction.

Machine C was loaded at excitation 1·8 amperes, and frequency 50, to give an output current of 22·5 amperes at power factor 0·67. The terminal volts were 94, and the load P was 1420 watts.

From the preceding equations R was found to be 2·8 and L 0·01. Also, r_a is nearly 0·25 and L_a 0·0024. Therefore $(R+r_a)^2=9·3$ and $\{(L_a+L)2\pi f\}^2=15·2$. So that

$$E_i=22·5\sqrt{15·2+9·3}=111 \text{ volts.}$$

Now the effective ampere turns producing 111 volts are, from Fig. 242, equal to 770; but the actual number used was $1·8 \times 500=900$. So for a load current of 22·5 amperes at power factor 0·67, the number of ampere turns per pole, neutralised by armature reaction, is

$$900-770=130.$$

For the same output current, speed, and excitation, this number for *unity* power factor is obtained from Fig. 243, and is equal to 96. Thus a decrease in power factor is accompanied by an increase in armature reaction.

A second case, in which the load current was 21 amperes at power factor 0·245, and the same speed and excitation, was worked out. In this case the terminal volts were 94 and the load 485 watts. R was found to be 1·1 ohms and L 0·0136 henry. By the same method as before, E_i was found to be 110 volts.

The effective ampere turns producing this internal voltage is, from Fig. 242, equal to 760, and 900 were used. Therefore the number of ampere turns per pole neutralised by armature reaction is 140. For the same current of 21 amperes, speed, and excitation this number for *unity* power factor is, from Fig. 243, equal to 89.

Therefore the extra ampere turns neutralised by armature reaction per pole, due to the lagging of the current, is 51. Thus armature reaction for a given output current increases as the power factor diminishes; the speed and excitation being constant.

Armature reaction in the short-circuited test.—Values of exciting current and armature short-circuit current are taken for rated speed. In *machine C* an ammeter and an extra rheostat were placed in the circuit of the field coils, and another ammeter across the terminals of the armature. The results obtained were:

I_f	0·38	0·54	0·62	0·72	0·84	0·93
I_{sh}	17·0	22·7	26·7	30·6	36·0	39·6

A *small* value of I_f should be started with, in this test. The frequency was 50.

The internal voltage may be calculated from

$$E_i=I_{sh}\sqrt{r_a^2+(L_a 2\pi f)^2},$$

in which r_a is nearly equal to 0·25, and $(L_a 2\pi f)^2$, as before, is nearly 0·6. Using these values,

$$E_i = I_{sh} \times 0{\cdot}81.$$

In this machine the short-circuit power factor is appreciable. For,

$$\tan\theta = \frac{L_a \cdot 2\pi f}{r_a} = \frac{\sqrt{0{\cdot}6}}{0{\cdot}25} = 3{\cdot}1,$$

and thus $\theta = 72°$ nearly. So $\cos\theta$ the power factor $= 0{\cdot}31$.

The table for armature reaction is as follows :

I_{sh} - - -	17·0	22·7	26·7	30·6	36·0	39·6
E_i - - -	13·8	18·4	21·7	24·8	29·2	32·1
Actual A.T. -	190	270	310	360	420	465
Effective A.T. -	70	105	124	140	165	180
Q - - -	120	165	186	220	255	285

The graph of Q and I_{sh} is given in Fig. 245, and the dotted curve, obtained from Fig. 243, shows the values of Q for the load currents at unity power factor; currents are expressed in amperes.

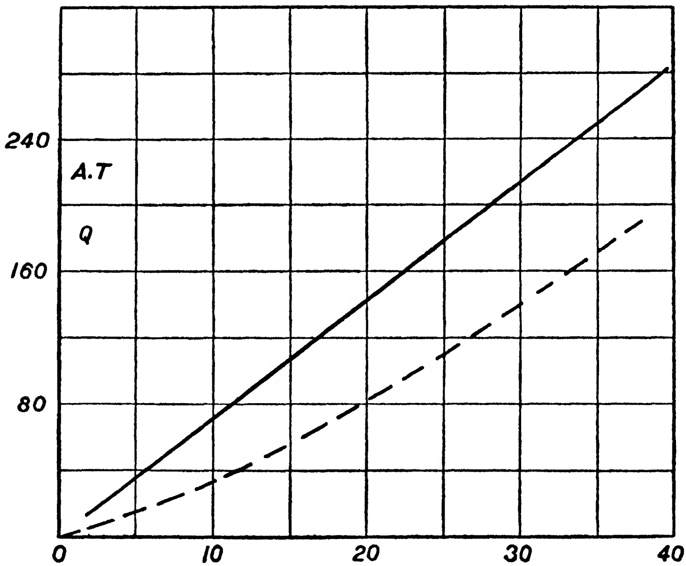

FIG. 245.—The ampere turns neutralised by armature reaction when the armature is short-circuited, and also when on load at unity power-factor. The curves are for the single phase alternator of output 3 K.W.

The iron loss, and windage and friction loss.—In this test the alternator is coupled to a direct-current shunt motor, which drives it

at rated speed on open circuit. A voltmeter is placed across the terminals of the alternator to give E_i the internal voltage, for different exciting currents. An extra rheostat may be placed in the circuit of the field coils to give a large range of exciting current, and also an ammeter to indicate the latter.

The shunt motor has a voltmeter and an ammeter to give P the input, which should be arranged to be at constant voltage. P is read

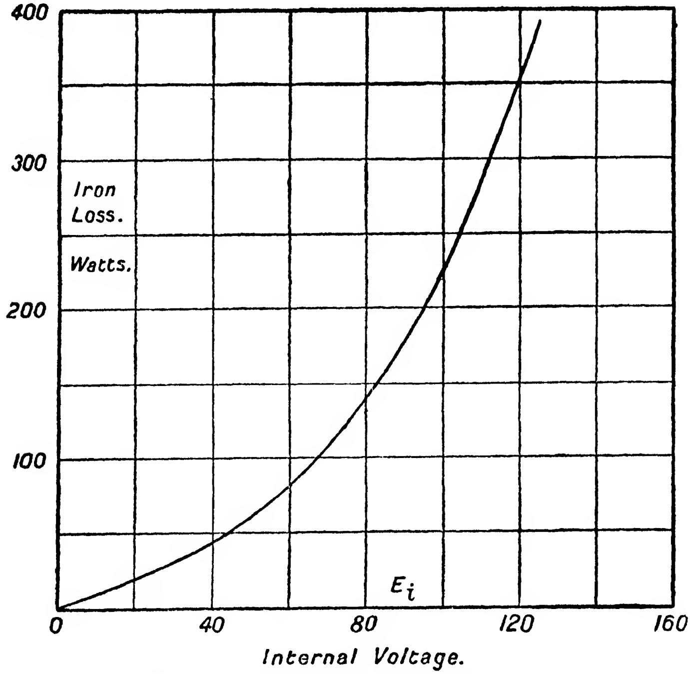

FIG. 246.—The iron loss and internal voltage of the single-phase alternator.

when the alternator is excited to give an internal voltage E_i. P_1 is then read after the exciting current is diminished and switched off. Neglecting the difference of the small copper losses in the armature of the motor, $P - P_1$ represents the iron loss of the alternator when its internal voltage is E_i. The alternator is next uncoupled, and P_2 the motor input is again read. $P_1 - P_2$ represents the windage and friction loss.

The following values were obtained for *machine C* at rated speed 1500 R.P.M. :

E_i - -	50	70	85	99	110	121
Iron loss	56	110	161	220	288	361

The *windage and friction loss* was 149 watts.

The graph of E_i and *iron loss* is shown in Fig. 246, and the values of this loss for the internal voltages of the regulation or load test, are obtained from this graph, and entered in the following table.

The detailed losses and efficiency table.—This is constructed for the regulation or load test values, and for *machine C* is as follows :

I - - -	0	6	12	17	21·6	26	30
E_t - - -	118·5	116	113	110	106·3	103·5	100
E_i - - -	118·5	117·6	116·4	114·5	113·5	112·1	110·0
Output - -	0	696	1356	1870	2300	2700	3000
Copper loss -	0	9	36	71	115	166	222
Copper loss -	131	131	131	131	131	131	131
Iron loss -	345	340	330	315	310	300	287
W & F - -	149	149	149	149	149	149	149
Tot. loss -	625	629	646	666	705	746	789
Input - -	625	1325	2002	2536	3005	3446	3789
η - - -	0	52·5	67·5	73·5	76·5	78 5	79·2

The first copper loss is that of the armature, the second that of the field coils.

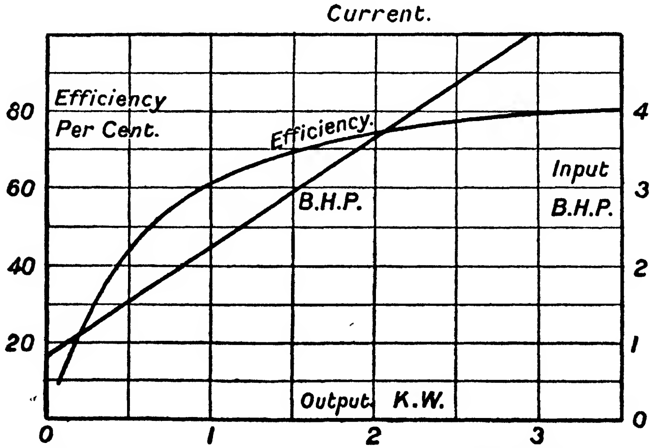

FIG. 247.—The efficiency and B.H.P. curves of the single-phase alternator.

The graphs of efficiency and horse-power input, against output are shown in Fig. 247.

The case of an inductive load may be worked out in a similar way. E_i, the internal voltage which has already been determined for several inductive loads, is somewhat more difficult to calculate than in the case of a non-inductive load ; otherwise there is practically no difference in working out the two cases.

TESTING A THREE-PHASE GENERATOR.

A similar set of tests as described for a single-phase alternator may be made on a two or three-phase machine. In the three-phase machine the resistance between any two terminals divided by two will give r_1, the resistance of each phase, if *star* connected ; and, if mesh connected, the terminal resistance will be two-thirds of r_1, because each phase is shunted by the other two in series.

The total armature or stator copper loss is $3I_l^2r_1$ if star, and $3\left(\frac{I_l}{\sqrt{3}}\right)^2 r_1 = I_l^2 r_1$ if mesh connected. This does not include the eddy-current loss in the conductors or the skin action, which together, may have an effect, equivalent to an increase in the copper loss of 30 to 50 per cent. Effective laminating of the conductors and the absence of harmonics in the rotating flux will considerably diminish this effect.

The impedance test.—In addition to the self inductance of one of the phases, there is the mutual inductance between each phase of the winding and the others. To find L_a, the equivalent self inductance of one section or phase of the winding, the armature is connected to a three-phase supply, as shown in Fig. 248.

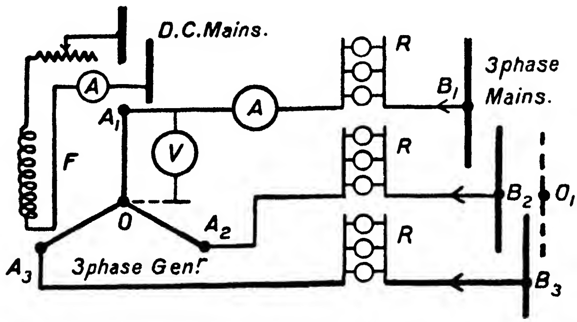

FIG. 248.—The arrangement for the impedance test of a three-phase alternator.

The resistance R is adjusted to have the same value in each line, that is, each phase to have an equal current. As each circuit, such as $OA_1B_1O_1$, may be regarded as independent of the other two, bearing in mind, however, the mutual induction, its impedance may be determined exactly as in the case of the single-phase alternator.

The line current I, and *e*, the voltage drop from line to neutral point O, are read. This should be done with rated exciting current in the field coils. The armature should also be turned by hand

through a small angle in order to obtain the average value of I and e each time.

Then the equivalent self inductance may be obtained from

$$L_a = \frac{1}{2\pi f I}\sqrt{e^2 - (r_a I^2)},$$

r_a being the resistance of one phase of the winding.

This was done for a 3·75 K.W. three-phase generator, which will be called *machine D*. The resistance between terminal and terminal was measured, and in each of the three determinations was found to be 0·15 ohm. This must be divided by two to give the resistance of each phase. To allow for skin and eddy action in the conductors, r_a was taken equal to 0·1 ohm.

The supply currents were at frequency 51·5. The values of L_a, the phase self inductance for currents in each phase, were calculated from the preceding formula and found to be :

I	e	L_a
13·85	6·75	0·00148
21·15	10·10	0·00144
28·05	12·95	0·00141

The no-load excitation test.—The arrangement for making this test is shown in Fig. 249, the internal voltage E_i of one phase being

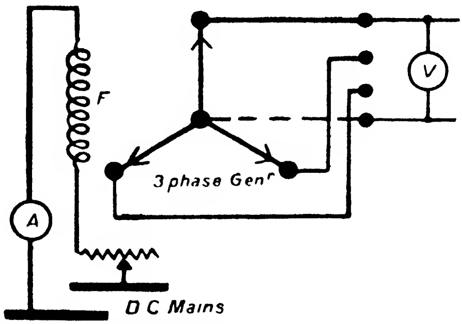

FIG. 249.—The arrangement for the no-load excitation test of a three-phase alternator.

read for different exciting currents in the field coils. This test was made on *machine D*, and the values obtained were :

E_i	21·5	27·0	33·0	37·0	42·0	45·0	48·0	52·0	57·0
I_f	0·38	0·50	0·63	0·76	0·89	1·00	1·08	1·22	1·39
A.T.	285	375	472	570	668	750	810	915	1042

The frequency was 50. A.T. is the number of ampere turns per field pole, on which is wound 750 turns.

The graph relating E_i and A.T. is given in Fig. 250.

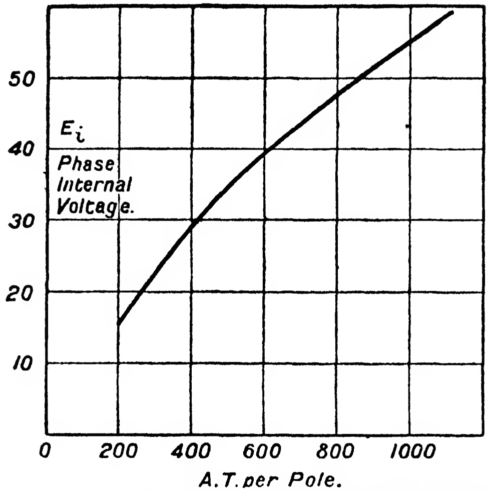

FIG. 250.—The no-load excitation curve of the three-phase generator.

The load or regulation test.—The connections for this test are shown in Fig. 251. A balanced load is used, R being the same value in each line throughout the test. If the load is non-inductive, an ammeter and voltmeter will give its value by their product; if inductive, the wattmeter shown should be used in addition to the ammeter and voltmeter.

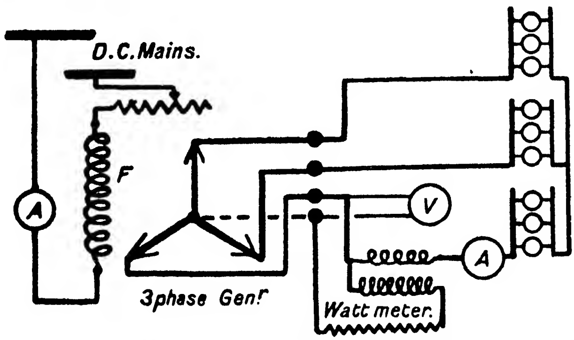

FIG. 251.—The arrangement for the load or regulation test of a three-phase alternator.

The values obtained for *machine D* on a balanced non-inductive load for speed 1000 R.P.M., that is, frequency 50, were as follows:

E_t	55	54	51	47	44·6
I	5·3	10·5	20·3	29	31·5
R	10·4	5·15	2·5	1·62	1·42

R was the non-inductive resistance of each phase load, and the current I_f in the field coils was 1·36 amperes.

The *internal voltage* per phase may be calculated from the formula

$$E_i = I\sqrt{(R + r_a)^2 + (L_a 2\pi f)^2}.$$

Taking L_a to be 0·00146 henry, f 50, and r_a 0·1 ohm, the following values of E_i were obtained :

I	5·3	10·5	20·3	29·0	31·5
E_i	56·0	55·0	53·5	51·5	50·0

The graphs of the regulation curve and the total characteristic curve are shown in Fig. 252.

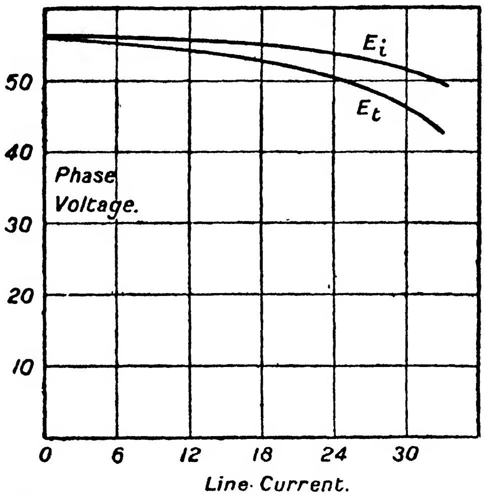

FIG. 252.—The voltage-current curves of the three-phase alternator.

Armature reaction.—The effective ampere turns per pole necessary to produce a given internal voltage is obtained from the excitation curve. For *machine D*, the actual ampere turns per pole used, were

$$1{\cdot}36 \times 750 = 1020.$$

Q, the number of ampere turns neutralised by armature reaction, was calculated and tabulated as follows :

I - - -	5·3	10·5	20·3	29·0	31·5
E_i - - -	56·0	55·0	53·5	51·5	50·0
Actual A.T. -	1020	1020	1020	1020	1020
Effective A.T. -	1015	993	954	905	870
Q - - -	5	27	66	115	150

The copper losses.—These include the copper loss in the field coils and rheostat, and the armature copper loss. The loss for the circuit

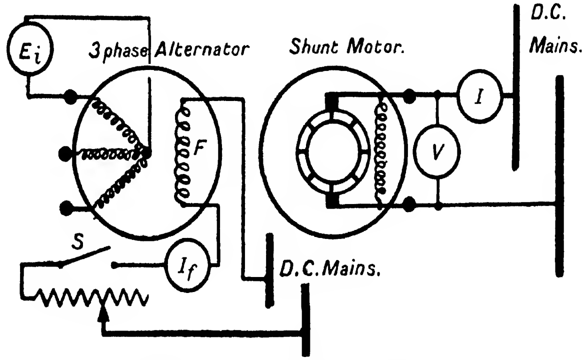

FIG. 253.—The arrangement for the determination of the iron, windage, and friction losses of a three phase alternator.

of the field coils is $I_f E$; I_f being the field current and E the voltage of the direct current supplying this exciting current. The armature

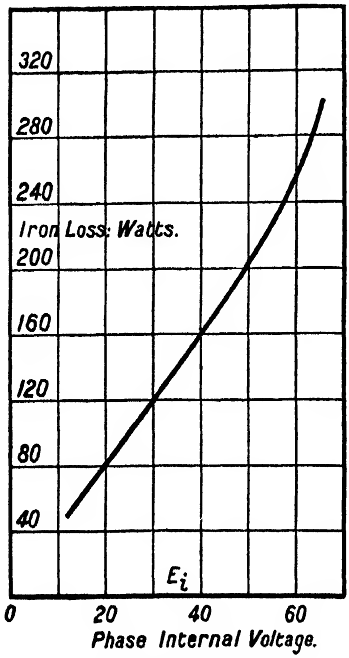

FIG. 254.—The iron loss of the three-phase generator for different internal voltages.

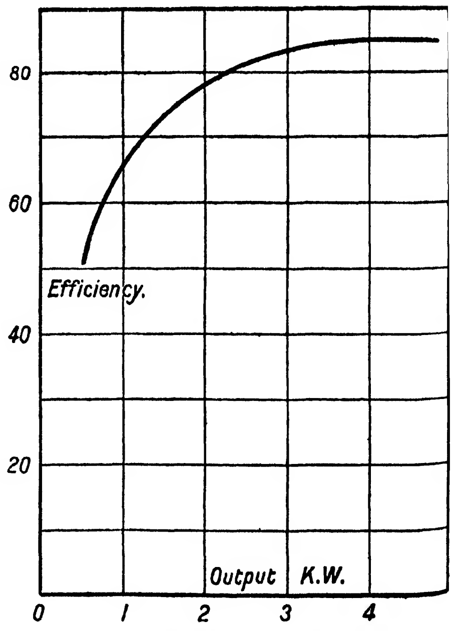

FIG. 255.—The efficiency curve for the three-phase generator.

copper loss is $3I^2r_1$, in which r_1 is the resistance corrected for eddy and skin action of one of the phases, and I is the line current.

For *machine D*, $I_f = 1{\cdot}36$, $E = 105$, and $r_1 = 0{\cdot}10$. The field loss is therefore 143 watts, and the armature loss $0{\cdot}3I^2$.

The iron loss, and windage and friction loss.—These losses are determined exactly as in the case of single-phase alternator or a direct-current generator, the arrangement for making the test being shown in Fig. 253.

The values obtained for *machine D* for frequency 50 were as follows:

E_i - -	40	45	50	55	60	65
Iron loss -	162	180	204	230	253	290

and the windage and friction loss was 115 watts. The graph of E_i and iron loss is shown in Fig. 254.

The efficiency and detailed loss table.—This table may now be constructed from the preceding data, and the efficiency of the three-phase generator determined for different outputs. The following table is for *machine D*:

I - -	5·3	10·5	20·3	29·0	31·5
E_t - -	55·0	54·0	51·0	47·0	44·6
E_i - -	56·0	55·0	53·5	51·5	50·0
Output -	876	1704	3105	4100	4200
Copper loss -	8	33	124	252	298
Copper loss -	143	143	143	143	143
Iron loss -	230	226	220	210	200
W and F -	115	115	115	115	115
Total losses -	496	517	602	720	756
Input - -	1372	2221	3707	4820	4956
Input B.H.P.	1·84	2·97	4·97	6·46	6·65
Efficiency -	63·5	76·5	83·5	85·0	85·0

The first copper loss is that of the armature, and the second that of the field circuit. W and F is the windage and friction loss. The table is for frequency 50. Fig. 255 gives the graph of the efficiency and output. The output is given by $3E_tI$.

TESTING A SYNCHRONOUS MOTOR.

The motor is first synchronised by running it up to speed, either by belting from another machine or by some auxiliary machine which may be directly coupled to it. The resistances of the armature and field coils, the determination of the iron, and windage and friction losses may be determined as in the case of a multiphase alternator. By reading the input power, current, and volts, when the machine is loaded, the efficiency table may be constructed.

In synchronous motors the power factor of the input may be altered for a given load, by varying the exciting current of the field

coils, and the effect of armature reaction on the value of the power factor may also be tested.

The following tests were made on a 3·75 K.W. three-phase generator used as a synchronous motor. It was star connected, and fed from a star-connected three-phase transformer. After synchronising, the machine was switched on to the transformer and loaded.

Load tests.—The arrangement used for these tests is shown in Fig. 256. S.M., the synchronous motor, is first brought up to speed by the shunt motor shown. Then S_2 is closed, and when the voltage and

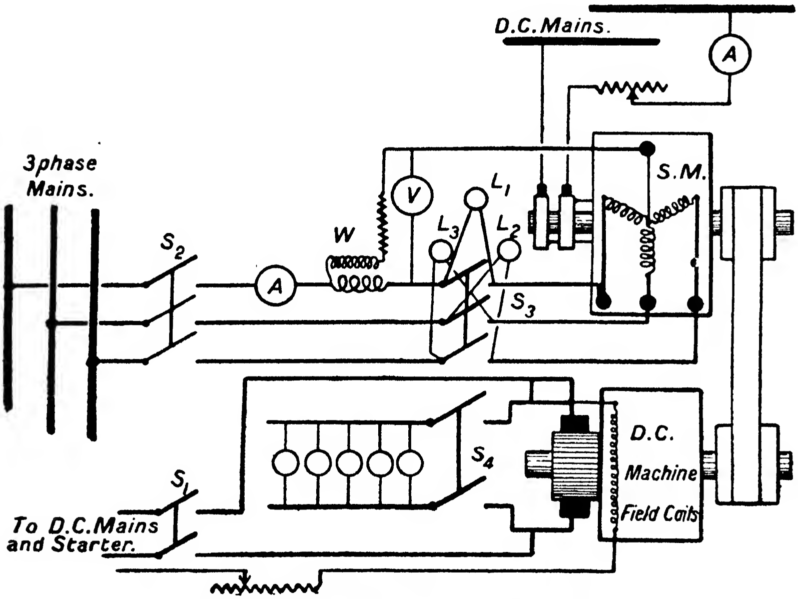

FIG. 256.—The arrangement for testing the three-phase synchronous motor.

speed of S.M. are right, according to the voltmeters and synchronising lamps L_1, L_2, and L_3, the switch S_3 is closed at the instant L_1 is quite dark; at the same time S_1 is opened.

S.M. is then loaded by driving the shunt motor as a generator, and the latter loaded step by step after closing S_4.

The following results were obtained for the machine under test, for different exciting currents in its field coils. In the table given, the line voltage is $\sqrt{3}\,E$, and the total input three times the phase input. The frequency of the supply mains was 51·5, and the machine has 6 poles. The speed of the motor was therefore

$$\frac{120 \times 51 \cdot 5}{6} = 1030 \text{ R.P.M.}$$

Exciting Current.	Phase E.	I.	Phase Input.	Cos θ.
0·61 ampere	47	17·2	400	0·50
	47	19·3	525	0·58
	47	21·5	686	0·68
	47	24·5	845	0·73
	47	30·6	1120	0·78
0·77 ampere	47	12·8	410	0·68
	47	16·0	600	0·80
	47	18·2	710	0·83
	47	24·0	1035	0·92
1·135 amperes	47	8·5	365	0·92
	47	11·4	500	0·93
	47	14·0	625	0·95
	47	16·8	775	0·98
	47	27·0	1265	1·00
1·35 amperes	47	14·0	350	0·53
	47	15·1	480	0·68
	47	17·3	640	0·78
	47	19·8	792	0·85
	47	24·8	1030	0·88
	47	28·8	1260	0·93

The graphs relating power factor and input are shown in Fig. 257, and it will be noted that there is a certain exciting current, which will

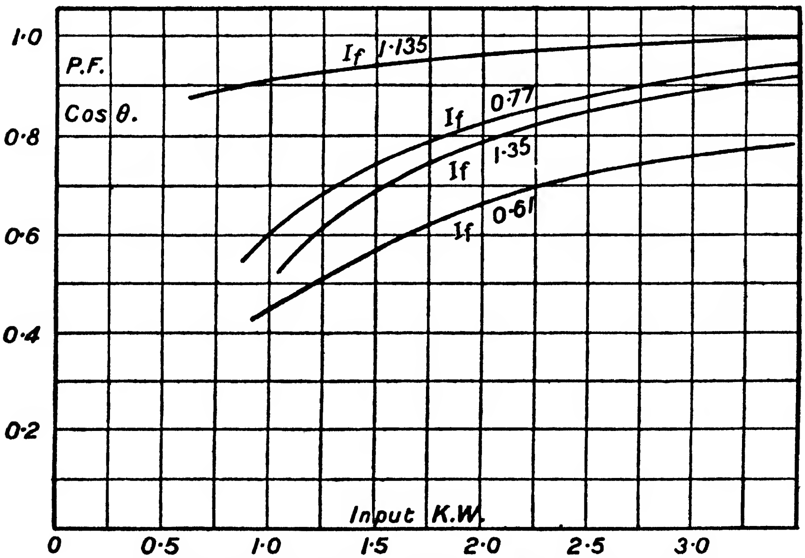

Fig. 257.—Curves showing the relation between power factor, exciting current, and load, for the three-phase synchronous motor.

give the best power factor when the machine is loaded. An increase or decrease of exciting current with respect to this value produces a lowering of the power factor.

The variation of *power factor* with *exciting current* I_f for different constant loads was then determined, and the results* were as follows:

Phase Input.	Phase E.	I.	Power Factor.	I_f.
410 watts	47	20·8	0·42	1·49
	47	13·0	0·67	1·26
	47	9·4	0·93	1·09
	47	10·3	0·84	0·87
	47	12·8	0·69	0·72
	47	15·5	0·56	0·61
	47	20·3	0·43	0·39
750 watts	47	22·5	0·71	1·45
	47	19·0	0·84	1·31
	47	16·0	1·00	1·15
	47	16·0	1·00	0·95
	47	17·5	0·91	0·79
	47	21·0	0·76	0·63
	47	25·5	0·63	0·46
1000 watts	47	26·0	0·82	1·45
	47	23·0	0·92	1·30
	47	21·2	1·00	1·15
	47	21·2	1·00	1·01
	47	21·5	0·99	0·90
	47	23·6	0·90	0·79
	47	26·0	0·82	0·68
	47	27·5	0·77	0·62
1415 watts	47	33·5	0·90	1·5
	47	30·0	1·00	1·15
	47	30·7	0·98	0·95
	47	32·0	0·94	0·86
	47	34·0	0·89	0·79
	47	36·0	0·84	0·73

The graphs relating *exciting current* and power factor for constant loads are shown in Fig. 258. The total input is three times the phase input, and the line voltage is $\sqrt{3}$ times the applied phase voltage E.

The no-load or excitation test.—In this test the motor was run as a generator, and different values of E_i, the phase internal voltage, taken for corresponding exciting currents. The values obtained at frequency 51·5 for the synchronous motor under consideration were:

I_f	0·38	0·5	0·63	0·76	0·89	1·00	1·08	1·22	1·39
E_i	22·0	27·7	33·5	38·0	43·0	46·0	49·3	53·5	58·5

The turns per pole were 750.

* The efficiency for the variation of power factor, given in each section of the table, does not appreciably change for this motor; the variations of the iron and copper losses practically neutralising each other.

Armature reaction, power factor, and exciting current.—When the machine was running as a synchronous motor on a load which required an input of 3 × 1000, that is, 3 K.W. (see table, page 322), the applied phase voltage was 47 volts. Now, the internal voltage will nearly balance this phase voltage, and may be approximately taken as 46 volts.

According to the excitation test, the effective ampere turns per pole necessary to produce this internal voltage were 750 × 1·00 = 750, the number of turns per pole being 750.

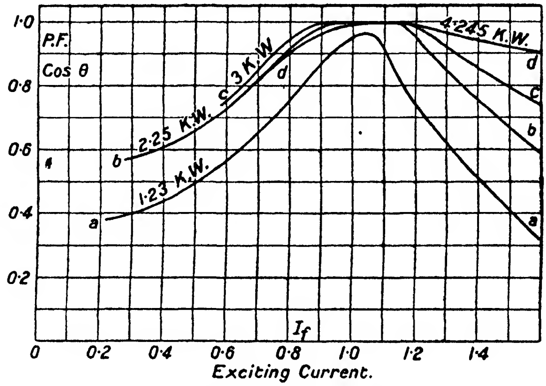

FIG. 258.—The graphs of power-factor variation with exciting current, for the three-phase synchronous motor, on different loads.

When the power factor for this given load is 0·82, the *actual* ampere turns used per pole were 750 × 0·68 = 510.

Therefore the armature winding has by its reaction *contributed* 750 − 510 = 240 ampere turns per pole. This effect has therefore been a magnetising one, produced by a lagging current.

For the same load and power factor 0·82, the actual ampere turns used were *also* 750 × 1·45 = 1090. So that the armature winding in this case, had, with a leading current, to neutralise by its reaction

1090 − 750 = 340 ampere turns per pole.

The following table has been derived for the load which requires an input of 3 K.W. from the table on page 322. Q is the number of ampere turns per pole neutralised or contributed as a result of the armature reaction. The former are distinguished by a negative sign before the value of Q.

I_f . . .	0·62	0·68	0·79	0·9	1·01	1·15	1·30	1·45
Actual A.T. .	465	510	592	677	760	863	978	1090
Effective A.T.	750	750	750	750	750	750	750	750
Q . . .	285	240	158	73	− 10	− 113	− 228	− 340
Cos θ . .	0·77	0·82	0·90	0·99	1·00	1·00	0·92	0·82

The graph relating Q and *cos θ* is shown in Fig. 259.

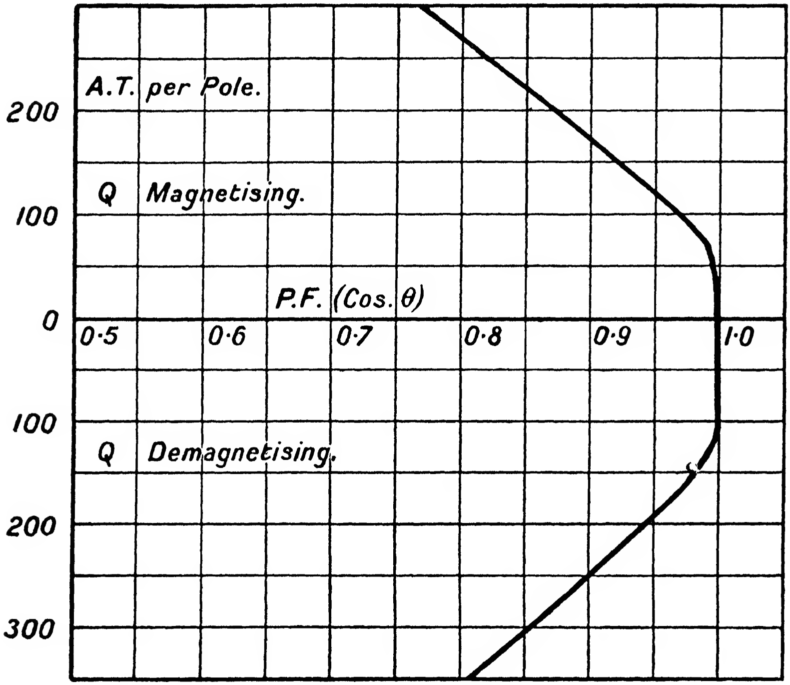

FIG. 259.—The ampere turns neutralised or contributed by armature reaction in the three-phase synchronous motor, on constant load, for input power at different power factors.

The input current leads with respect to the applied voltage for exciting currents above about 1·1 amperes, and lags for values below this.

The copper losses of the motor.—These include the field loss and the armature loss $3I^2r_1$, in which r_1 is the resistance of one phase increased by 20 or 30 per cent., and sometimes more, to allow for eddy and skin action. Without this correction, r_1 for the machine under consideration is 0·075 ohm, so that 0·1 ohm may be taken as the corrected value. The field coils were fed from mains at 105 volts, so that their loss is $105 I_f$ watts.

The iron loss, and windage and friction loss.—The motor is run as a generator by being belted or direct coupled to a direct-current shunt motor. A voltmeter is placed across one of the phases of the synchronous motor, and an ammeter in its field circuit. The voltmeter reads E_i, the internal voltage of the phase.

The shunt motor is fed at constant voltage, and its input is read for different exciting currents in the field coils of the synchronous

motor; the latter being run at rated speed on open circuit. The input is also read when the exciting current is switched off. The small copper loss in each case, if necessary, may be subtracted from the input, its value being the square of the armature current of the shunt motor multiplied by the armature resistance.

The difference between these two corrected inputs gives the *iron loss* for the particular internal voltage E_i. By reading the input when the synchronous motor is uncoupled, and taking its corrected value from that, when no exciting current was used, will give the windage and friction loss. This test is similar to that of finding the iron loss, and windage and friction loss, of a direct-current generator.

The values obtained are tabulated as follows:

Shunt Motor.					Synchronous Motor.	
E.	I.	Input.	$I_a^2 r_a$.	Difference.	Iron Loss.	Phase Voltage.
110	13·6	1500	20	1480	117	29
110	14·2	1560	22	1538	175	42
110	14·5	1600	23	1577	214	50
110	15·0	1650	25	1625	262	58
110	15·2	1670	26	1644	281	63
110	12·55	1380	17	1363	0	0
110	11·4	1255	14	1241	0	0

In the table, the last but one set of values, is for the case of no exciting current in the synchronous motor, and the last set is for the case when it is uncoupled. The *windage* and *friction* loss is therefore

$$1363 - 1241 = 122 \text{ watts.}$$

The graph relating E_i and iron loss is given in Fig. 260.

Assuming that the internal voltage when the machine is running as a synchronous motor is 46 volts, the corresponding iron loss from the graph in Fig. 260 is 190 watts.

The efficiency and detailed loss table.—The efficiency will be worked out for the values obtained in the load test when the exciting current was 1·135 amperes; these values are given in the table on page 321. Thus:

I - - -	8·5	11·4	14·0	16·8	27·0
Input - -	1095	1500	1875	2325	3795
Copper loss -	22	39	59	85	219
Copper loss -	119	119	119	119	119
Iron loss -	190	190	190	190	190
W and F -	122	122	122	122	122
Tot. loss - -	453	470	490	516	650
Output - -	642	1030	1385	1809	3145
B.H P. - -	0·86	1·38	1·86	2·42	4·21
Torque - -	44	70	95	123	215
Efficiency -	58·5	68·7	74·0	78·0	83·0
Power factor -	0·92	0·93	0·95	0·98	1·00

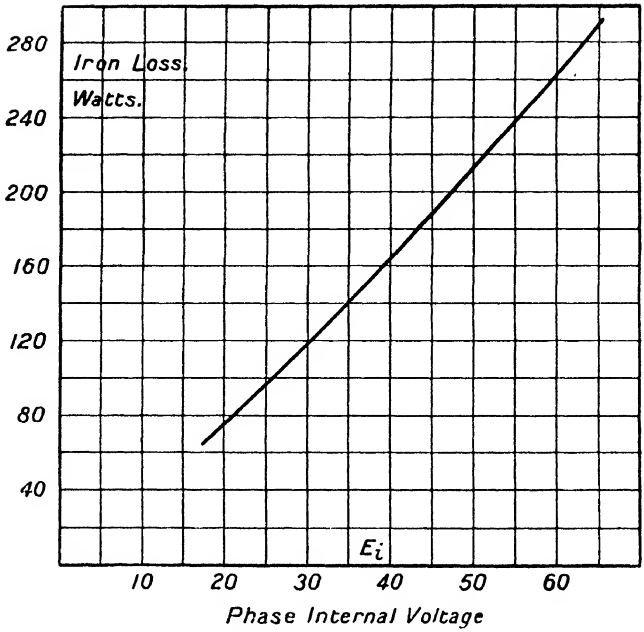

FIG. 260.—The iron-loss curve of the three-phase synchronous motor, for different internal voltages.

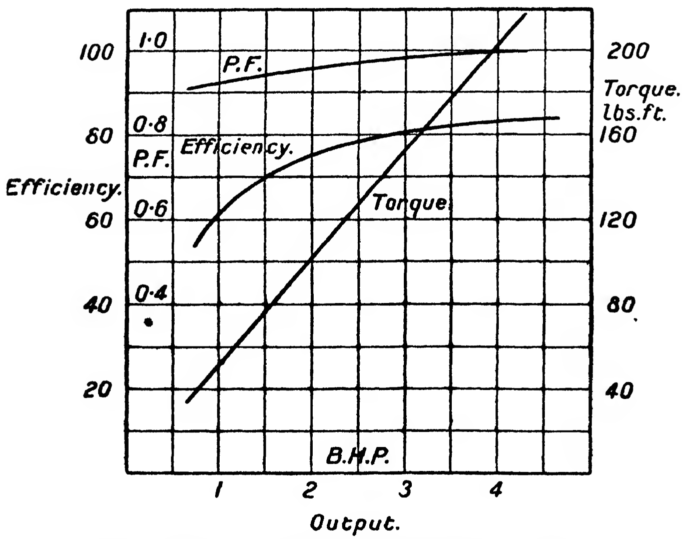

FIG. 261.—The characteristic curves of the three-phase synchronous motor, of rating about 4 B.H.P.

The first copper loss in the table is that of the armature winding, and the second that of the field coils and rheostat.

The graphs of efficiency, power factor, and torque against the output are given in Fig. 261. The speed was 1030 R.P.M., so that the torque in lbs.-ft. is given by

$$T \,.\, 2\pi N = \text{B.H.P.} \times 33000,$$

$$T = 5{\cdot}1 \text{ B.H.P.}$$

In exactly the same way the efficiency table may be derived for the other cases of different exciting currents given in table, page 321. This should if possible be done by the student, and the curves obtained for each set compared with each other.

CHAPTER XVI.

TESTING OF ALTERNATING-CURRENT MACHINES—Continued.

In this chapter the testing of a rotary converter, induction motor, transformer, repulsion motor, and the principle and use of the oscillograph will be considered.

TESTING A ROTARY CONVERTER.

A rotary converter may be run from its direct-current side and loaded on its alternating-current side, or *vice versa*. It is more generally used the latter way; in the former case, as stated before, it is termed an *inverted* rotary converter.

The voltage-ratio test.—The machine is run as an unloaded direct-current shunt motor, and the voltage measured from ring to ring on the one side and brush to brush on the other side. For single-phase machines the ratio of alternating to direct voltage should be very nearly 0·707, for three-phase 0·612, for six-phase 0·354, and for twelve-phase 0·183.

This ratio was found for a small three-phase rotary converter of output about 4 K.W. to be $\frac{80}{131}=0·611$. For reference this converter will be termed *machine E*.

The drop test and phase resistance.—In the drop test the machine is regarded as a direct-current generator, and tested accordingly by sending different currents through the armature and reading the drop, the alternating side being open. The values for *machine E* were found to be:

I	6	12·3	19·0	29·0	38·2
e	1·3	2·6	4·1	6·25	8·42
r_a	0·216	0·211	0·216	0·216	0·215

r_a includes the brush contact resistance with that of the armature.

The phase resistance is 1·5 the value of the resistance between terminal and terminal on the three-phase side, the winding being mesh connected.

For *machine E*, r_1 the phase resistance was $\frac{3}{2}\times 0·15=0·225$. This value also includes the brush-contact resistance.

The iron loss, windage and friction loss, and excitation test.—This test is performed exactly as in the case of a direct-current machine. The rotary converter is belted or direct coupled to a shunt motor, and the input read for different exciting currents; the field coils being separately excited.

The armature is open, and the volts are read on the direct-current side, and the field current is read. In this case the iron loss will be plotted against the field current and not against E_i, as the internal voltage of the rotary converter when running normally is difficult to estimate with reasonable accuracy.

For most practical purposes, in tabulating the detailed losses of the machine, it is sufficient to take the iron loss from the graph relating it and the exciting current, for each of the exciting currents used in the load test.

The results obtained for *machine E* are given in the following table. A set of values was taken for a speed of 1040 R.P.M. The converter was separately excited and driven by a small induction motor whose input current, voltage, and power were read by ammeter, voltmeter, and wattmeter. The applied phase voltage of this motor was 200 and line voltage 346.

Motor.			Rotary Converter.		
I.	E.	W.	Iron Loss.	E.	I_f.
3·7	346	1500	450	140	2·32
3·7	346	1395	345	128	1·92
3·65	346	1347	297	118	1·68
3·55	346	1200	150	93	1·24
3·60	346	1125	75	74	0·95
3·52	346	1110	60	72	0·92
3·47	346	1050	0	0	0·00
3·3	346	420	Uncoupled.		

The windage and friction loss for this speed of 1040 R.P.M. is $1050 - 420 = 630$ watts, which is large on account of the extra friction due to the brushes on the alternating-current side.

The iron, friction, and windage losses were also determined for a higher speed, namely 1250 R.P.M., and the following values obtained:

I_f - -	1·10	1·20	1·32	1·48	1·72
Iron loss -	250	260	285	339	420
W and F -	800	800	800	800	800

The graphs of iron loss and internal voltage against field current is shown in Fig. 262, the excitation curve being for speed 1040 R.P.M.

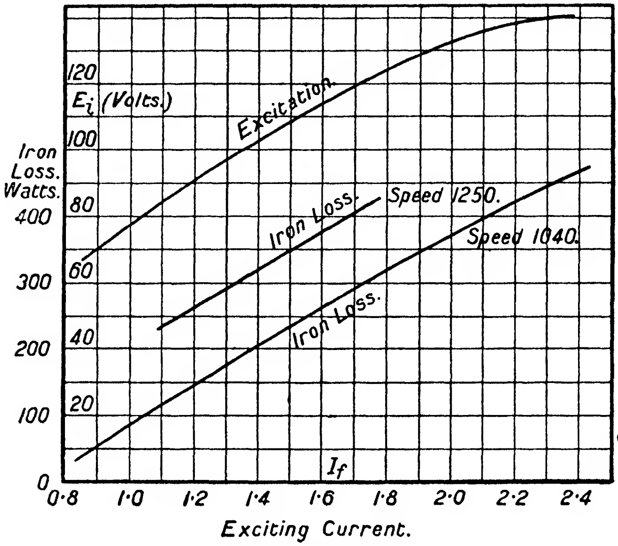

FIG. 262.—The iron loss and exciting cur.ent of the rotary converter.

The increase of iron loss for an exciting field of 1·7 amperes due to changing the speed from 1040 to 1250, is 410 − 295 = 115 watts, that is, 39 per cent. increase for a speed increase of 20 per cent. For the same increase of speed, the windage and friction losses increase 27 per cent.

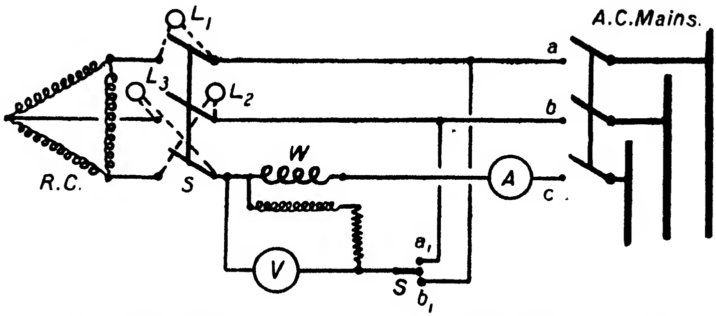

FIG. 263.—The arrangement for the load test of the rotary converter, when fed from alternating-current mains.

The load test: the converter used for producing direct current.—The arrangement for making this test is shown in Fig. 239. M_1 are

the supply mains, S the main switch of the converter, m_1 direct-current mains for running the machine up to synchronous speed, and m_2 are the direct-current load mains.

The mains *a*, *b*, and *c* are supplied with a wattmeter, ammeter, and voltmeter, as shown in Fig. 263. Two readings, W_1 and W_2, of the wattmeter give by their sum the total input power. The wattmeter is read first with switch *s* on a_1, then with *s* on b_1. The supply switch is first closed, the rotary converter synchronised, and then S is closed; at the same time the switch of the direct-current mains used for driving the machine is opened.

The machine will run most efficiently for unity power factor, and the field rheostat should be adjusted to give *minimum* input current for full load; this being the condition for obtaining unity power factor at full or any other load. For other exciting currents the machine will not be so efficient.

The following results were obtained for *machine E* after adjusting the field rheostat to give unity power factor for full load, or, what is the same thing, minimum line current for full load. After this adjustment the rheostat was left unchanged.

A.C.	E	80	80	80	80	80	80	80
	I	10·5	12·8	16·8	21·8	25·5	30·4	34·7
	W_1	850	1050	1335	1650	1885	2200	2435
	W_2	200	475	865	1325	1645	2000	2350
	Input	1050	1525	2200	2975	3530	4200	4785
	$\sqrt{3}$ EI	1455	1775	2325	3020	3540	4200	4800
	Cos θ	0·72	0·86	0·945	0·985	0·996	1·00	1·00
D.C.	E	128	127	124·5	123	121	117	116
	I	0	3·0	8·5	14·3	19·0	24·5	28·7
	Output	0	381	1060	1760	2300	2870	3330
I_f		2·10	2·10	2·10	2·07	2·05	1·99	1·95
Field loss		270	267	262	255	248	233	226
Speed		1058	1055	1045	1040	1035	1030	1025
Efficiency		0	25	48	59	65	68	69·5

E for the alternating side is the line-to-line voltage and I the line current. The input power is the sum of the wattmeter readings.

To obtain the power factor, the formula for input power was used, namely

$$W_1 + W_2 = \sqrt{3}\, EI \cos\theta.$$

The *efficiency*, *power factor*, and *speed* are shown plotted against the *output* in Fig. 264.

To show the decrease of efficiency due to using a field excitation which does not give unity power factor, a load test was made with a larger exciting current than that required for unity power factor. For an input of 4·6 K.W. an exciting current of 2·55 amperes gave a power factor of 0·8. The field rheostat was then left unchanged and

the load taken off in steps, and the values taken as before. The results worked out were :

A.C. Input -	1430	2480	3600	4270	4580
D.C. Output -	0	975	2050	2600	2790
Efficiency -	0	39·0	57·0	61·0	61·0

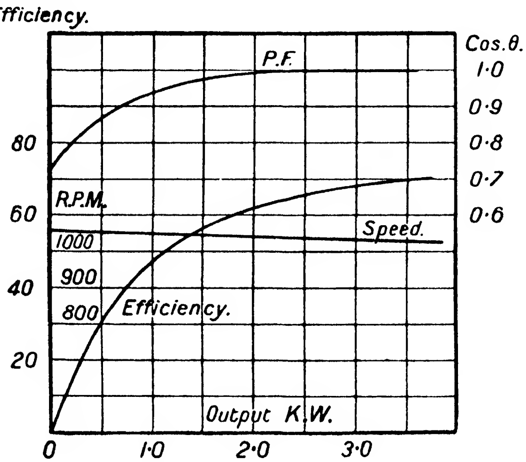

FIG. 264.—The characteristic curves of the rotary converter, when fed from alternating-current mains.

The efficiency curve for power factors 0·8 and unity are shown plotted against the input in Fig. 265.

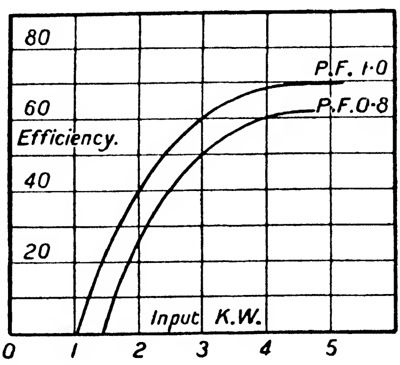

FIG. 265.—The efficiency curves of the rotary converter, for feeding currents at two different power factors.

Load test as an inverted rotary converter.—In this case the converter is fed by direct current, and its load is on the alternating-

current side. The diagram of connections for the load is shown in Fig. 266; *a*, *b*, and *c* being the three-phase leads from the rotary

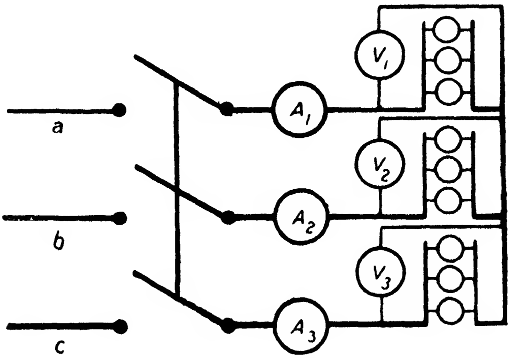

FIG. 266.—The arrangement for the load test of the rotary converter, when fed from direct-current mains.

converter. If the load is inductive, one, or two wattmeters will be required.

The values obtained for *machine E* loaded on the three-phase side with a non-inductive load were:

A.C.	I_1	0	6·7	11·5	16·1	20	27·6
	I_2	0	6·7	12·5	16·8	20·7	27·7
	I_3	0	6·5	12·0	16·2	20·3	27·7
	E_1	39·0	42·0	46·0	45·0	44·5	44·0
	E_2	44·0	45·6	43·0	43·0	42·5	42·6
	E_3	42·0	44·0	41·7	42·0	42·0	40·0
	I_1E_1	0	282	530	723	890	1220
	I_2E_2	0	305	538	723	880	1180
	I_3E_3	0	287	500	680	855	1110
	Output	0	874	1568	2126	2625	3510
	Frequency	53·2	53·0	52·5	52·0	52·0	51·5
	Speed	1064	1060	1050	1040	1040	1030
D.C.	E	131	131	131	131	131	131
	I	11·6	18·3	23·9	27·9	31·7	38·5
	Input	1520	2400	3140	3650	4150	5050
	Efficiency	0	36·5	50·0	58·0	63·0	69·5

The field current remained constant at 1·9 amperes, so that the loss in the circuit of the field coils was $131 \times 1·9 = 250$ watts.

The graphs of efficiency and speed against input power are shown in Fig. 267.

THE INDUCTION MOTOR.

The three chief tests generally applied to an induction motor are the no-load or excitation test, the short-circuit test, and the load test. In addition the magnetic leakage coefficients of the stator and rotor windings may be determined. From the results the circle diagram of the motor may be drawn, and the overload capacity and pull-out torque of the machine obtained.

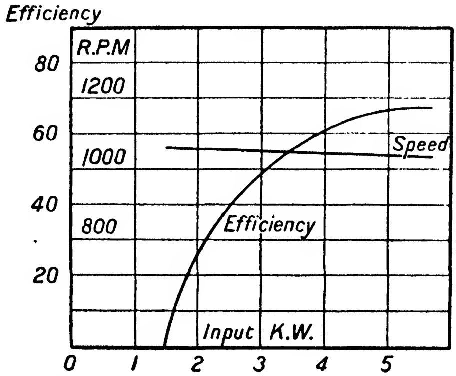

FIG. 267.—The efficiency and speed curves for the rotary converter, when fed from direct-current mains.

An induction motor is designed for a given frequency, and if used on mains of higher frequency, its input current efficiency and over-load capacity will be smaller. It should not be fed from mains of much lower frequency than rated value, as an excess input current will be produced for even moderate loads.

The no-load or excitation test.—In this test the motor is unloaded and fed at rated frequency with different voltages. The latter may be obtained from an alternator whose range of voltage is considerable, or from a three-phase transformer whose secondaries are supplied with numerous tappings. The range of voltage for this test may begin at about 50 per cent. above rated value, and end with a value at which the speed of the motor begins to appreciably fall. Values of voltage, line current, and input power are read on voltmeter, ammeter, and wattmeter.

Any change of speed may be readily observed by using a *stroboscopic* disc screwed to one end of the shaft. This disc made of thin sheet metal is painted into sectors alternately black and white. The total number of sectors is twice the number of poles of the stator winding, or four times the number of stator coils per phase. The disc is illuminated by placing near it an electric lamp, with metallic filament, of about 50 candle-power or greater, fed from the *same* mains supplying the motor.

When the rotor revolves at a lower speed than that of the stator rotating field, that is, synchronous speed, the disc will show an apparent rotation of the sectors in an opposite direction to that of the rotation of the rotor. The actual speed of the rotor is the synchronous speed minus the speed of this apparent rotation. At no-load and rated voltage this apparent rotation will be extremely slow. The synchronous speed is then practically equal to no-load speed at rated voltage; its exact value may be calculated from

$$\text{R.P.M.} = \frac{120f}{p}.$$

p is the number of stator poles, and is equal to twice the number of stator coils per phase, which may be counted.

At *no load* the input is equal to the sum of the iron, windage, friction losses, and a small copper loss in the stator winding. The *windage* and *friction* loss may be found by driving the machine, whose stator switch is open, with a direct-current shunt motor or a small induction motor of the right speed, and reading the input of the latter before and after coupling. The difference will give the windage and friction loss of the machine under test. The small no-load copper loss may be calculated from $3I_0^2r_1$, r_1 being one-half the resistance measured from terminal to terminal, corrected for eddy and skin action. The value of the *iron loss* for different feeding voltages may then be determined.

The following values were obtained for a 46 B.H.P. three-phase induction motor fed from 346 volt mains at frequency 51·2. This motor will be called *machine F*.

r_1, the stator phase resistance, was found to be $\frac{1}{2} \times 0{\cdot}15 = 0{\cdot}075$ ohm. Corrected for skin and eddy action, the value $r_1 = 0{\cdot}10$ ohm will be used.

The windage and friction loss at synchronous speed was found to be 990 watts.

The unloaded motor was then fed at different terminal volts, and the current, voltage, and input were read and found to be as follows:

E (line) -	152	203	253	301	346
I_0 (line) -	6·0	7·0	8·5	10·2	12·0
Input watts	1080	1190	1500	1800	2110
$3I_0^2r_1(a)$ -	11	15	22	31	43
W and F (b)	990	990	990	990	990
$(a)+(b)$ -	1001	1005	1012	1021	1033
Iron loss -	79	185	488	779	1077

The frequency was found to be 51·2. The last column is for rated voltage and no-load, and its data will give the no-load point of the circle diagram.

The graph relating E and I_0, or the excitation curve, is given in Fig. 268, and that of the iron loss and E in Fig. 269.

The short-circuit test.—The rotor winding is short circuited, and either clamped down, or preferably loaded with a brake.

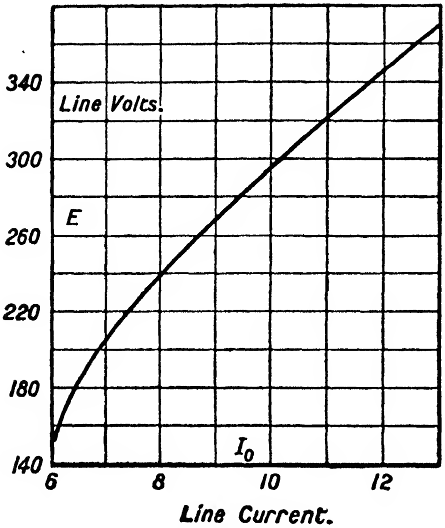

FIG. 268.—The excitation curve of the induction motor.

The stator is fed at different voltages very much lower than rated value ; the highest voltage giving an input stator current about 50 per cent. of full-load value, or somewhat greater if the test is made quickly. For each voltage, the brake is adjusted so that the rotor will just crawl round, and the brake value observed. An ammeter, voltmeter, and wattmeter give the stator current, voltage, and input power.

By means of this test an approximate value at rated voltage of the stator short-circuit current and also its effective component may be determined. Thus, a second point on the circle diagram may be obtained and the semicircle drawn.

The input, as read on the wattmeter, is the sum of the copper losses in the windings plus a small iron loss. The latter is small, because the feeding voltage even for full-load currents is very much smaller than rated voltage. An allowance for the iron loss may be determined by the following test.

The iron-loss test : rotor winding open.—In this case the rotor will remain stationary. The stator is fed at different voltages, and the input current, voltage, and power are read. The input power is the iron loss in both stator and rotor cores plus a small stator copper loss, $3I^2r$, which may be deducted.

Copper loss.—From the two preceding tests the copper loss may be determined by subtracting from the input in the first test, the corresponding iron loss of the second.

The short-circuit stator current, its effective component, and the starting torque, at rated voltage.—These values are derived as follows. Theoretically, the short-circuit stator current is approximately proportional to the applied voltage, because the equivalent reactance per phase of the stator winding for stand-still condition is very nearly constant.

Therefore, to find I_{sh}, the *short-circuit current* at *rated voltage,*

E is plotted against I, and the nearest straight line is drawn through the points obtained in the short-circuit test and continued onward until rated voltage is reached. Or, I_{sh} may be calculated by proportion from the limited length of the line obtained from the results of the experiment.

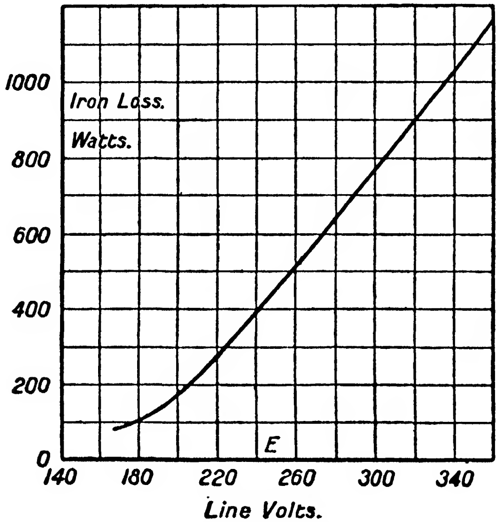

FIG. 269.—The iron-loss curve of the induction motor.

The *effective component* of the short-circuit stator current is nearly proportional to the short-circuit current. For the input P in the short-circuit test is $\sqrt{3}\,EI\cos\theta$, which is equal to $\sqrt{3}\,EI_e$, and P is the sum of the copper losses in the windings plus a very small iron loss, which in this consideration may be neglected.

Also, the machine being a static transformer in this test, the stator and rotor currents are nearly proportional, so that the total copper losses are nearly proportional to I^2, the square of the short-circuit stator current. Therefore

$$EI_e \propto I^2 \quad \text{and} \quad E \propto I\,; \quad \therefore\; I_e \propto I.$$

Thus, by plotting I_e, which is equal to $\frac{P}{\sqrt{3}\,E}$, against I, and drawing the nearest straight line through the points, an approximate value of I_e at rated voltage may be found.

The *starting torque* at rated voltage may be determined from a straight line drawn through the points obtained for $\frac{T}{E}$ and I, because these values are nearly proportional. This follows from the fact that

in the circle diagram the starting torque is proportional to $3I_2^2r_2$, the rotor copper loss for the rotor short-circuit current. Now, I_2 is nearly proportional to I, the short-circuit stator current, and I is proportional to E. Therefore

$$\frac{T}{E} \propto I.$$

The starting torque at rated voltage may thus be approximately determined, and the torque line of the circle diagram be drawn.

From the results of the no-load and short-circuit tests the complete circle diagram may therefore be constructed.

The results obtained for *machine F*, with the short-circuited rotor creeping round very slowly, were as follows:

E (line)	40	59	78	88	98	102	115
I (line) -	32	48	65·5	74·2	84	89	100
Input P	640	1470	2640	3600	4470	5100	6630
I_e -	9·4	14·4	19·5	23	26·3	29	33·3
T lbs. ft.	1·79	3·85	7·2	8·7	11·2	12	15·3
$\frac{T}{E}$ -	0·045	0·065	0·092	0·099	0·114	0·118	0·133

For *open* rotor circuits, the following results were obtained:

E (line) -	83	105	154	208	255
I (line) -	2·75	3·5	5·25	7·0	8·75
Input P -	75	120	240	383	540
$3I^2r_1$ - -	2	4	8	15	23
Iron loss -	73	116	232	368	517

The iron loss is plotted against E in Fig. 270.

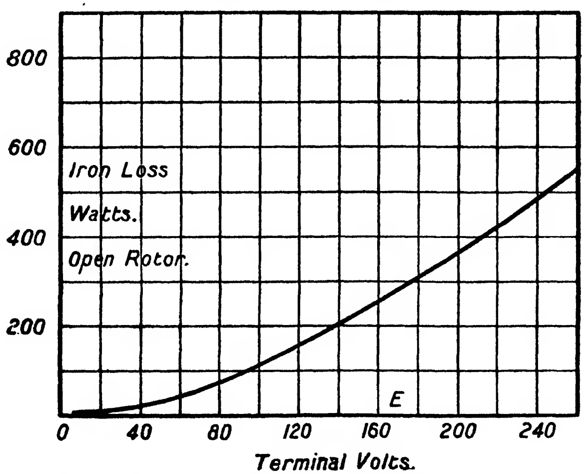

FIG. 270.—The iron-loss curve of the induction motor, for use in the short-circuit test.

The total *copper loss* for different input currents may now be determined thus:

I (line)	32	48	65·5	74·2	84	89	100
P	640	1470	2640	3600	4470	5100	6630
Iron loss	30	40	65	80	100	108	145
Copper loss	610	1430	2575	3520	4370	4992	6485

Fig. 271 gives the graph of copper loss and line current.

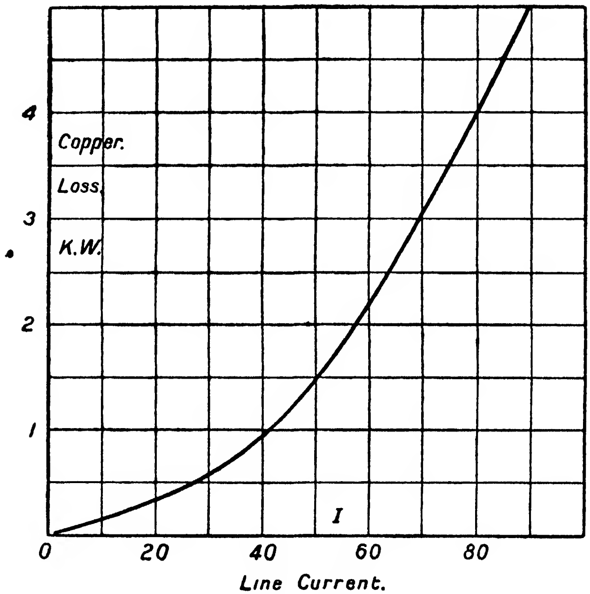

FIG. 271.—The curve of copper loss and line current, for the induction motor.

The value of I_{sh}, the *short-circuit stator current*, at rated voltage 346, may be obtained by proportion from the graph relating E and I in Fig. 272. Thus, by simple proportion, I_{sh} is found to be approximately equal to 285 amperes.

The value of I_e, the effective component of I_{sh}, is similarly obtained from the graph relating I_e and I in the same figure, I_e for I equal to 285 amperes being 90 amperes.

The value of the *starting torque* at rated voltage is found from the graph of $\frac{T}{E}$ and I in Fig. 272. Thus, $\frac{T}{E}$ is 0·38 for I equal to 285 amperes. Therefore

$$T = 346 \times 0{\cdot}38 = 132 \text{ lbs.-ft.}$$

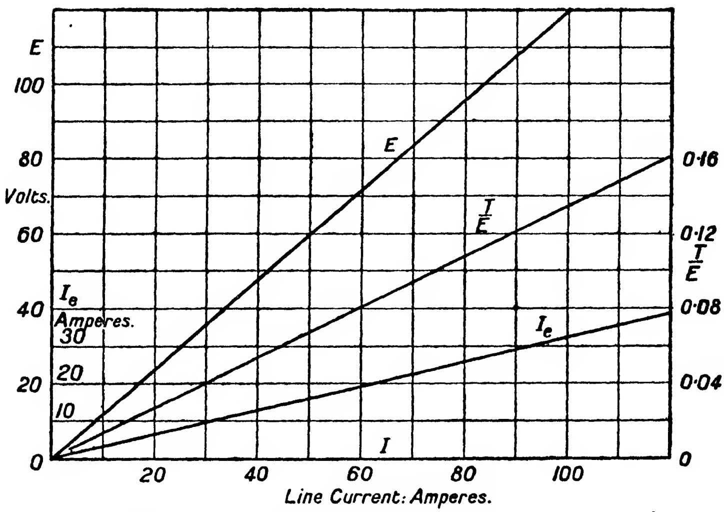

FIG. 272.—The graphs for obtaining the short-circuit current, and the starting torque of the induction motor.

In the circle diagram (Fig. 273), HZ may then be obtained by means of the relation on page 292, namely,

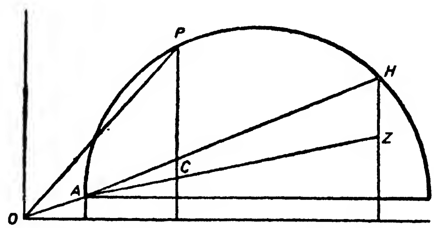

FIG. 273.—The circle diagram of an induction motor, in which HZ represents the starting torque.

$$T = \frac{7\sqrt{3}}{N_1} . E_l . PC.$$

$$\therefore \ HZ = \frac{N_1 T}{7\sqrt{3}\, E_l}.$$

For *machine F*, $N_1 = \frac{120 f}{p} = \frac{120 \times 51{\cdot}2}{6} = 1024$, and E_l is 346 volts. Therefore

$$HZ = \frac{1024 \times 132}{7\sqrt{3} \times 346} = 32{\cdot}5 \text{ amperes.}$$

The load test.—The motor is loaded and the input current, volts, and power are read. The speed is best measured by a stroboscope. This test was made on *machine F*, which was direct coupled to a direct-current generator, and different loads were put on the latter. The results were :

Terminal volts	356	353	353	345	345	342	340	340
Phase volts	205	204	204	199	199	197	196	196
Line current	15	23·7	34·5	46	56	65	76	85
K.W. per phase	1·8	4·04	6·4	8·4	10·2	11·8	13·6	15·2
Power factor	0·59	0·83	0·91	0·92	0·92	0·92	0·91	0·91
Input K.W.	5·4	12·1	19·2	25·2	30·6	35·4	40·8	45·6
R.P.M.	1021	1017	1011	1006	1002	997	992	988

The table of B.H.P. and efficiency.—The following table is worked out for *machine F* :

Stator current	15	23·7	34·5	46	56	65	76	85
Stator input	5·4	12·1	19·2	25·2	30·6	35·4	40·8	45·6
Iron loss	1·08	1·08	1·08	1 08	1·08	1·08	1·08	1·08
F and W loss	0·99	0·99	0·99	0 99	0·99	0·99	0·99	0·99
Copper loss	0·25	0·50	0·75	1·25	1·80	2·60	3·60	4·50
Total loss	2·32	2·57	2·82	3·32	3·87	4·67	5·67	6·57
Motor output	3·08	9·53	16·38	21·88	26·73	30·73	35·13	39·03
B.H.P.	4·2	12·8	22·0	29·3	36·0	41·3	47·0	52·3
Efficiency	58	79	85·5	87	88	87·2	86	86
Power factor	0·59	0·83	0·91	0·92	0·92	0·92	0·91	0·91
Torque lbs.-ft.	21·6	66	114	154	188	218	250	280

The graphs of efficiency, power factor, speed, and torque against B.H.P. are given in Fig. 274

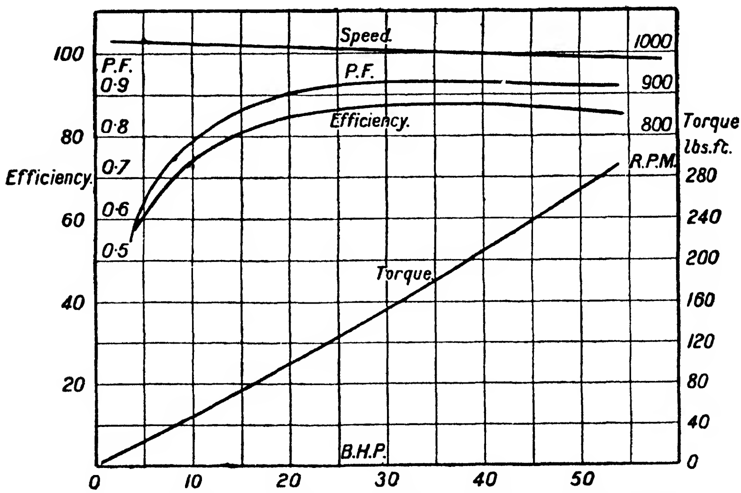

FIG. 274.—The characteristic curves of the induction motor for normal operation.

Determination of λ_1, λ_2, and σ.—The leakage coefficients of the stator and rotor windings, and the dispersion coefficient $\sigma = \lambda_1\lambda_2 - 1$, may be determined thus.

The rotor circuits are opened, and an accurate voltmeter is placed across two of the opened terminals; another voltmeter is placed across two of the terminals of the stator. These voltmeters are read when the stator is connected to mains of rated voltage. Let E_1 be the reading for the stator and E_2 for the rotor. Then

$$E_1 = Ak_1\phi_1 Z_1,$$

$$E_2 = Ak_2\frac{\phi_1}{\lambda_1}Z_2,$$

A being a constant proportional to the frequency, k_1 and k_2 the breadth coefficients of the respective windings, Z_1 and Z_2 the numbers of respective conductors, ϕ_1 the flux threading the stator winding, and λ_1 the coefficient of magnetic leakage of the stator.

Next, the stator terminals are left open, and a voltage applied to the rotor terminals of such value that it gives about rated voltage across the stator terminals. Let E_1' and E_2' be the respective voltages for stator and rotor. Then

$$E_2' = Ak_2\phi_2 Z_2,$$

$$E_1' = Ak_1\frac{\phi_2}{\lambda_2}Z_1.$$

Therefore
$$\lambda_1 = \frac{k_2}{k_1}\cdot\frac{Z_2}{Z_1}\cdot\frac{E_1}{E_2},$$

$$\lambda_2 = \frac{k_1}{k_2}\cdot\frac{Z_1}{Z_2}\cdot\frac{E_2'}{E_1'}.$$

By multiplication,
$$\lambda_1\lambda_2 = \frac{E_1}{E_1'}\cdot\frac{E_2'}{E_2},$$

and $\sigma = \lambda_1\lambda_2 - 1$ may be determined.

In order to find λ_1 and λ_2, the ratio $\frac{k_1 Z_1}{k_2 Z_2}$ must be known. Values of k_1 and k_2 are given in the table on page 272.

For trustworthy results it is necessary to use accurate and sensitive voltmeters, preferably electrostatic, which must be read carefully.

The values found for *machine F* were:

$$E_1 = 342, \qquad E_2 = 140,$$

$$E_1' = 353{\cdot}2, \quad E_2' = 151{\cdot}4,$$

as read by an electrostatic voltmeter. So that

$$\lambda_1\lambda_2 = \frac{342}{140}\times\frac{151{\cdot}4}{353{\cdot}2} = 1{\cdot}0472.$$

In this machine $\frac{Z_1}{Z_2}=2\cdot4$, and $k_1=0\cdot958$, $k_2=0\cdot96$. So that

$$\lambda_1=1\cdot0203, \quad \lambda_2=1\cdot0263,$$

$$\sigma=\lambda_1\lambda_2-1=0\cdot0472.$$

The circle diagram.—The circle diagram of the machine may now be constructed, and approximate values found for the over-load capacity and maximum or pull-out torque. Owing to the limited range of values obtained in the short-circuit test, the value of the short-circuit stator current and its effective component at *rated* voltage may only be regarded as roughly approximate.

In some cases the higher input values of the load test, in conjunction with the no-load data, will be more satisfactory to use for finding the diameter of the circle. The value of σ is also an important check on the results of the preceding methods.

The values of the diameter of the circle by these three methods will now be determined for *machine F*.

From the no-load test, OA, that is I_0, was found to be 12; and AM (Fig. 275) is given by

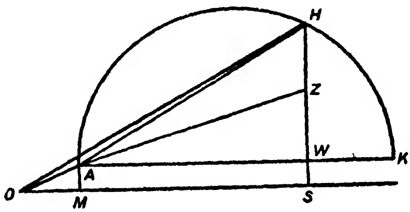

FIG. 275.—A circle diagram of an induction motor for determining the value of the diameter of the circle.

$$AM=\frac{2110}{\sqrt{3}\times346}=3\cdot52 \text{ amperes.}$$

From the short-circuit test, OH, that is I_{sh}, was found to be about 285, HZ 32·5, and HW 90 amperes.

The diameter of the circle may now be found. Thus,

$$OS=\sqrt{285^2-93\cdot5^2}=269.$$

So that

$$AW=257.$$

Also

$$WK=\frac{HW^2}{AW}=\frac{90^2}{257}=31\cdot5.$$

Therefore the diameter is equal to $257+31\cdot5$, that is, about 289 amperes.

From the load test it is similarly found to be 260, 240, 258, 240, and 253 for the respective input currents 46, 56, 65, 76, and 85 amperes.

From the coefficient of dispersion $\sigma=0\cdot0472$, the diameter is given at once from

$$\sigma=\frac{OM}{AK}=\frac{11\cdot9}{AK},$$

that is, the diameter equals 252 amperes.

This last value agrees very closely with the values obtained from the second method, while the value from the short-circuit test is somewhat higher.

The fairest policy will be to draw the circle diagram with a diameter of about 260 amperes, and deduce the over-load characteristics on this basis rather than use one of 289 amperes, which is probably a too optimistic value.

This has been done in Fig. 276. In the diagram, OQ is taken as 85 amperes, the highest input current of the load test. QS represents 52·3 B.H.P., and expressed in current has value

$$\frac{52{\cdot}3 \times 746}{\sqrt{3} \times 346} = 65{\cdot}2 \text{ amperes.}$$

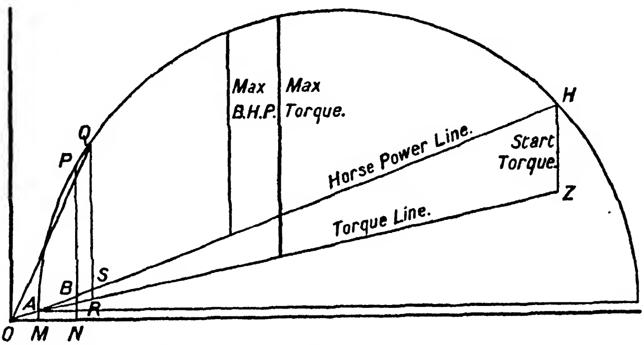

FIG. 276.—The circle diagram of the induction motor, with some of its important lines.

AS continued, gives the horse-power line.

QR is drawn equal to $\dfrac{1024 \times 280}{7\sqrt{3} \times 346}$,

280 being the torque in lbs.-ft. for the input current of 85 amperes and 1024 the synchronous speed. AR continued, gives the torque line.

PB is set up to represent *full* load brake horse-power of value 46, and OP, the full-load current, is then found to be 74 amperes. The full-load power factor is $\dfrac{PN}{OP} = 0{\cdot}91$.

The *over-load capacity* is the maximum horse-power divided by the full-load value, and is found to be 1·56. This maximum value is 72 B.H.P.

The *pull-out torque* is the maximum torque, and equals 1·75 times full-load torque. Its value is $240 \times 1{\cdot}75 = 420$ lbs.-ft.

The *starting torque* is 0·65 times full-load torque for short-circuited rotor; that is, its value is $240 \times 0{\cdot}65 = 156$ lbs.-ft.

Other over-load characteristics of the motor may be obtained from the circle diagram, such as the variation of speed, power factor, efficiency, and torque against B.H.P. The graphs of these have

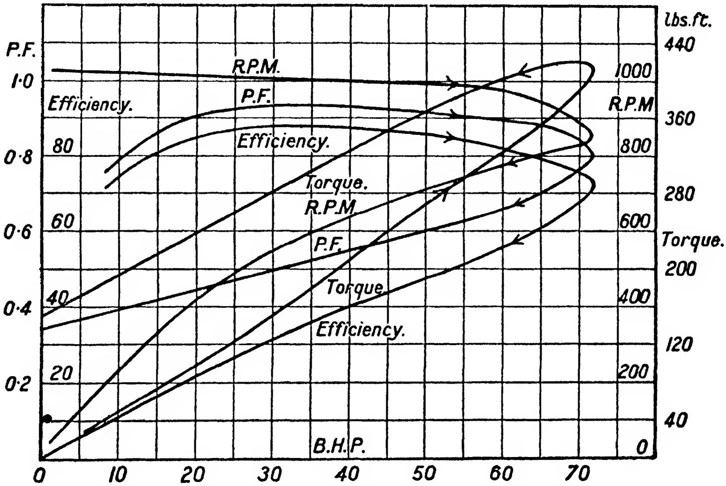

FIG. 277.—The complete characteristic curves of the induction motor.

already been drawn for the *load* test. Fig. 277 gives the complete curves derived from the circle diagram of Fig. 276 for over-load performance as well.

TESTING A TRANSFORMER.

In many respects a transformer resembles an induction motor in its operation and testing. It has a flux diagram similar to that of the motor. In the case of the transformer, the dispersive coefficient σ, given by

$$\sigma = \lambda_1 \lambda_2 - 1,$$

is much smaller on account of the absence of air gaps which promote leakage. Also, the load is electrical and not mechanical.

The no-load input power gives the iron loss of the transformer plus a very small no-load copper loss $I_0^2 r_1$ if single phase, and $3 I_0^2 r_1$ if three phase; I_1 being the no-load primary current and r_1 the phase resistance.

The iron loss for all loads up to full load is fairly constant for a given frequency. For this loss depends only upon ϕ_m, the maximum flux in the core, and ϕ_m is nearly constant for the following reasons. The back voltage in the primary coil is proportional to ϕ_m for a given frequency, and it balances the applied voltage on account of the primary drop being almost negligible in comparison with the latter.

Therefore, since the applied voltage is constant, the back voltage and consequently ϕ_m, are nearly constant.

This may be readily shown experimentally by wrapping a layer of wire of A turns around the core or one of the transformer coils, and placing a voltmeter across the ends of this new coil. The transformer is then loaded up and the readings E_A of the voltmeter observed. These readings are found to be nearly constant for all loads, and show a slight decrease as the load increases; this decrease being more pronounced in transformers of small output. Since

$$E_A = 4 \cdot 44 \phi_m A f 10^{-8},$$

it follows that ϕ_m, and consequently the iron loss, remains nearly constant. The no-load iron loss is therefore nearly representative for all loads.

The iron loss at full load, for instance, may be more accurately determined by noting the value of E_A in the preceding experiment for this load, and then feeding the transformer with a voltage which gives the same value of E_A at *no* load. From the no-load input the iron loss at rated voltage for full load may be obtained.

In the load test the input and output quantities are read, namely I_1, E_1, P_1 for the primary, and I_2, E_2, and P_2 for the secondary. The respective power factors are

$$\frac{P_1}{I_1E_1} \quad \text{and} \quad \frac{P_2}{I_2E_2}.$$

The efficiency is $\frac{P_2}{P_1}$, and $P_1 - P_2$ will give the sum of the iron and copper losses. The latter may be obtained by subtracting the iron loss from $P_1 - P_2$.

The difference between no-load and full-load values of E_2, divided by the latter when multiplied by a hundred, gives the *percentage regulation* of the transformer.

The copper losses may also be determined by the short-circuit test. In this test the secondary is short circuited and the primary is fed at much lower voltages than rated value. A wattmeter gives the input and an ammeter the primary current.

A set of values are taken for input currents ranging from small value up to full or about thirty per cent. over-load value. As the feeding voltage is small, the iron loss will also be small, so that the input is nearly equal to the sum of the copper losses.

This iron loss, if necessary, may be obtained by reading the input when the secondary is on open circuit, and the primary is fed by the voltages used in the short-circuit test.

A choking coil with an adjustable liquid resistance in series, or a number of such arranged in parallel, each controlled by a switch, may be used as an inductive load. Small transformers with their secondaries loaded with an adjustable liquid resistance are also useful as inductive circuits.

The following results were obtained for a 4 K.W. single-phase transformer, designed for a frequency of 50, and to step down from 346 to 80 volts :

Primary.				Secondary.			
E_1.	I_1.	W_1.	P.F.	E_2.	I_2.	W_2.	Efficiency.
348	2·5	175	0·2	82	0	0	0
348	3·15	810	0·74	82	7	572	71
348	4·98	1500	0·865	82	16	1310	87
346	6·5	2150	0·955	81	23·5	1890	88
345	8·25	2720	0·96	81	31	2500	92
344	9·98	3300	0·965	80	38	3040	92
344	11·6	3880	0·97	80	45·5	3640	94
341	12·4	4100	0·97	79	49	3870	94

This table is for the case of a non-inductive load. Fig. 278 gives the efficiency, power factor, and terminal voltage plotted against the output load. In this case $E_2 = E_t$, the terminal voltage.

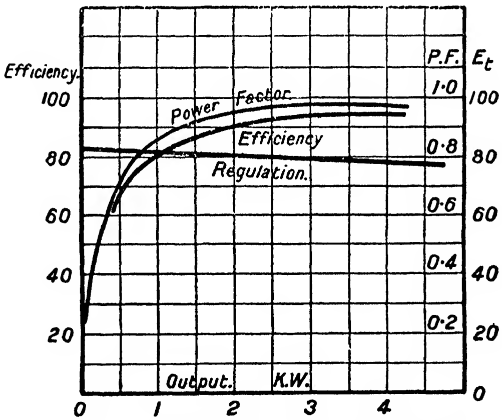

FIG. 278.—The characteristic curves of the transformer.

A three-phase transformer may be similarly tested. If the load is balanced, one wattmeter may be used for measuring the input power, and a second, the output power when the load is inductive. In the case of an unbalanced load, two wattmeters will be needed to measure the input power and two for the output power.

In the latter case the term power factor has no definite meaning, but in the case of a balanced load it is represented by

$$\frac{P}{\sqrt{3}\,E_l I_l},$$

provided the currents and voltages are sinusoidal, and that P is the input or output power, E_l the line voltage, and I_l the line current.

THE REPULSION MOTOR.

This machine is like the direct-current series motor in quickly running up to a dangerous speed when unloaded, and also in its capacity of exerting a considerable starting torque which enables it to be started on load.

In testing the repulsion motor it is necessary to load it up with an accurate form of brake, such as a magnetic brake. The input quantities are read by ammeter, voltmeter, and wattmeter. The efficiency, power factor, horse-power, speed, and torque may then be determined.

Both speed and torque may be changed by adjusting the position of the brushes. This adjustment is provided for, and may easily be made.

A test was made on a 6 K.W. repulsion motor fed from a single-phase transformer at frequency 51·2. It was loaded with a magnetic brake. Four sets of values were taken at different speeds, each set for the same speed. The speed was adjusted by changing the position of the brushes.

Speed 800 *R.P.M.*

Voltage - -	98·5	96·5	92	89·3	88
Current - -	41·2	49	61	67	71
Input K.W. -	2·25	2·9	3·8	4·5	4 82
Torque lbs.-ft.	10·2	15	22·5	26	28·3
B.H.P. - -	1·55	2·28	3·42	3·95	4·3
Power factor -	0·55	0·61	0·68	0·75	0·77
Efficiency -	51·5	58·5	67·0	65·5	67·0

Speed 1000 *R.P.M.*

Voltage - -	98	96·5	94	91	89	86·5
Current - -	36·5	42·5	51·2	62	68·5	78
Input K.W. -	2·1	2·72	3·6	4·65	5·1	6·0
Torque lbs.-ft. -	7·5	11	16·8	22·5	25·5	29·5
B.H.P. - -	1·42	2·1	3·2	4·28	4·85	5·6
Power factor -	0·59	0·66	0·75	0·82	0·84	0·89
Efficiency -	50·5	57·6	66·4	68·7	71·0	69·6

Speed 1200 *R.P.M.*

Voltage - -	96	95	93·7	91	90
Current - -	32	35·5	41·4	50·6	66·5
Input K.W. -	1·9	2·25	2·91	3·9	5·26
Torque lbs.-ft.	5·4	7·5	10·7	15	22·5
B.H.P. - -	1·24	1·71	2·45	3·43	5·13
Power factor -	0·62	0·67	0·75	0·85	0·88
Efficiency -	48·6	57·0	62·8	65·7	72·7

Speed 1400 *R.P.M.*

Voltage - -	96·2	94	91	84
Current - -	36·3	44	54·5	73·6
Input K.W. -	2·62	3·5	4·42	5·74
Torque lbs.-ft.	7·5	11·2	14·8	19·5
B.H.P. - -	2·0	3·0	3·95	5·2
Power factor -	0·75	0·85	0·90	0·93
Efficiency -	57	64	67	68

A set of values were also taken for *fixed* brush position, and are as follows :

R.P.M. - -	976	1076	1355	1600	2000
Voltage - -	87·3	89	9·25	95·5	96·3
Current - -	77	71	56	46	37
Input K.W. -	5·75	5·5	4·58	3 85	3·1
Torque lbs.-ft.	29·6	25·7	15·8	9·8	4.0
B.H.P. - -	5·5	5·27	4·07	3·0	1·52
Power factor -	0·85	0·87	0·88	0·88	0·87
Efficiency -	71	71·5	66·5	58	36·5

The graph of efficiency against B.H.P. for speed 1400 is shown in Fig. 279. The corresponding graphs for the other three speeds 1200, 1000, and 800 lie very close to the one given, and are not drawn.

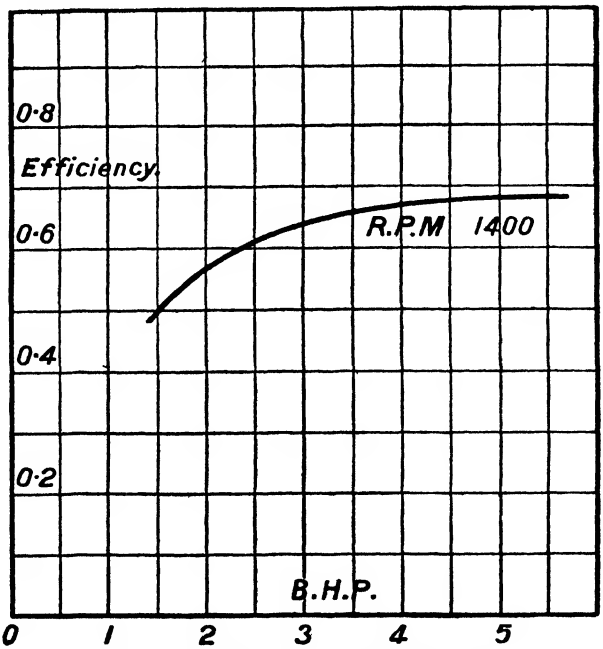

FIG. 279.—Efficiency curve of the repulsion motor.

The graphs of power factor against B.H.P. for these four speeds are given in Fig. 280, and it is seen that the higher the speed the

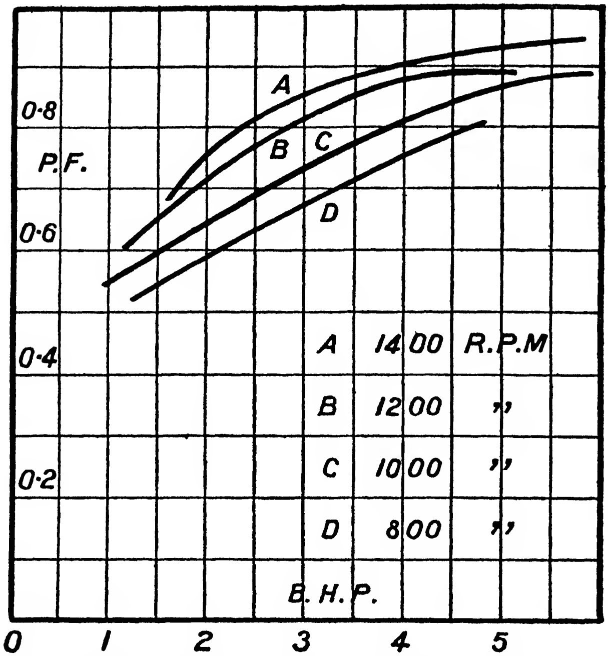

FIG. 280.—Variation of power factor with that of speed and load, in the case of the repulsion motor.

greater is the power factor. This also applies to the speeds 1600 and 2000 of the last table, their corresponding points lying above the curves shown.

Fig. 281 gives a set of derived curves obtained from Fig. 280, and shows the increase of power factor with load and also with speed.

For the test with brushes in a *fixed* position, the graphs of efficiency, power factor, and speed against B.H.P. are given in Fig. 282, and that of torque and speed in Fig. 283.

From these curves it will be noted that for *fixed* brush position the power factor is nearly independent of the load, and thus its operation at various loads would not appreciably disturb the power factor of the supply mains to which it is connected.

Its efficiency, however, drops quickly for loads below 4 B.H.P., and this position of the brushes is therefore not the best for obtaining the maximum efficiency at these lower loads.

Maximum efficiency for a given load may be obtained by adjusting the brushes, that is, altering the driving torque of the motor.

Fig. 283 indicates the very large starting torque which this particular position of the brushes would give, and the rise in speed as the torque diminishes.

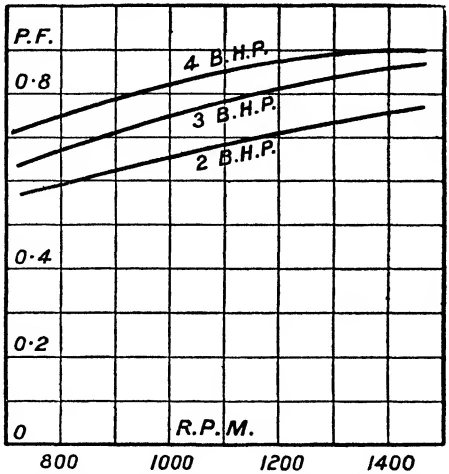

FIG. 281.—Curves showing the increase of power factor with increase of load, and also with increase of speed, in the case of the repulsion motor.

In Fig. 284 are shown the graphs relating efficiency and torque for different speeds. The graph for 1200 R.P.M. practically coincides

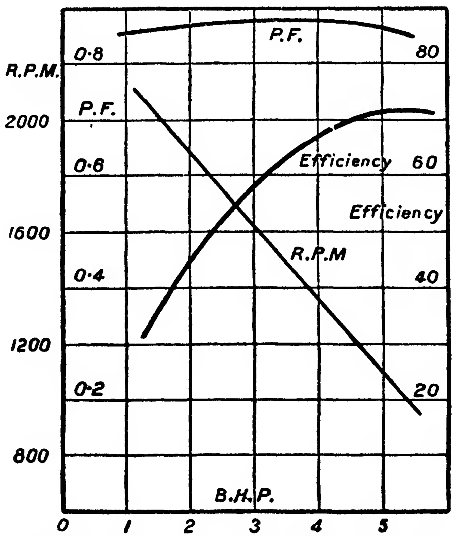

FIG. 282.—The characteristic curves for the repulsion motor; brushes fixed.

with that for 1400 R.P.M. For still higher speeds the curves would be expected to gradually descend. This is confirmed by the torque and efficiency for 1600 R.P.M. given in the last table, the corresponding point of which, falling below the curve for speed 1400.

The efficiency therefore increases with the torque, and for a given torque with the speed up to a certain limiting value, about 1300 in this case, and then decreases for higher speeds.

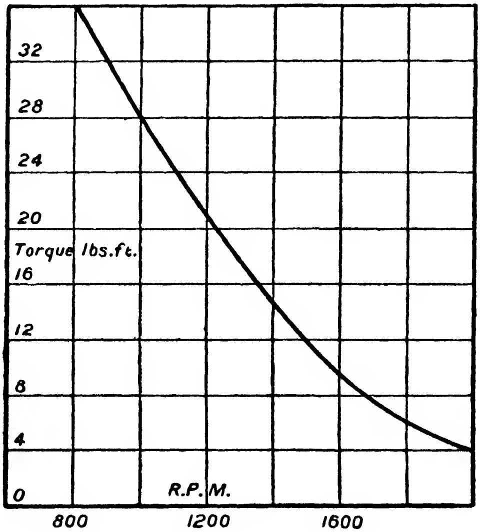

FIG. 283.—The torque-speed curve of the repulsion motor; brushes fixed.

It will be noted that in the last test for a given position of the brushes, a load of 2 B.H.P. may be obtained at a speed of 1870. The efficiency is then 45 per cent. and power factor 0·87. For another position of the brushes the same load is operated at a speed of 1400, and the efficiency is 57 per cent. and power factor 0·75. A third position for the same load gives a speed of 800, and the efficiency is nearly 57 per cent. at a power factor 0·59.

Of these three positions the one corresponding to speed 1400 is economically the best, for though the last position gives about the same efficiency, the machine is not so well cooled, and is subject to more internal heating.

For since, in these two cases, the efficiencies are about the same, the sum of the copper, iron, windage, and friction losses will also be about equal.

For speed 1400, the axle and brush friction will be greater than in the case of the speed 800, and the windage loss will be much greater for the higher speed, especially as the machine is fitted with fan blades for forced ventilation.

Therefore the sum of the iron and copper losses will be considerably greater for the case of lower speed. Also, the input current was larger in the latter case than for that of the higher speed. Thus the internal heating will be greater for the case of speed 800, and the cooling facilities will be much less than when the speed is 1400.

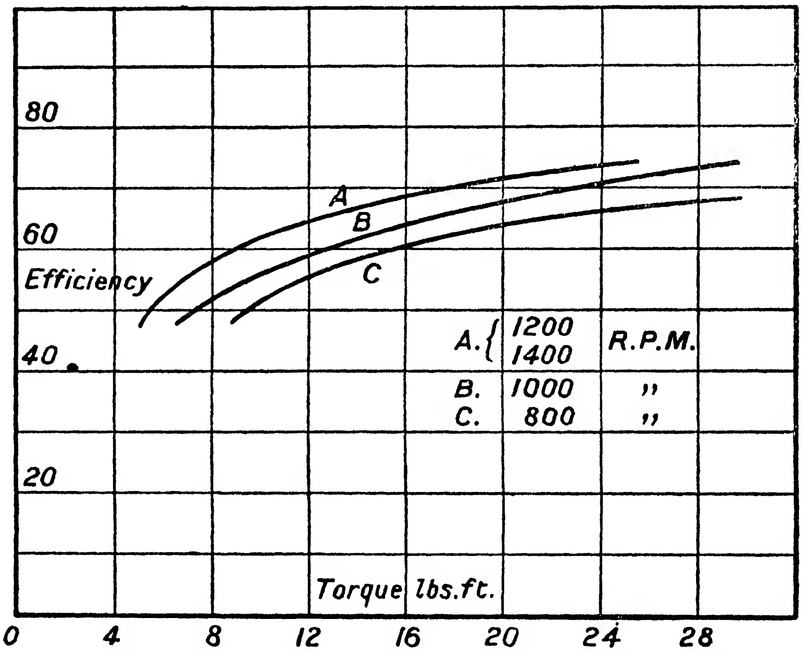

FIG. 284.—Efficiency and torque of a repulsion motor when operating at different speeds.

It follows, then, that for a continuous run for some time the third position of the brush is not so good as the second.

A given load may therefore be operated at different speeds, or at a particular speed to give maximum efficiency, or at a particular speed which gives maximum power factor.

THE OSCILLOGRAPH.

This is one of the most important instruments required in the testing room, and has an extensive use in the testing of alternating-current machines. Some of the tests which may be made with this instrument are as follows. A study of

1. The rush of current when switching on metallic filament lamps.
2. The voltage and current waves of an alternating-current circuit, taken singly or together in true relation to one another. Thus the

presence of harmonics may be detected, and the wave form indicating them, analysed.

3. The phase and line voltages of a star-connected alternator. From theory, if a third, ninth, fifteenth, twenty-first, etc., harmonic appears in the phase voltage, it should disappear in the line voltage. Also, the same should happen for the phase and line *currents* of a mesh-connected alternator.

4. The harmonics produced in the voltage curve of an alternator due to wide slots. When a machine has x armature slots per pole, the harmonics $2x+1$ and $2x-1$ are produced in the voltage and current waves if the slots are wide and open, that is, if the magnetic pulsations due to the slots and teeth are sufficiently large. Thus, if x is 9, the seventeenth and nineteenth harmonics would be present. These would have a frequency of 850 and 950, if that of the fundamental was 50.

5. The current wave of an alternator with harmonics in its voltage curve when connected first to a non-inductive, then to an inductive, and finally to a capacity load. Self inductance smooths out the harmonics in the current wave, while capacity increases their amplitudes in the current wave.

6. The current wave of the primary of a transformer on different loads ranging from no load to full load. At light loads the wave may indicate the presence of a third harmonic, and perhaps others due to the hysteresis of the core. As the load increases, the indications of their presence become fainter.

7. The simultaneous phase voltage and current waves of a synchronous motor or rotary converter during the change of exciting current. As the exciting current is raised from a low to a high value, the current wave will gradually move from a lagging displacement, with respect to the voltage wave, to a leading displacement, passing through the position of phase coincidence, a position indicating unity power factor.

8. Transitory actions, such as the discharge of a condenser and the behaviour of the current at the blowing of fuses.

9. The form and frequency of the current in a circuit in which an electrolytic interrupter is used.

A single glance at an oscillogram may sometimes show a defect in a machine, which otherwise would take a long time to discover.

One of the best known types of this instrument is *Duddell's* oscillograph. It is constructed on a principle first suggested by *M. A. Blondel.* Two parallel conductors formed by bending a thin strip of phosphor bronze back on itself over an ivory pulley are placed between the poles of a strong magnet. These conductors are vertical, and their plane is normally parallel to the direction of the magnetic field, that is, as the plane of a galvanometer coil. A small mirror is fixed across these narrowly separated conductors.

When a current passes through them, one is displaced forward, the other backward, and the mirror is deflected about a vertical axis.

This element is immersed in oil to damp the movement and make the instrument dead-beat. One element is used for current and a second for voltage waves.

This instrument is made in several forms. One form is made to especially deal with high frequency currents at moderate voltages, such as those in telephone circuits. A second form is constructed for high-voltage circuits, and is insulated for 50,000 volts or more. The third form is for general use in laboratory and testing room.

Another type of instrument used is *Irwin's* hot-wire oscillograph. The principle of this type is illustrated in Fig. 285. AB and CD are two phosphor bronze strips, RR two equal non-inductive resistances very large compared with the resistance of the strips.

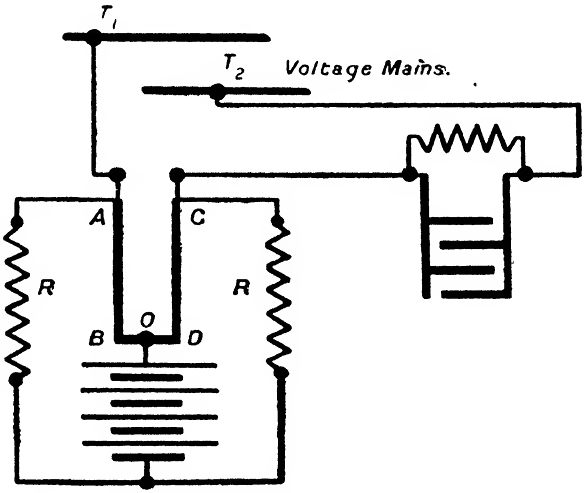

FIG. 285.—The volt-element of Irwin's hot-wire oscillograph.

A steady direct current passes into the junction of the strips, and passes from O to A and O to C through RR back to the battery. Thus, if an alternating current is passed from A to C through the strips, AB at one instant will carry a larger current than CD. AB will therefore be hotter, and its extension will be longer than in the case of CD. This difference of extension causes a mirror attached to the strips to be deflected.

If i is the instantaneous value of the alternating current and I the steady direct current, $(i+\text{I})$ is the current carried by one strip and $(i-\text{I})$ by the other. Let r be the resistance of each strip. Then the heat energy developed in the strips for a very small interval dt will be respectively

$$(i+\text{I})^2 r\,dt \quad \text{and} \quad (i-\text{I})^2 r\,dt.$$

Now, this heat energy does two things, it raises the temperature of the strip by a small amount, and expends the remainder of itself in the form of radiation from the strip.

Let θ_1 be the excess temperature of one strip above the surrounding medium, which is oil, and θ_2 that of the other. Then, if M is the product of the mass of each strip multiplied by its specific heat, and k the radiation coefficient,

$$(i+\mathrm{I})^2 r\,dt = \mathrm{M}\,d\theta_1 + k\theta_1\,dt,$$

$$(i-\mathrm{I})^2 r\,dt = \mathrm{M}\,d\theta_2 + k\theta_2\,dt.$$

By subtraction

$$4i\mathrm{I}r = \mathrm{M}\cdot\frac{d(\theta_1-\theta_2)}{dt} + k(\theta_1-\theta_2).$$

Let
$$\theta_1 - \theta_2 = \theta,$$

so that
$$\mathrm{M}\frac{d\theta}{dt} + k\theta = 4i\mathrm{I}r;$$

$$\therefore\ \frac{d\theta}{dt} + \frac{k}{\mathrm{M}}\theta = \frac{4\mathrm{I}r}{\mathrm{M}}\cdot i.$$

If i obeys a simple sine law, such as $i = \mathrm{I}_m \sin \omega t$, and K is taken as $\frac{4\mathrm{I}r}{\mathrm{M}}\cdot\mathrm{I}_m$,

$$\frac{d\theta}{dt} + \frac{k}{\mathrm{M}}\theta = \mathrm{K}\sin\omega t.$$

Now, this equation is of exactly the same form as that for a circuit, containing self inductance and resistance, fed by a simple sinusoidal voltage. Therefore the lag of θ behind the alternating current through the strips, according to page 53, is

$$\tan^{-1}\frac{\mathrm{M}\omega}{k} = \tan^{-1}\frac{2\pi f\mathrm{M}}{k},$$

that is, the difference of temperature between the strips, and therefore the motion of the mirror, lags behind the alternating current flowing through the strips.

In order that this motion may not lag behind the voltage supplying the current, a shunted condenser is placed in series with the strips as shown. This causes the current to *lead* the voltage across T_1 and T_2 by an amount equal to the lag of the motion of the mirror behind the current. Thus the motion of the mirror is in phase with the voltage wave under test.

This compensation also holds good for harmonics in the voltage wave, because the value of the shunted condenser required is independent of the frequency. Each component harmonic may be regarded as acting alone and contributing its share in producing the difference of temperature θ.

In this oscillograph the arrangement of the strips for the current wave is different, and so is the method of getting the movement of

the mirror in phase with it. The arrangement is shown in Fig. 286. P is the primary of a small current transformer, which carries the current whose wave is required. This primary is in series with R, a non-inductive resistance. T_1 and T_2 are the current terminals of the oscillograph. The total resistance of P and R is very small.

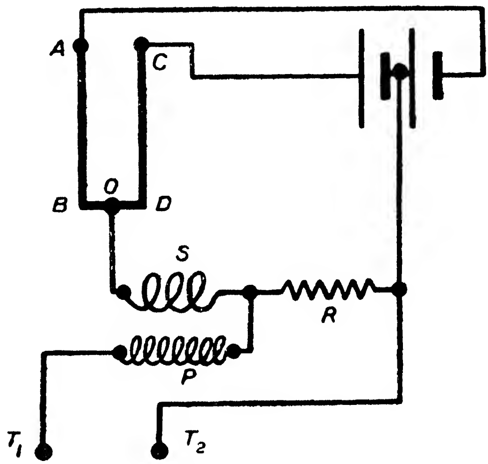

FIG. 286.—The current-element of Irwin's hot-wire oscillograph.

S, the secondary coil of the transformer, has only a few turns, and produces very little reaction on the flux produced by P. Thus, the flux in the core is produced by P, and it is in phase with the current through P, that is, the test current.

The voltage produced in S is therefore either 90 degrees behind the flux in the core, that is, behind the test current, or 90 degrees ahead, according to the direction of the spirality of S or how its ends are connected.

Thus the voltage of S may be made to lead the voltage of R, which is in phase with the test current by 90 degrees. Each of these voltages are proportional to the test current.

The current i through the strips, may therefore be arranged to lead the test current, by an amount equal to the lag of the movement of the mirror behind itself.

The same equations for heating given for the volt-element also apply to the current-element.

Full constructional details and methods of handling these delicate instruments are supplied by the makers.

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